

**Tentamen i LÅGTEMPERATURFYSIK för F, Kf, GU och forskarstuderanden**

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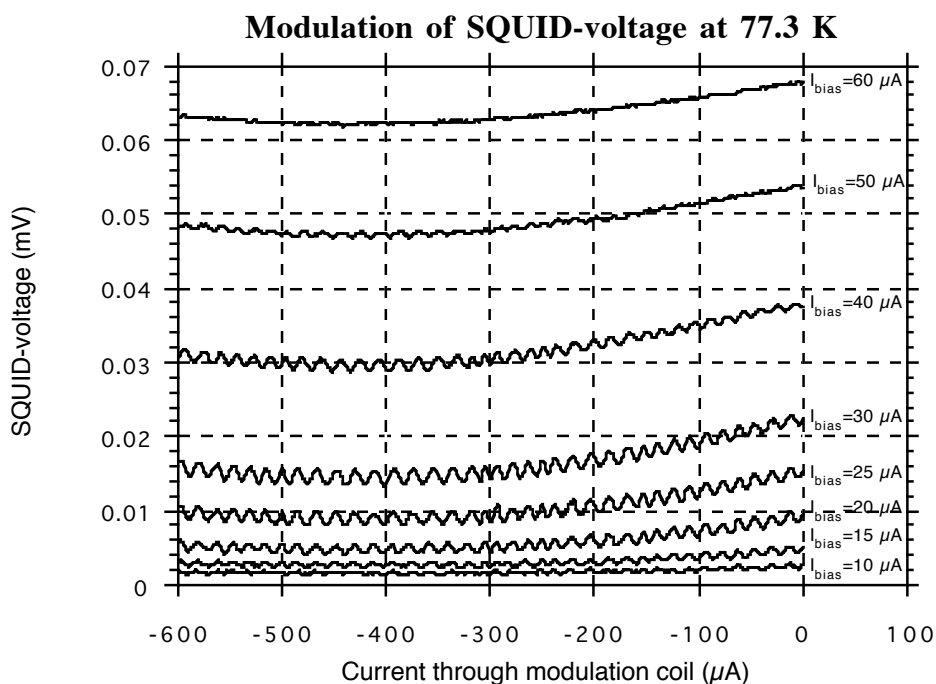
**Hjälpmedel:** Tefyma, Physics Handbook, Stand Math Tables och liknande handböcker, valfri räknedosa.

Svaren på fem av de följande problemen räknas som tentamen (fler får besvaras). Ge logiska beskrivningar, illustrera gärna med diagram eller figurer, skriv läsligt. Svara på svenska eller engelska, frågorna ges på engelska så att de kan användas i undervisningen kommande år.

1. You have measured the velocity of second sound in superfluid helium
  - (a) What is meant by sound of second order in superfluid helium? What kind of wave motion is it and why is it obtained? Suggest an experiment to measure the velocity of 2nd sound. What result do you expect as the temperature approaches  $T_\lambda$ ? Well below  $T_\lambda$ ? What happens at very low temperature? Can you explain the very high thermal conductivity of superfluid helium in a region below  $T_\lambda$  with the aid of second sound? What happens to the heat conductivity as the temperature becomes much smaller?(3.5p)
  - (b) What is meant by 3rd and 4th order sound in helium? Sound of 0th order in  $^3\text{He}$ ?(0.5p)
  
2. We have come across the concept of Bose-Einstein condensation (BEC) several times within the course.
  - (a) What characterizes a boson and what is a Bose-Einstein condensation? When and under what conditions does the latter occur? The superfluid state in  $^4\text{He}$  is considered to be a Bose-Einstein condensate. What are the properties of the atoms of the condensate? How does it differ from a condensate of an ideal gas of atoms? What is the temperature dependence of the density of the condensate? (2p)
  - (b)  $^3\text{He}$  and superconductors can also be considered as some kind of Bose-Einstein condensates despite the fact that those atoms and electrons are fermions. Why? What is characteristic of the constituents that form the characteristic wave functions of those condensates? (1p)
  - (c) It has lately been possible to observe the condensation of H, Rb, Na, and Li atom gases. How have these atoms been cooled to the low temperatures of Bose-Einstein condensation? Describe the principles of the cooling methods used. (1p)
  
3. The penetration depth  $\lambda$  and the coherence length  $\xi$  are two important lengths in superconductors that follow from the Ginzburg-Landau equations, which can be written:
 
$$\alpha\Psi + \beta|\Psi|^2\Psi + (1/2m^*)(-i\hbar\nabla + e^*A)^2\Psi = 0$$

$$J_S(r) = i(e^*\hbar/2m^*)(\Psi\nabla\Psi^* - \Psi^*\nabla\Psi) - (e^*2/m^*)\Psi^*\Psi A$$
  - (a) Define and describe the two lengths. (1p)

- (b) What are typical values of these  
 (i) in a pure type I superconductor (like Al)?  
 (ii) in a typical high  $T_c$  (cuprate) superconductor (type II superconductor)?  
 What does this imply for the extension of the wavefunction describing a Cooper pair in the two types of superconductors (i and ii)? (2p)
- (c) What happens to the two lengths  $\lambda$  and  $\xi$  as you decrease the mean free path of the normal electrons by alloying a pure superconductor? What are the temperature dependencies of  $\lambda$  and  $\xi$  close to  $T_c$ ? (1p)
4. Superconducting Quantum Interference Device (SQUID). You have investigated such a device in one of the laboratory exercises.
- a) Discuss the principle of a dc-SQUID, show a typical I-V curve, define a bias and sketch the resulting flux response. How does one couple the field to be measured to the SQUID? How is it possible to arrange the pick-up to measure a field gradient? (2p)
- b) What is the difference between a dc-SQUID and an ac-SQUID? Describe shortly how the latter works. (1p)
- c) A current is fed through a coil which is inductively coupled to a SQUID and the voltage over the SQUID is measured. The result is shown below for different bias currents  $I_{\text{bias}}$  (10-60  $\mu\text{A}$ ). Calculate the mutual inductance between the coil and the SQUID. (1p)



5. There are similarities between the current transport in Josephson junctions and in ultrasmall tunnel junctions dominated by charging phenomena (Coulomb blockade). The charge  $Q$  and the phase  $\Phi$  in a small superconducting tunnel junction (or number and phase) are conjugate variables.
- (a) Consider an SIS (superconductor/insulator/superconductor) tunnel junction. Discuss and compare the Josephson zero voltage current and the Coulomb blockade (threshold voltage)

in the two regimes (Josephson and charging, resp.) of a small junction. What conditions have to be fulfilled in order to observe the two phenomena, in particular the charging phenomenon? (2p)

- (b) In a laboratory exercise, you have studied a recently proposed primary thermometer based upon the temperature dependence of the zero bias conductance of a small tunnel junction. Discuss in principle how the thermometer functions. (1p)
- (c) What is the difference between a primary and a secondary thermometer? (1p)

6.  $^3\text{He}$  and high  $T_C$  superconductors.

Both  $^3\text{He}$  and high  $T_C$  superconductors can be fundamentally different from the "low  $T_C$  superconductors" described by the BCS theory. This may be valid for both the symmetry of the wave function, describing the superconducting order, and the interaction leading to pairing.

Describe (shortly) the main properties of the two phenomena ( $^3\text{He}$  and high  $T_C$  superconductors) with an emphasis on those properties that indicate that a simple singlet BCS pairing is not sufficient to describe the states. Describe how the attractive interactions may differ from those of "ordinary" low- $T_C$  superconductors. (4p)

7. Assume that you want to deposit a thin film upon a glass that is cooled by a heat reservoir at 1 K. The glass substrate is 2 mm thick, has an area of  $1\text{ cm}^2$  and "sees" a black body at room temperature through a hole that also can be considered to be  $1\text{ cm}^2$ . What temperature do you expect at the surface of the glass if you assume that its back side is at 1 K? The average thermal conductivity of the glass can be considered as  $\lambda=0.0002\text{ W/cm}^2\text{K}$  within a temperature range of 1 to 2 K,  $\lambda=0.0006\text{ W/cm}^2\text{K}$  within a temperature range of 1 to 4 K,  $\lambda=0.001\text{ W/cm}^2\text{K}$  within a temperature range of 1 to 20 K. The emissivity of the glass is 0.9, Stefan's constant  $\sigma = 5.67 \times 10^{-12}\text{ W/cm}^2\text{K}^4$ . The accommodation coefficient of He on glass is about 0.67 at 10 K, 1 below 4 K. The pressure is assumed to be  $<10^{-10}$  torr. Suggest some measures to decrease the temperature at the substrate surface. Plus question: How would you check the temperature at the surface? (4p)

8. Order of magnitude values of parameters. It is always of value to know rough estimates of different parameter values. Give approximate values of the following parameters:

- Boiling temperatures at 1 atm. of  $^4\text{He}$ ,  $\text{H}_2$  and  $\text{N}_2$
- Lambda temperature of  $^4\text{He}$ .
- Poly critical point of  $^3\text{He}$  (equilibrium between superfluids A and B and Fermi liquid)
- Fermi temperature of liquid  $^3\text{He}$
- Ratio  $2\Delta(0)/k_B T_C$  according to the BCS theory
- $\kappa$ -value which separates Type I and Type II superconductors
- Superconducting transition temperature of  $\text{YBa}_2\text{Cu}_3\text{O}_{7-\delta}$
- Energy (in eV) corresponding to a temperature of 1 K
- Quantized magnetic flux,  $\Phi_0$
- Josephson frequency corresponding to a bias voltage of  $20\text{ }\mu\text{V}$
- Temperature range in which the platinum resistance thermometer is used within the international temperature scale (Sum 4p)