P9.56 6.28×10^7 kg

P9.58 1.47 km

P9.60 (a) $\frac{2d}{c+v}$; (b) $\frac{2d}{c}\sqrt{\frac{c-v}{c+v}}$

P9.62 See the solution for the graph.
$K_c = 0.990K_r$ when $u = 0.115c$,
$K_c = 0.950K_r$ when $u = 0.257c$,
$K_c = 0.500K_r$ when $u = 0.786c$.

Chapter 10
Rotational Motion

CHAPTER OUTLINE

ANSWERS TO QUESTIONS

Q10.1 1 rev/min, or $\frac{\pi}{30}$ rad/s. Into the wall (clockwise rotation). $\alpha = 0$.

FIG. Q10.1

Q10.2 The speedometer will be inaccurate. The speedometer measures the number of revolutions per second of the tires. A larger tire will travel more distance in one full revolution as $2\pi r$.

Q10.3 The object will start to rotate if the two forces act along different lines. Then the torques of the forces will not be equal in magnitude and opposite in direction.

Q10.4 A quick flip will set the hard-boiled egg spinning faster and more smoothly. The raw egg loses mechanical energy to internal fluid friction.

Q10.5 No, only if its angular momentum changes.

Q10.6 $I_{CM} = MR^2$, $I_{CM} = MR^2$, $I = \frac{1}{3}MR^2$, $I_{CM} = \frac{1}{2}MR^2$

Q10.7 Since the source reel stops almost instantly when the tape stops playing, the friction on the source reel axle must be fairly large. Since the source reel appears to us to rotate at almost constant angular velocity, the angular acceleration must be very small. Therefore, the torque on the source reel due to the tension in the tape must almost exactly balance the frictional torque. In turn, the frictional torque is nearly constant because kinetic friction forces don't depend on velocity, and the radius of the axle where the friction is applied is constant. Thus we conclude that the torque exerted by the tape on the source reel is essentially constant in time as the tape plays.

continued on next page

As the source reel radius R shrinks, the reel's angular speed $\omega = \frac{v}{R}$ must increase to keep the tape speed v constant. But the biggest change is to the reel's moment of inertia. We model the reel as a roll of tape, ignoring any spool or platter carrying the tape. If we think of the roll of tape as a uniform disk, then its moment of inertia is $I = \frac{1}{2}MR^2$. But the roll's mass is proportional to its base area πR^2. Thus, on the whole the moment of inertia is proportional to R^4. The moment of inertia decreases very rapidly as the reel shrinks!

The tension in the tape coming into the read-and-write heads is normally dominated by balancing frictional torque on the source reel, according to $TR \approx \tau_{\text{friction}}$. Therefore, as the tape plays the tension is largest when the reel is smallest. However, in the case of a sudden jerk on the tape, the rotational dynamics of the source reel becomes important. If the source reel is full, then the moment of inertia, proportional to R^4, will be so large that higher tension in the tape will be required to give the source reel its angular acceleration. If the reel is nearly empty, then the same tape acceleration will require a smaller tension. Thus, the tape will be more likely to break when the source reel is nearly full. One sees the same effect in the case of paper towels; it is easier to snap a towel free when the roll is new than when it is nearly empty.

Q10.8 (a) Down–cross–left is away from you: $-\hat{j}\times(-\hat{i}) = -\hat{k}$

(b) Left–cross–down is toward you: $-\hat{i}\times(-\hat{j}) = \hat{k}$

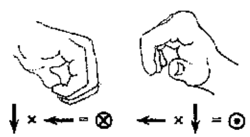

FIG. Q10.8

Q10.9 There must be two rotors to balance the torques on the body of the helicopter. If it had only one rotor, the engine would cause the body of the helicopter to swing around rapidly with angular momentum opposite to the rotor.

Q10.10 The diver leaves the platform with some angular momentum about a horizontal axis through her center of mass. When she draws up her legs, her moment of inertia decreases and her angular speed increases for conservation of angular momentum. Straightening out again slows her rotation.

Q10.11 The long pole has a large moment of inertia about an axis along the rope. An unbalanced torque will then produce only a small angular acceleration of the performer-pole system, to extend the time available for getting back in balance. To keep the center of mass above the rope, the performer can shift the pole left or right, instead of having to bend his body around. The pole sags down at the ends to lower the system center of gravity.

Q10.12 Suppose we look at the motorcycle moving to the right. Its drive wheel is turning clockwise. The wheel speeds up when it leaves the ground. No outside torque about its center of mass acts on the airborne cycle, so its angular momentum is conserved. As the drive wheel's clockwise angular momentum increases, the frame of the cycle acquires counterclockwise angular momentum. The cycle's front end moves up and its back end moves down.

Q10.13 Mass moves away from axis of rotation, so moment of inertia increases, angular speed decreases, and period increases.

Q10.14 Try it first and then explain it! If there is no air resistance and no rolling resistance, Example 10.15 shows that the races will be ties. The race results can reveal a small amount of air resistance.

Q10.15 To win the race, you want to decrease the moment of inertia of the wheels as much as possible. Small, light, solid disk-like wheels would be best!

Q10.16 When you bend over, your center of gravity shifts forward. Once your CG is no longer over your feet, gravity contributes to a nonzero net torque on your body and you begin to rotate.

Q10.17 The free body diagram demonstrates that it is necessary to have friction on the ground to counterbalance the normal force of the wall and to keep the base of the ladder from sliding. Interestingly enough, if there is friction on the floor *and* on the wall, it is not possible to determine whether the ladder will slip from the equilibrium conditions alone.

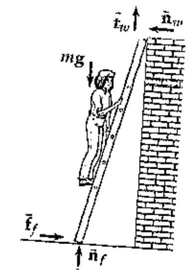

FIG. Q10.17

Q10.18 (a) Consider pushing up with one hand on one side of a steering wheel and pulling down equally hard with the other hand on the other side. A pair of equal-magnitude oppositely-directed forces applied at different points is called a couple.

(b) An object in free fall has a non-zero net force acting on it, but a net torque of zero about its center of mass.

SOLUTIONS TO PROBLEMS

Section 10.1 Angular Position, Speed, and Acceleration

P10.1 (a) $\theta|_{t=0} = \boxed{5.00 \text{ rad}}$

$$\omega|_{t=0} = \left.\frac{d\theta}{dt}\right|_{t=0} = 10.0 + 4.00t|_{t=0} = \boxed{10.0 \text{ rad/s}}$$

$$\alpha_{t=0} = \left.\frac{d\omega}{dt}\right|_{t=0} = \boxed{4.00 \text{ rad/s}^2}$$

(b) $\theta|_{t=3.00 \text{ s}} = 5.00 + 30.0 + 18.0 = \boxed{53.0 \text{ rad}}$

$$\omega|_{t=3.00 \text{ s}} = \left.\frac{d\theta}{dt}\right|_{t=3.00 \text{ s}} = 10.0 + 4.00t|_{t=3.00 \text{ s}} = \boxed{22.0 \text{ rad/s}}$$

$$\alpha|_{t=3.00 \text{ s}} = \left.\frac{d\omega}{dt}\right|_{t=3.00 \text{ s}} = \boxed{4.00 \text{ rad/s}^2}$$

Section 10.2 Rotational Kinematics—The Rigid Object Under Constant Angular Acceleration

P10.2 $\omega_f = 2.51\times10^4 \text{ rev/min} = 2.63\times10^3 \text{ rad/s}$

(a) $$\alpha = \frac{\omega_f - \omega_i}{t} = \frac{2.63\times10^3 \text{ rad/s} - 0}{3.2 \text{ s}} = \boxed{8.21\times10^2 \text{ rad/s}^2}$$

(b) $$\theta_f = \omega_i t + \frac{1}{2}\alpha t^2 = 0 + \frac{1}{2}\left(8.21\times10^2 \text{ rad/s}^2\right)(3.2 \text{ s})^2 = \boxed{4.21\times10^3 \text{ rad}}$$

P10.3 $$\omega_i = \frac{100 \text{ rev}}{1.00 \text{ min}}\left(\frac{1 \text{ min}}{60.0 \text{ s}}\right)\left(\frac{2\pi \text{ rad}}{1.00 \text{ rev}}\right) = \frac{10\pi}{3} \text{ rad/s}, \ \omega_f = 0$$

(a) $$t = \frac{\omega_f - \omega_i}{\alpha} = \frac{0 - \frac{10\pi}{3}}{-2.00} \text{ s} = \boxed{5.24 \text{ s}}$$

(b) $$\theta_f = \bar{\omega}t = \left(\frac{\omega_f + \omega_i}{2}\right)t = \left(\frac{10\pi}{6} \text{ rad/s}\right)\left(\frac{10\pi}{6} \text{ s}\right) = \boxed{27.4 \text{ rad}}$$

*P10.4 $\omega_i = 3\,600 \text{ rev/min} = 3.77\times10^2 \text{ rad/s}$

$\theta = 50.0 \text{ rev} = 3.14\times10^2 \text{ rad}$ and $\omega_f = 0$

$\omega_f^2 = \omega_i^2 + 2\alpha\theta$

$0 = \left(3.77\times10^2 \text{ rad/s}\right)^2 + 2\alpha\left(3.14\times10^2 \text{ rad}\right)$

$\alpha = \boxed{-2.26\times10^2 \text{ rad/s}^2}$

P10.5 $\omega = 5.00 \text{ rev/s} = 10.0\pi \text{ rad/s}$. We will break the motion into two stages: (1) a period during which the tub speeds up and (2) a period during which it slows down.

While speeding up, $$\theta_1 = \bar{\omega}t = \frac{0 + 10.0\pi \text{ rad/s}}{2}(8.00 \text{ s}) = 40.0\pi \text{ rad}$$

While slowing down, $$\theta_2 = \bar{\omega}t = \frac{10.0\pi \text{ rad/s} + 0}{2}(12.0 \text{ s}) = 60.0\pi \text{ rad}$$

So, $$\theta_{\text{total}} = \theta_1 + \theta_2 = 100\pi \text{ rad} = \boxed{50.0 \text{ rev}}$$

P10.6 $\theta_f - \theta_i = \omega_i t + \frac{1}{2}\alpha t^2$ and $\omega_f = \omega_i + \alpha t$ are two equations in two unknowns ω_i and α

$\omega_i = \omega_f - \alpha t$: $$\theta_f - \theta_i = (\omega_f - \alpha t)t + \frac{1}{2}\alpha t^2 = \omega_f t - \frac{1}{2}\alpha t^2$$

$$37.0 \text{ rev}\left(\frac{2\pi \text{ rad}}{1 \text{ rev}}\right) = 98.0 \text{ rad/s}(3.00 \text{ s}) - \frac{1}{2}\alpha(3.00 \text{ s})^2$$

$232 \text{ rad} = 294 \text{ rad} - \left(4.50 \text{ s}^2\right)\alpha$: $$\alpha = \frac{61.5 \text{ rad}}{4.50 \text{ s}^2} = \boxed{13.7 \text{ rad/s}^2}$$

P10.7 (a) $$\omega = \frac{\Delta\theta}{\Delta t} = \frac{1 \text{ rev}}{1 \text{ day}} = \frac{2\pi \text{ rad}}{86\,400 \text{ s}} = \boxed{7.27\times10^{-5} \text{ rad/s}}$$

(b) $$\Delta t = \frac{\Delta\theta}{\omega} = \frac{107°}{7.27\times10^{-5} \text{ rad/s}}\left(\frac{2\pi \text{ rad}}{360°}\right) = \boxed{2.57\times10^4 \text{ s}} \text{ or } 428 \text{ min}$$

Section 10.3 Relations Between Rotational and Translational Quantities

P10.8 Estimate the tire's radius at 0.250 m and miles driven as 10 000 per year.

$$\theta = \frac{s}{r} = \frac{1.00\times10^4 \text{ mi}}{0.250 \text{ m}}\left(\frac{1\,609 \text{ m}}{1 \text{ mi}}\right) = 6.44\times10^7 \text{ rad/yr}$$

$$\theta = 6.44\times10^7 \text{ rad/yr}\left(\frac{1 \text{ rev}}{2\pi \text{ rad}}\right) = 1.02\times10^7 \text{ rev/yr or } \boxed{\sim10^7 \text{ rev/yr}}$$

P10.9 (a) $$\omega = 2\pi f = \frac{2\pi \text{ rad}}{1 \text{ rev}}\left(\frac{1\,200 \text{ rev}}{60.0 \text{ s}}\right) = \boxed{126 \text{ rad/s}}$$

(b) $$v = \omega r = (126 \text{ rad/s})\left(3.00\times10^{-2} \text{ m}\right) = \boxed{3.77 \text{ m/s}}$$

(c) $$a_c = \omega^2 r = (126)^2\left(8.00\times10^{-2}\right) = 1\,260 \text{ m/s}^2 \text{ so } \vec{a}_r = \boxed{1.26 \text{ km/s}^2 \text{ toward the center}}$$

(d) $$s = r\theta = \omega r t = (126 \text{ rad/s})\left(8.00\times10^{-2} \text{ m}\right)(2.00 \text{ s}) = \boxed{20.1 \text{ m}}$$

P10.10 Given $r = 1.00\text{ m}$, $\alpha = 4.00\text{ rad/s}^2$, $\omega_i = 0$ and $\theta_i = 57.3° = 1.00\text{ rad}$

(a) $\omega_f = \omega_i + \alpha t = 0 + \alpha t$

At $t = 2.00\text{ s}$, $\omega_f = 4.00\text{ rad/s}^2(2.00\text{ s}) = \boxed{8.00\text{ rad/s}}$

(b) $v = r\omega = 1.00\text{ m}(8.00\text{ rad/s}) = \boxed{8.00\text{ m/s}}$

$|a_r| = a_c = r\omega^2 = 1.00\text{ m}(8.00\text{ rad/s})^2 = 64.0\text{ m/s}^2$

$a_t = r\alpha = 1.00\text{ m}(4.00\text{ rad/s}^2) = 4.00\text{ m/s}^2$

The magnitude of the total acceleration is:

$$a = \sqrt{a_r^2 + a_t^2} = \sqrt{(64.0\text{ m/s}^2)^2 + (4.00\text{ m/s}^2)^2} = \boxed{64.1\text{ m/s}^2}$$

The direction of the total acceleration vector makes an angle ϕ with respect to the radius to point P:

$$\phi = \tan^{-1}\left(\frac{a_t}{a_c}\right) = \tan^{-1}\left(\frac{4.00}{64.0}\right) = \boxed{3.58°}$$

(c) $\theta_f = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2 = (1.00\text{ rad}) + \frac{1}{2}(4.00\text{ rad/s}^2)(2.00\text{ s})^2 = \boxed{9.00\text{ rad}}$

***P10.11** (a) Consider a tooth on the front sprocket. It gives this speed, relative to the frame, to the link of the chain it engages:

$$v = r\omega = \left(\frac{0.152\text{ m}}{2}\right)76\text{ rev/min}\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right) = \boxed{0.605\text{ m/s}}$$

(b) Consider the chain link engaging a tooth on the rear sprocket:

$$\omega = \frac{v}{r} = \frac{0.605\text{ m/s}}{\left(\frac{0.07\text{ m}}{2}\right)} = \boxed{17.3\text{ rad/s}}$$

(c) Consider the wheel tread and the road. A thread could be unwinding from the tire with this speed relative to the frame:

$$v = r\omega = \left(\frac{0.673\text{ m}}{2}\right)17.3\text{ rad/s} = \boxed{5.82\text{ m/s}}$$

(d) We did not need to know the length of the pedal cranks, but we could use that information to find the linear speed of the pedals:

$$v = r\omega = 0.175\text{ m }7.96\text{ rad/s}\left(\frac{1}{1\text{ rad}}\right) = 1.39\text{ m/s}$$

P10.12 (a) $\omega_i = \frac{v}{r} = \frac{1.30\text{ m/s}}{0.023\text{ m}} = \boxed{56.5\text{ rad/s}}$

(b) $\omega_f = \frac{1.30\text{ m/s}}{0.058\text{ m}} = \boxed{22.4\text{ rad/s}}$

(c) $\omega_f = \omega_i + \alpha t:\ \alpha = \frac{22.4\text{ rad/s} - 56.5\text{ rad/s}}{(74(60)+33)\text{ s}} = \frac{-34.1\text{ rad/s}}{4473\text{ s}} = \boxed{-7.63\times10^{-3}\text{ rad/s}^2}$

(d) $\theta_f - \theta_i = \frac{1}{2}(\omega_f + \omega_i)t = \frac{1}{2}((56.5 + 22.4)\text{ rad/s})(4473\text{ s}) = \boxed{1.77\times10^5\text{ rad}}$

(e) $x = vt = (1.30\text{ m/s})(4\,473\text{ s}) = \boxed{5.81\times10^3\text{ m}}$

***P10.13** The force of static friction must act forward and then more and more inward on the tires, to produce both tangential and centripetal acceleration. Its tangential component is $m(1.70\text{ m/s}^2)$. Its radially inward component is $\frac{mv^2}{r}$. This takes the maximum value

$$m\omega_f^2 r = mr(\omega_i^2 + 2\alpha\Delta\theta) = mr\left(0 + 2\alpha\frac{\pi}{2}\right) = m\pi r\alpha = m\pi a_t = m\pi(1.70\text{ m/s}^2).$$

With skidding impending we have $\sum F_y = ma_y$, $+n - mg = 0$, $n = mg$

$$f_s = \mu_s n = \mu_s mg = \sqrt{m^2(1.70\text{ m/s}^2)^2 + m^2\pi^2(1.70\text{ m/s}^2)^2}$$
$$\mu_s = \frac{1.70\text{ m/s}^2}{g}\sqrt{1+\pi^2} = \boxed{0.572}$$

Section 10.4 Rotational Kinetic Energy

P10.14 $m_1 = 4.00\text{ kg}$, $r_1 = |y_1| = 3.00\text{ m}$;

$m_2 = 2.00\text{ kg}$, $r_2 = |y_2| = 2.00\text{ m}$;

$m_3 = 3.00\text{ kg}$, $r_3 = |y_3| = 4.00\text{ m}$;

$\omega = 2.00\text{ rad/s}$ about the x-axis

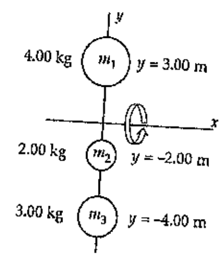

FIG. P10.14

(a) $I_x = m_1r_1^2 + m_2r_2^2 + m_3r_3^2$

$I_x = 4.00(3.00)^2 + 2.00(2.00)^2 + 3.00(4.00)^2 = \boxed{92.0\text{ kg}\cdot\text{m}^2}$

$K_R = \frac{1}{2}I_x\omega^2 = \frac{1}{2}(92.0)(2.00)^2 = \boxed{184\text{ J}}$

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(b) $v_1 = r_1\omega = 3.00(2.00) = \boxed{6.00\text{ m/s}}$ $\quad K_1 = \frac{1}{2}m_1v_1^2 = \frac{1}{2}(4.00)(6.00)^2 = 72.0\text{ J}$

$v_2 = r_2\omega = 2.00(2.00) = \boxed{4.00\text{ m/s}}$ $\quad K_2 = \frac{1}{2}m_2v_2^2 = \frac{1}{2}(2.00)(4.00)^2 = 16.0\text{ J}$

$v_3 = r_3\omega = 4.00(2.00) = \boxed{8.00\text{ m/s}}$ $\quad K_3 = \frac{1}{2}m_3v_3^2 = \frac{1}{2}(3.00)(8.00)^2 = 96.0\text{ J}$

$$K = K_1 + K_2 + K_3 = 72.0 + 16.0 + 96.0 = \boxed{184\text{ J}} = \frac{1}{2}I_x\omega^2$$

P10.15 From conservation of energy for the object-turntable-cylinder-Earth system,

$$\frac{1}{2}I\left(\frac{v}{r}\right)^2 + \frac{1}{2}mv^2 = mgh$$

$$I\frac{v^2}{r^2} = 2mgh - mv^2$$

$$I = \boxed{mr^2\left(\frac{2gh}{v^2} - 1\right)}$$

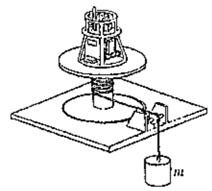

FIG. P10.15

P10.16 The moment of inertia of a thin rod about an axis through one end is $I = \frac{1}{3}ML^2$. The total rotational kinetic energy is given as $K_R = \frac{1}{2}I_h\omega_h^2 + \frac{1}{2}I_m\omega_m^2$

with $\quad I_h = \frac{m_hL_h^2}{3} = \frac{60.0\text{ kg}(2.70\text{ m})^2}{3} = 146\text{ kg}\cdot\text{m}^2$

and $\quad I_m = \frac{m_mL_m^2}{3} = \frac{100\text{ kg}(4.50\text{ m})^2}{3} = 675\text{ kg}\cdot\text{m}^2$

In addition, $\quad \omega_h = \frac{2\pi\text{ rad}}{12\text{ h}}\left(\frac{1\text{ h}}{3\,600\text{ s}}\right) = 1.45\times10^{-4}\text{ rad/s}$

while $\quad \omega_m = \frac{2\pi\text{ rad}}{1\text{ h}}\left(\frac{1\text{ h}}{3\,600\text{ s}}\right) = 1.75\times10^{-3}\text{ rad/s}$

Therefore, $\quad K_R = \frac{1}{2}(146)\left(1.45\times10^{-4}\right)^2 + \frac{1}{2}(675)\left(1.75\times10^{-3}\right)^2 = \boxed{1.03\times10^{-3}\text{ J}}$

P10.17 Take the two objects, pulley, and Earth as the system. If we neglect friction in the system, then mechanical energy is conserved and we can state that the increase in kinetic energy of the system equals the decrease in potential energy. Since $K_i = 0$ (the system is initially at rest), we have $\Delta K = K_f - K_i = \frac{1}{2}m_1v^2 + \frac{1}{2}m_2v^2 + \frac{1}{2}I\omega^2$ where m_1 and m_2 have a common speed. But $v = R\omega$ so that $\Delta K = \frac{1}{2}\left(m_1 + m_2 + \frac{I}{R^2}\right)v^2$. From Figure P10.17, we see that a loss of gravitational energy is associated with the motion of m_1 and a gain with the motion of m_2. Applying the law of conservation of energy $\Delta K + \Delta U = 0$ gives $\frac{1}{2}\left(m_1 + m_2 + \frac{I}{R^2}\right)v^2 + m_2gh - m_1gh = 0$.

$$\boxed{v = \sqrt{\frac{2(m_1 - m_2)gh}{\left(m_1 + m_2 + \frac{I}{R^2}\right)}}}$$

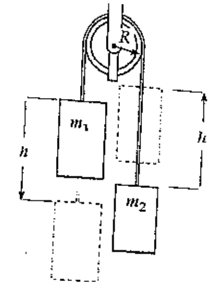

FIG. P10.17

Since $v = R\omega$, the angular speed of the pulley at this instant is given by

$$\boxed{\omega = \frac{v}{R} = \sqrt{\frac{2(m_1 - m_2)gh}{\left(m_1R^2 + m_2R^2 + I\right)}}}$$

*P10.18 For large energy storage at a particular rotation rate, we want a large moment of inertia. To combine this requirement with small mass, we place the mass as far away from the axis as possible.

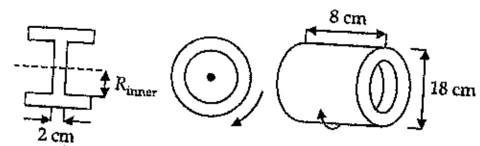

FIG. P10.18

We choose to make the flywheel as a hollow cylinder 18 cm in diameter and 8 cm long. To support this rim, we place a disk across its center. We assume that a disk 2 cm thick will be sturdy enough to support the hollow cylinder securely. The one remaining adjustable parameter is the thickness of the wall of the hollow cylinder. From Table 10.2, the moment of inertia can be written as

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$$I_{\text{disk}} + I_{\text{hollow cylinder}} = \frac{1}{2}M_{\text{disk}}R_{\text{disk}}^2 + \frac{1}{2}M_{\text{wall}}\left(R_{\text{outer}}^2 + R_{\text{inner}}^2\right)$$
$$= \frac{1}{2}\rho V_{\text{disk}}R_{\text{outer}}^2 + \frac{1}{2}\rho V_{\text{wall}}\left(R_{\text{outer}}^2 + R_{\text{inner}}^2\right)$$
$$= \frac{\rho}{2}\pi R_{\text{outer}}^2(2\text{ cm})R_{\text{outer}}^2 + \frac{\rho}{2}\left[\pi R_{\text{outer}}^2 - \pi R_{\text{inner}}^2\right](6\text{ cm})\left(R_{\text{outer}}^2 + R_{\text{inner}}^2\right)$$
$$= \frac{\rho\pi}{2}\left[(9\text{ cm})^4(2\text{ cm}) + (6\text{ cm})\left((9\text{ cm})^2 - R_{\text{inner}}^2\right)\left((9\text{ cm})^2 + R_{\text{inner}}^2\right)\right]$$
$$= \rho\pi\left[6\,561\text{ cm}^5 + (3\text{ cm})\left((9\text{ cm})^4 - R_{\text{inner}}^4\right)\right]$$
$$= \rho\pi\left[26\,244\text{ cm}^5 - (3\text{ cm})R_{\text{inner}}^4\right]$$

For the required energy storage,

$$\frac{1}{2}I\omega_1^2 = \frac{1}{2}I\omega_2^2 + W_{\text{out}}$$
$$\frac{1}{2}I\left[(800\text{ rev/min})\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right)\right]^2 - \frac{1}{2}I\left[(600)\left(\frac{2\pi\text{ rad}}{60\text{ s}}\right)\right]^2 = 60\text{ J}$$
$$I = \frac{60\text{ J}}{1\,535/\text{s}^2} = \left(7.86\times10^3\text{ kg/m}^3\right)\pi\left[26\,244\text{ cm}^5 - (3\text{ cm})R_{\text{inner}}^4\right]$$
$$1.58\times10^{-6}\text{ m}^5\left(\frac{100\text{ cm}}{1\text{ m}}\right)^5 = 26\,244\text{ cm}^5 - (3\text{ cm})R_{\text{inner}}^4$$
$$R_{\text{inner}} = \left(\frac{26\,244\text{ cm}^4 - 15\,827\text{ cm}^4}{3}\right)^{1/4} = 7.68\text{ cm}$$

The mass of the flywheel is then

$$M_{\text{disk}} + M_{\text{wall}} = \rho\pi R_{\text{outer}}^2(2\text{ cm}) + \rho\left[\pi R_{\text{outer}}^2 - \pi R_{\text{inner}}^2\right](6\text{ cm})$$
$$= \left(7.86\times10^3\text{ kg/m}^3\right)\pi\left[(0.09\text{ m})^2(0.02\text{ m}) + \left[(0.09\text{ m})^2 - (0.076\,8\text{ m})^2\right](0.06\text{ m})\right]$$
$$= 7.27\text{ kg}$$

If we made the thickness of the disk somewhat less than 2 cm and the inner radius of the cylindrical wall less than 7.68 cm to compensate, the mass could be a bit less than 7.27 kg.

*P10.19 Note that the torque on the trebuchet is not constant, so its angular acceleration changes in time. At our mathematical level it would be unproductive to calculate values for α on the way to find ω_f. Instead, we consider that gravitational energy of the 60-kg-Earth system becomes gravitational energy of the lighter mass plus kinetic energy of both masses. The maximum speed appears as the rod passes through the vertical. Let v_1 represent the speed of the small-mass particle m_1. Then here the rod is turning at $\omega_1 = \frac{v_1}{2.86\text{ m}}$. The larger-mass particle is moving at

$$v_2 = (0.14\text{ m})\omega_1 = \frac{0.14v_1}{2.86}$$

Now the energy-conservation equation becomes

$$\left(K_1 + K_2 + U_{g1} + U_{g2}\right)_i = \left(K_1 + K_2 + U_{g1} + U_{g2}\right)_f$$
$$0 + 0 + 0 + m_2gy_{2i} = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 + m_1gy_{1f} + 0$$
$$(60\text{ kg})(9.8\text{ m/s}^2)(0.14\text{ m}) = \frac{1}{2}(0.12\text{ kg})v_1^2 + \frac{1}{2}(60\text{ kg})\left(\frac{0.14v_1}{2.86}\right)^2 + (0.12\text{ kg})(9.8\text{ m/s}^2)(2.86\text{ m})$$
$$82.32\text{ J} = \frac{1}{2}(0.12\text{ kg})v_1^2 + \frac{1}{2}(0.144\text{ kg})v_1^2 + 3.36\text{ J}$$
$$v_1 = \left(\frac{2(79.0\text{ J})}{0.264\text{ kg}}\right)^{1/2} = \boxed{24.5\text{ m/s}}$$

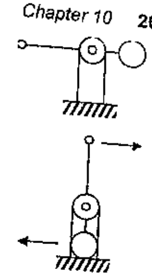

FIG. P10.19

Section 10.5 Torque and the Vector Product

P10.20 Resolve the 100 N force into components perpendicular to and arallel to the rod, as

$$F_{\text{par}} = (100\text{ N})\cos 57.0° = 54.5\text{ N}$$

and $F_{\text{perp}} = (100\text{ N})\sin 57.0° = 83.9\text{ N}$

The torque of F_{par} is zero since its line of action passes through he pivot point.

The torque of F_{perp} is $\tau = 83.9\text{ N}(2.00\text{ m}) = \boxed{168\text{ N}\cdot\text{m}}$ (clockwise)

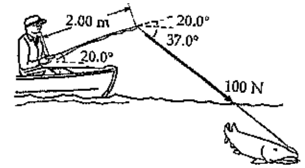

FIG. P10.20

P10.21 $\sum\tau = 0.100\text{ m}(12.0\text{ N}) - 0.250\text{ m}(9.00\text{ N}) - 0.250\text{ m}(10.0\text{ N}) = \boxed{-3.55\text{ N}\cdot\text{m}}$

The thirty-degree angle is unnecessary information.

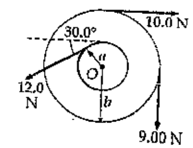

FIG. P10.21

P10.22 $\vec{M}\times\vec{N}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 6 & 2 & -1\\ 2 & -1 & -3\end{vmatrix}=\boxed{-7.00\hat{i}+16.0\hat{j}-10.0\hat{k}}$

P10.23 (a) $\vec{\tau}=\vec{r}\times\vec{F}=(4.00\hat{i}+5.00\hat{j})\times(2.00\hat{i}+3.00\hat{j})$

$|\vec{\tau}|=|12.00\hat{k}-10.0\hat{k}|=|2.00\hat{k}|=\boxed{2.00\text{ N}\cdot\text{m}}$

(b) The torque vector is in the direction of the unit vector $\hat{k}$, or in the $\boxed{+z\text{ direction}}$.

P10.24 $\vec{A}\cdot\vec{B}=-3.00(6.00)+7.00(-10.0)+(-4.00)(9.00)=-124$

$AB=\sqrt{(-3.00)^2+(7.00)^2+(-4.00)^2}\cdot\sqrt{(6.00)^2+(-10.0)^2+(9.00)^2}=127$

(a) $\cos^{-1}\left(\frac{\vec{A}\cdot\vec{B}}{AB}\right)=\cos^{-1}(-0.979)=\boxed{168°}$

(b) $\vec{A}\times\vec{B}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ -3.00 & 7.00 & -4.00\\ 6.00 & -10.0 & 9.00\end{vmatrix}=23.0\hat{i}+3.00\hat{j}-12.0\hat{k}$

$|\vec{A}\times\vec{B}|=\sqrt{(23.0)^2+(3.00)^2+(-12.0)^2}=26.1$

$\sin^{-1}\left(\frac{|\vec{A}\times\vec{B}|}{AB}\right)=\sin^{-1}(0.206)=\boxed{11.9°}$ or 168°

(c) Only $\boxed{\text{the first method}}$ gives the angle between the vectors unambiguously.

P10.25 $|\hat{i}\times\hat{i}|=1\cdot1\cdot\sin 0°=0$

$\hat{j}\times\hat{j}$ and $\hat{k}\times\hat{k}$ are zero similarly since the vectors being multiplied are parallel.

$|\hat{i}\times\hat{j}|=1\cdot1\cdot\sin 90°=1$

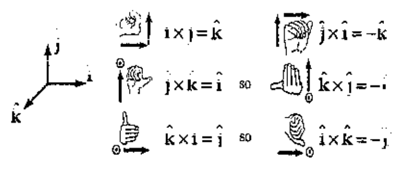

FIG. P10.25

Section 10.6 The Rigid Object in Equilibrium

P10.26 $\sum F_y=0$: $+380\text{ N}-F_g+320\text{ N}=0$

$F_g=700\text{ N}$

Take torques about her feet:

$\sum\tau=0$: $-380\text{ N}(2.00\text{ m})+(700\text{ N})x+(320\text{ N})0=0$

$x=\boxed{1.09\text{ m}}$

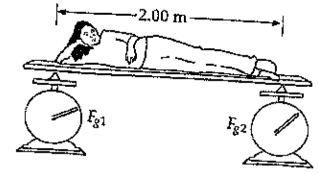

FIG. P10.26

P10.27 Take torques about P.

$$\sum\tau_p=-n_0\left[\frac{\ell}{2}+d\right]+m_1g\left[\frac{\ell}{2}+d\right]+m_bgd-m_2gx=0$$

We want to find x for which $n_0=0$.

$$x=\frac{(m_1g+m_bg)d+m_1g\frac{\ell}{2}}{m_2g}=\boxed{\frac{(m_1+m_b)d+m_1\ell/2}{m_2}}$$

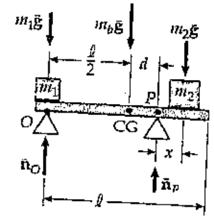

FIG. P10.27

***P10.28** See the free-body diagram at the right. When the plank is on the verge of tipping about point P, the normal force n_1 goes to zero. Then, summing torques about point P gives

$\sum\tau_p=-mgd+Mgx=0$ or $x=\left(\frac{m}{M}\right)d$.

From the dimensions given on the free-body diagram, observe that $d=1.50$ m. Thus, when the plank is about to tip,

$$x=\left(\frac{30.0\text{ kg}}{70.0\text{ kg}}\right)(1.50\text{ m})=\boxed{0.643\text{ m}}.$$

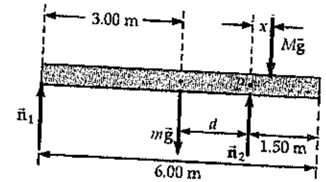

FIG. P10.28

P10.29 (a) Taking moments about P,

$$(R\sin 30.0°)0+(R\cos 30.0°)(5.00\text{ cm})-(150\text{ N})(30.0\text{ cm})=0$$
$$R=1\,039.2\text{ N}=1.04\text{ kN}$$

The force exerted by the hammer on the nail is equal in magnitude and opposite in direction:

$\boxed{1.04\text{ kN at }60°\text{ upward and to the right.}}$

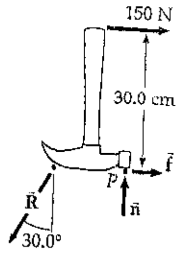

FIG. P10.29

(b) $f=R\sin 30.0°-150\text{ N}=370\text{ N}$

$n=R\cos 30.0°=900\text{ N}$

$\boxed{\vec{F}_{\text{surface}}=(370\text{ N})\hat{i}+(900\text{ N})\hat{j}}$

P10.30 (a)
$$\sum F_x=f-n_w=0 \quad (1)$$
$$\sum F_y=n_g-m_1g-m_2g=0 \quad (2)$$
$$\sum \tau_A=-m_1g\left(\frac{L}{2}\right)\cos\theta-m_2gx\cos\theta+n_wL\sin\theta=0$$

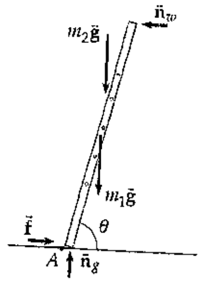

FIG. P10.30

From the torque equation,

$$n_w=\left[\frac{1}{2}m_1g+\left(\frac{x}{L}\right)m_2g\right]\cot\theta$$

Then, from equation (1): $f=n_w=\boxed{\left[\frac{1}{2}m_1g+\left(\frac{x}{L}\right)m_2g\right]\cot\theta}$

and from equation (2): $n_g=\boxed{(m_1+m_2)g}$

(b) If the ladder is on the verge of slipping when $x=d$,

then $\mu=\dfrac{f|_{x=d}}{n_g}=\boxed{\dfrac{(m_1/2+m_2d/L)\cot\theta}{m_1+m_2}}$.

P10.31 Using $\sum F_x=\sum F_y=\sum\tau=0$, choosing the origin at the left end of the beam, we have (neglecting the weight of the beam)

$$\sum F_x=R_x-T\cos\theta=0,$$
$$\sum F_y=R_y+T\sin\theta-F_g=0,$$

and $\sum\tau=-F_g(L+d)+T\sin\theta(2L+d)=0$.

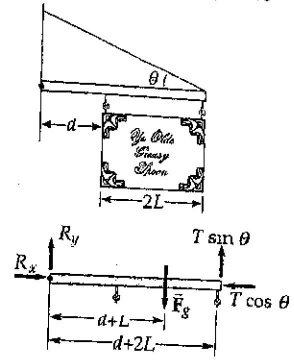

FIG. P10.31

Solving these equations, we find:

(a) $T=\boxed{\dfrac{F_g(L+d)}{\sin\theta(2L+d)}}$

(b) $R_x=\boxed{\dfrac{F_g(L+d)\cot\theta}{2L+d}}$ $\quad R_y=\boxed{\dfrac{F_gL}{2L+d}}$

P10.32 We interpret the problem to mean that the support at point B is frictionless. Then the support exerts a force in the x direction and

$$\boxed{F_{By}=0}$$
$$\sum F_x=F_{Bx}-F_{Ax}=0$$
$$F_{Ay}-(3\,000+10\,000)g=0$$

and $\sum\tau=-(3\,000g)(2.00)-(10\,000g)(6.00)+F_{Bx}(1.00)=0$.

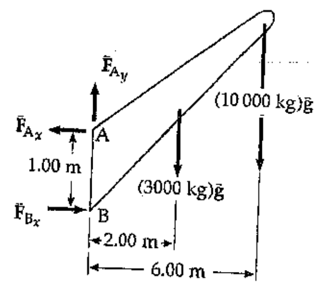

FIG. P10.32

These equations combine to give

$$F_{Ax}=F_{Bx}=\boxed{6.47\times10^5\text{ N}}$$
$$F_{Ay}=\boxed{1.27\times10^5\text{ N}}$$

Section 10.7 The Rigid Object Under a Net Torque

P10.33 $\omega_f = \omega_i + \alpha t$: $10.0\ \text{rad/s} = 0 + \alpha(6.00\ \text{s})$

$$\alpha = \frac{10.00}{6.00}\ \text{rad/s}^2 = 1.67\ \text{rad/s}^2$$

(a) $\sum \tau = 36.0\ \text{N}\cdot\text{m} = I\alpha$: $I = \frac{\sum \tau}{\alpha} = \frac{36.0\ \text{N}\cdot\text{m}}{1.67\ \text{rad/s}^2} = \boxed{21.6\ \text{kg}\cdot\text{m}^2}$

(b) $\omega_f = \omega_i + \alpha t$: $0 = 10.0 + \alpha(60.0)$

$$\alpha = -0.167\ \text{rad/s}^2$$

$$|\tau| = |I\alpha| = (21.6\ \text{kg}\cdot\text{m}^2)(0.167\ \text{rad/s}^2) = \boxed{3.60\ \text{N}\cdot\text{m}}$$

(c) Number of revolutions $\theta_f = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2$

During first 6.00 s $\theta_f = \frac{1}{2}(1.67)(6.00)^2 = 30.1\ \text{rad}$

During next 60.0 s $\theta_f = 10.0(60.0) - \frac{1}{2}(0.167)(60.0)^2 = 299\ \text{rad}$

$$\theta_{\text{total}} = 329\ \text{rad}\left(\frac{1\ \text{rev}}{2\pi\ \text{rad}}\right) = \boxed{52.4\ \text{rev}}$$

P10.34 $I = \frac{1}{2}mR^2 = \frac{1}{2}(100\ \text{kg})(0.500\ \text{m})^2 = 12.5\ \text{kg}\cdot\text{m}^2$

$\omega_i = 50.0\ \text{rev/min} = 5.24\ \text{rad/s}$

$$\alpha = \frac{\omega_f - \omega_i}{t} = \frac{0 - 5.24\ \text{rad/s}}{6.00\ \text{s}} = -0.873\ \text{rad/s}^2$$

$$\tau = I\alpha = 12.5\ \text{kg}\cdot\text{m}^2(-0.873\ \text{rad/s}^2) = -10.9\ \text{N}\cdot\text{m}$$

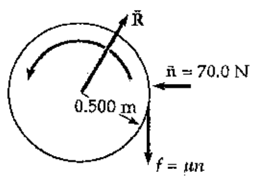

FIG. P10.34

The magnitude of the torque is given by $fR = 10.9\ \text{N}\cdot\text{m}$, where f is the force of friction.

Therefore, $f = \frac{10.9\ \text{N}\cdot\text{m}}{0.500\ \text{m}}$ and $f = \mu_k n$

yields $\mu_k = \frac{f}{n} = \frac{21.8\ \text{N}}{70.0\ \text{N}} = \boxed{0.312}$

P10.35 $\sum \tau = I\alpha = \frac{1}{2}MR^2\alpha$

$$-135\ \text{N}(0.230\ \text{m}) + T(0.230\ \text{m}) = \frac{1}{2}(80\ \text{kg})\left(\frac{1.25}{2}\ \text{m}\right)^2(-1.67\ \text{rad/s}^2)$$

$T = \boxed{21.5\ \text{N}}$

P10.36 (a) $I = \frac{1}{2}M(R_1^2 + R_2^2) = \frac{1}{2}(0.35\ \text{kg})\left[(0.02\ \text{m})^2 + (0.03\ \text{m})^2\right] = 2.28\times10^{-4}\ \text{kg}\cdot\text{m}^2$

$$(K_1 + K_2 + K_{\text{rot}} + U_{g2})_i - f_k d = (K_1 + K_2 + K_{\text{rot}})_f$$

$$\frac{1}{2}(0.850\ \text{kg})(0.82\ \text{m/s})^2 + \frac{1}{2}(0.42\ \text{kg})(0.82\ \text{m/s})^2 + \frac{1}{2}(2.28\times10^{-4}\ \text{kg}\cdot\text{m}^2)\left(\frac{0.82\ \text{m/s}}{0.03\ \text{m}}\right)^2$$
$$+0.42\ \text{kg}(9.8\ \text{m/s}^2)(0.7\ \text{m}) - 0.25(0.85\ \text{kg})(9.8\ \text{m/s}^2)(0.7\ \text{m})$$
$$= \frac{1}{2}(0.85\ \text{kg})v_f^2 + \frac{1}{2}(0.42\ \text{kg})v_f^2 + \frac{1}{2}(2.28\times10^{-4}\ \text{kg}\cdot\text{m}^2)\left(\frac{v_f}{0.03\ \text{m}}\right)^2$$

$$0.512\ \text{J} + 2.88\ \text{J} - 1.46\ \text{J} = (0.761\ \text{kg})v_f^2$$

$$v_f = \sqrt{\frac{1.94\ \text{J}}{0.761\ \text{kg}}} = \boxed{1.59\ \text{m/s}}$$

(b) $\omega = \frac{v}{r} = \frac{1.59\ \text{m/s}}{0.03\ \text{m}} = \boxed{53.1\ \text{rad/s}}$

P10.37 (a) $m_2 g - T_2 = m_2 a$

$$T_2 = m_2(g - a) = 20.0\ \text{kg}(9.80\ \text{m/s}^2 - 2.00\ \text{m/s}^2) = \boxed{156\ \text{N}}$$

$$T_1 - m_1 g \sin 37.0° = m_1 a$$

$$T_1 = (15.0\ \text{kg})(9.80 \sin 37.0° + 2.00)\ \text{m/s}^2 = \boxed{118\ \text{N}}$$

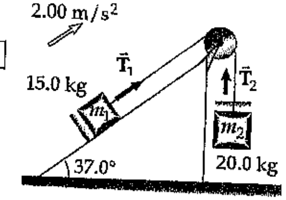

FIG. P10.37

(b) $(T_2 - T_1)R = I\alpha = I\left(\frac{a}{R}\right)$

$$I = \frac{(T_2 - T_1)R^2}{a} = \frac{(156\ \text{N} - 118\ \text{N})(0.250\ \text{m})^2}{2.00\ \text{m/s}^2} = \boxed{1.17\ \text{kg}\cdot\text{m}^2}$$

P10.38 For the purpose of computing the torque on the rod, we recall that the whole weight Mg can be modeled as acting at the center of mass of the rod. Because the rod is uniform, its center of mass is at its geometric center. The magnitude of the torque due to this force about an axis through the pivot is

$$\tau = \frac{MgL}{2}$$

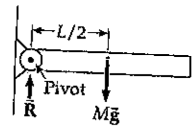

FIG. P10.38

The support force at the hinge has zero torque about an axis through the pivot, because this force passes through the axis (hence $r = 0$). Since $\tau = I\alpha$, where $I = \frac{1}{3}ML^2$ for this axis of rotation (see Table 10.2), we get $I\alpha = Mg\frac{L}{2}$: $\alpha = \frac{Mg(L/2)}{(1/3)ML^2} = \boxed{\frac{3g}{2L}}$.

The angular acceleration is common to *all* points on the rod.

continued on next page

To find the linear acceleration of the right end of the rod, we use the relation $a_t = R\alpha$ with $R = L$: this gives

$$a_t = L\alpha = \boxed{\frac{3}{2}g}$$

This result is rather interesting, since $a_t > g$. That is, the end of the rod has an acceleration *greater* than the acceleration due to gravity. Therefore, if a coin were placed at the end of the rod, the end of the rod would fall faster than the coin when released.

Other points on the rod have a linear acceleration less than $\frac{3}{2}g$. For example, the middle of the rod has an acceleration $\frac{3}{4}g$.

P10.39 (a) For the counterweight,

$\sum F_y = ma_y$ becomes: $\quad 50.0 - T = \left(\frac{50.0}{9.80}\right)a$

For the reel $\sum \tau = I\alpha$ reads $TR = I\alpha = I\frac{a}{R}$

where $\quad I = \frac{1}{2}MR^2 = 0.093\,8\ \text{kg}\cdot\text{m}^2$

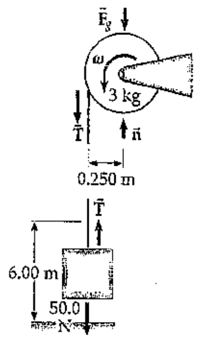

FIG. P10.39

We substitute to eliminate the acceleration:

$$50.0 - T = 5.10\left(\frac{TR^2}{I}\right)$$

$$T = \boxed{11.4\ \text{N}}$$

and $\quad a = \frac{50.0 - 11.4}{5.10} = \boxed{7.57\ \text{m/s}^2}$

$v_f^2 = v_i^2 + 2a(x_f - x_i)$: $\quad v_f = \sqrt{2(7.57)6.00} = \boxed{9.53\ \text{m/s}}$

(b) Use conservation of energy for the system of the object, the reel, and the Earth:

$(K+U)_i = (K+U)_f$: $\quad mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$

$$2mgh = mv^2 + I\left(\frac{v^2}{R^2}\right) = v^2\left(m + \frac{I}{R^2}\right)$$

$$v = \sqrt{\frac{2mgh}{m + \frac{I}{R^2}}} = \sqrt{\frac{2(50.0\ \text{N})(6.00\ \text{m})}{5.10\ \text{kg} + \frac{0.093\,8}{(0.250)^2}}} = \boxed{9.53\ \text{m/s}}$$

Section 10.8 Angular Momentum

P10.40 Whether we think of the Earth's surface as curved or flat, we interpret the problem to mean that the plane's line of flight extended is precisely tangent to the mountain at its peak, and nearly parallel to the wheat field. Let the positive x direction be eastward, positive y be northward, and positive z be vertically upward.

(a) $\vec{r} = (4.30\ \text{km})\hat{k} = (4.30 \times 10^3\ \text{m})\hat{k}$

$\vec{p} = m\vec{v} = 12\,000\ \text{kg}(-175\hat{i}\ \text{m/s}) = -2.10 \times 10^6\hat{i}\ \text{kg}\cdot\text{m/s}$

$\vec{L} = \vec{r} \times \vec{p} = (4.30 \times 10^3\hat{k}\ \text{m}) \times (-2.10 \times 10^6\hat{i}\ \text{kg}\cdot\text{m/s}) = \boxed{(-9.03 \times 10^9\ \text{kg}\cdot\text{m}^2/\text{s})\hat{j}}$

(b) $\boxed{\text{No}}$. $L = |\vec{r}||\vec{p}|\sin\theta = mv(r\sin\theta)$, and $r\sin\theta$ is the altitude of the plane. Therefore, $L =$ *constant* as the plane moves in level flight with constant velocity.

(c) $\boxed{\text{Zero}}$. The position vector from Pike's Peak to the plane is anti-parallel to the velocity of the plane. That is, it is directed along the same line and opposite in direction.

Thus, $L = mvr\sin 180° = 0$.

P10.41 $\vec{r} = (6.00\hat{i} + 5.00t\hat{j}\ \text{m})$ $\qquad \vec{v} = \frac{d\vec{r}}{dt} = 5.00\hat{j}\ \text{m/s}$

so $\quad \vec{p} = m\vec{v} = 2.00\ \text{kg}(5.00\hat{j}\ \text{m/s}) = 10.0\hat{j}\ \text{kg}\cdot\text{m/s}$

and $\quad \vec{L} = \vec{r} \times \vec{p} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 6.00 & 5.00t & 0 \\ 0 & 10.0 & 0 \end{vmatrix} = \boxed{(60.0\ \text{kg}\cdot\text{m}^2/\text{s})\hat{k}}$

P10.42 The total angular momentum about the center point is given by $L = I_h\omega_h + I_m\omega_m$

with $\quad I_h = \frac{m_h L_h^2}{3} = \frac{60.0\ \text{kg}(2.70\ \text{m})^2}{3} = 146\ \text{kg}\cdot\text{m}^2$

and $\quad I_{m3} = \frac{m_m L_m^2}{3} = \frac{100\ \text{kg}(4.50\ \text{m})^2}{3} = 675\ \text{kg}\cdot\text{m}^2$

In addition, $\quad \omega_h = \frac{2\pi\ \text{rad}}{12\ \text{h}}\left(\frac{1\ \text{h}}{3\,600\ \text{s}}\right) = 1.45 \times 10^{-4}\ \text{rad/s}$

while $\quad \omega_m = \frac{2\pi\ \text{rad}}{1\ \text{h}}\left(\frac{1\ \text{h}}{3\,600\ \text{s}}\right) = 1.75 \times 10^{-3}\ \text{rad/s}$

Thus, $\quad L = 146\ \text{kg}\cdot\text{m}^2(1.45 \times 10^{-4}\ \text{rad/s}) + 675\ \text{kg}\cdot\text{m}^2(1.75 \times 10^{-3}\ \text{rad/s})$

or $\quad \boxed{L = 1.20\ \text{kg}\cdot\text{m}^2/\text{s}}$

P10.43 (a) $I = \frac{1}{12}m_1L^2 + m_2(0.500)^2 = \frac{1}{12}(0.100)(1.00)^2 + 0.400(0.500)^2 = 0.108\,3\ \text{kg}\cdot\text{m}^2$

$$L = I\omega = 0.108\,3(4.00) = \boxed{0.433\ \text{kg}\cdot\text{m}^2/\text{s}}$$

(b) $I = \frac{1}{3}m_1L^2 + m_2R^2 = \frac{1}{3}(0.100)(1.00)^2 + 0.400(1.00)^2 = 0.433$

$$L = I\omega = 0.433(4.00) = \boxed{1.73\ \text{kg}\cdot\text{m}^2/\text{s}}$$

*P10.44 We require $a_r = g = \frac{v^2}{r} = \omega^2 r$

$$\omega = \sqrt{\frac{g}{r}} = \sqrt{\frac{(9.80\ \text{m/s}^2)}{100\ \text{m}}} = 0.313\ \text{rad/s}$$

$$I = Mr^2 = 5\times10^4\ \text{kg}(100\ \text{m})^2 = 5\times10^8\ \text{kg}\cdot\text{m}^2$$

(a) $L = I\omega = 5\times10^8\ \text{kg}\cdot\text{m}^2\,0.313/\text{s} = \boxed{1.57\times10^8\ \text{kg}\cdot\text{m}^2/\text{s}}$

(c) $\sum\tau = I\alpha = \frac{I(\omega_f - \omega_i)}{\Delta t}$

$$\sum\tau\Delta t = I\omega_f - I\omega_i = L_f - L_i$$

This is the angular impulse-angular momentum theorem.

(b) $\Delta t = \frac{L_f - 0}{\sum\tau} = \frac{1.57\times10^8\ \text{kg}\cdot\text{m}^2/\text{s}}{2(125\ \text{N})(100\ \text{m})} = \boxed{6.26\times10^3\ \text{s}} = 1.74\ \text{h}$

Section 10.9 Conservation of Angular Momentum

P10.45 (a) From conservation of angular momentum for the system of two cylinders:

$$(I_1 + I_2)\omega_f = I_1\omega_i \quad \text{or} \quad \omega_f = \boxed{\frac{I_1}{I_1 + I_2}\omega_i}$$

(b) $K_f = \frac{1}{2}(I_1 + I_2)\omega_f^2$ and $K_i = \frac{1}{2}I_1\omega_i^2$

so $\frac{K_f}{K_i} = \frac{\frac{1}{2}(I_1 + I_2)}{\frac{1}{2}I_1\omega_i^2}\left(\frac{I_1}{I_1 + I_2}\omega_i\right)^2 = \boxed{\frac{I_1}{I_1 + I_2}\ \text{which is less than 1}}$.

P10.46 $I_i\omega_i = I_f\omega_f$: $(250\ \text{kg}\cdot\text{m}^2)(10.0\ \text{rev/min}) = \left[250\ \text{kg}\cdot\text{m}^2 + 25.0\ \text{kg}(2.00\ \text{m})^2\right]\omega_2$

$$\omega_2 = \boxed{7.14\ \text{rev/min}}$$

P10.47 (a) The table turns opposite to the way the woman walks, so its angular momentum cancels that of the woman. From conservation of angular momentum for the system of the woman and the turntable, we have $L_f = L_i = 0$

so, $L_f = I_{\text{woman}}\omega_{\text{woman}} + I_{\text{table}}\omega_{\text{table}} = 0$

and $\omega_{\text{table}} = \left(-\frac{I_{\text{woman}}}{I_{\text{table}}}\right)\omega_{\text{woman}} = \left(-\frac{m_{\text{woman}}r^2}{I_{\text{table}}}\right)\left(\frac{v_{\text{woman}}}{r}\right) = -\frac{m_{\text{woman}}rv_{\text{woman}}}{I_{\text{table}}}$

$$\omega_{\text{table}} = -\frac{60.0\ \text{kg}(2.00\ \text{m})(1.50\ \text{m/s})}{500\ \text{kg}\cdot\text{m}^2} = -0.360\ \text{rad/s}$$

or $\omega_{\text{table}} = \boxed{0.360\ \text{rad/s (counterclockwise)}}$

(b) work done $= \Delta K = K_f - 0 = \frac{1}{2}m_{\text{woman}}v_{\text{woman}}^2 + \frac{1}{2}I\omega_{\text{table}}^2$

$$W = \frac{1}{2}(60\ \text{kg})(1.50\ \text{m/s})^2 + \frac{1}{2}(500\ \text{kg}\cdot\text{m}^2)(0.360\ \text{rad/s})^2 = \boxed{99.9\ \text{J}}$$

P10.48 (a) The total angular momentum of the system of the student, the stool, and the weights about the axis of rotation is given by

$$I_{\text{total}} = I_{\text{weights}} + I_{\text{student}} = 2(mr^2) + 3.00\ \text{kg}\cdot\text{m}^2$$

Before: $r = 1.00$ m.

Thus, $I_i = 2(3.00\ \text{kg})(1.00\ \text{m})^2 + 3.00\ \text{kg}\cdot\text{m}^2 = 9.00\ \text{kg}\cdot\text{m}^2$

After: $r = 0.300$ m

Thus, $I_f = 2(3.00\ \text{kg})(0.300\ \text{m})^2 + 3.00\ \text{kg}\cdot\text{m}^2 = 3.54\ \text{kg}\cdot\text{m}^2$

We now use conservation of angular momentum.

$$I_f\omega_f = I_i\omega_i$$

or $\omega_f = \left(\frac{I_i}{I_f}\right)\omega_i = \left(\frac{9.00}{3.54}\right)(0.750\ \text{rad/s}) = \boxed{1.91\ \text{rad/s}}$

(b) $K_i = \frac{1}{2}I_i\omega_i^2 = \frac{1}{2}(9.00\ \text{kg}\cdot\text{m}^2)(0.750\ \text{rad/s})^2 = \boxed{2.53\ \text{J}}$

$$K_f = \frac{1}{2}I_f\omega_f^2 = \frac{1}{2}(3.54\ \text{kg}\cdot\text{m}^2)(1.91\ \text{rad/s})^2 = \boxed{6.44\ \text{J}}$$

P10.49 When they touch, the center of mass is distant from the center of the larger puck by

$$y_{CM} = \frac{0 + 80.0\text{ g}(4.00\text{ cm} + 6.00\text{ cm})}{120\text{ g} + 80.0\text{ g}} = 4.00\text{ cm}$$

(a) $L = r_1 m_1 v_1 + r_2 m_2 v_2 = 0 + (6.00\times10^{-2}\text{ m})(80.0\times10^{-3}\text{ kg})(1.50\text{ m/s}) = \boxed{7.20\times10^{-3}\text{ kg}\cdot\text{m}^2/\text{s}}$

(b) The moment of inertia about the CM is

$$I = \left(\frac{1}{2}m_1 r_1^2 + m_1 d_1^2\right) + \left(\frac{1}{2}m_2 r_2^2 + m_2 d_2^2\right)$$

$$I = \frac{1}{2}(0.120\text{ kg})(6.00\times10^{-2}\text{ m})^2 + (0.120\text{ kg})(4.00\times10^{-2})^2$$

$$+\frac{1}{2}(80.0\times10^{-3}\text{ kg})(4.00\times10^{-2}\text{ m})^2 + (80.0\times10^{-3}\text{ kg})(6.00\times10^{-2}\text{ m})^2$$

$$I = 7.60\times10^{-4}\text{ kg}\cdot\text{m}^2$$

Angular momentum of the two-puck system is conserved: $L = I\omega$

$$\omega = \frac{L}{I} = \frac{7.20\times10^{-3}\text{ kg}\cdot\text{m}^2/\text{s}}{7.60\times10^{-4}\text{ kg}\cdot\text{m}^2} = \boxed{9.47\text{ rad/s}}$$

P10.50 For one of the crew,

$\sum F_r = ma_r$: $\quad n = \dfrac{mv^2}{r} = m\omega_i^2 r$

We require $\quad n = mg$, so $\omega_i = \sqrt{\dfrac{g}{r}}$

Now, $\quad I_i\omega_i = I_f\omega_f$

$$\left[5.00\times10^8\text{ kg}\cdot\text{m}^2 + 150\times65.0\text{ kg}\times(100\text{ m})^2\right]\sqrt{\frac{g}{r}} = \left[5.00\times10^8\text{ kg}\cdot\text{m}^2 + 50\times65.0\text{ kg}(100\text{ m})^2\right]\omega_f$$

$$\left(\frac{5.98\times10^8}{5.32\times10^8}\right)\sqrt{\frac{g}{r}} = \omega_f = 1.12\sqrt{\frac{g}{r}}$$

Now, $\quad |a_r| = \omega_f^2 r = 1.26g = \boxed{12.3\text{ m/s}^2}$

P10.51 $I_i = mr_i^2 = 0.12\text{ kg}(0.4\text{ m})^2 = 1.92\times10^{-2}\text{ kg}\cdot\text{m}^2$

$I_f = mr_f^2 = 0.12\text{ kg}(0.25\text{ m})^2 = 7.5\times10^{-3}\text{ kg}\cdot\text{m}^2$

$\omega_i = \dfrac{v_i}{r_i} = \dfrac{0.8\text{ m/s}}{0.4\text{ m}} = 2\text{ rad/s}$

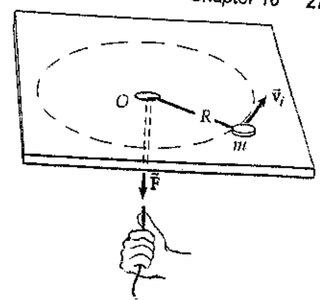

FIG. P10.51

Now, use conservation of angular momentum for the system of the ball

$$\omega_f = \omega_i\left(\frac{I_i}{I_f}\right) = (2\text{ rad/s})\left(\frac{1.92\times10^{-2}\text{ kg}\cdot\text{m}^2}{7.5\times10^{-3}\text{ kg}\cdot\text{m}^2}\right) = 5.12\text{ rad/s}$$

The work done $= \Delta K = \frac{1}{2}I_f\omega_f^2 - \frac{1}{2}I_i\omega_i^2$. Substituting the appropriate values found earlier, we have the work done: $\boxed{5.99\times10^{-2}\text{ J}}$.

Section 10.10 Precessional Motion of Gyroscopes

P10.52 $I = \dfrac{2}{5}MR^2 = \dfrac{2}{5}(5.98\times10^{24}\text{ kg})(6.37\times10^6\text{ m})^2 = 9.71\times10^{37}\text{ kg}\cdot\text{m}^2$

$L = I\omega = 9.71\times10^{37}\text{ kg}\cdot\text{m}^2\left(\dfrac{2\pi\text{ rad}}{86\,400\text{ s}}\right) = 7.06\times10^{33}\text{ kg}\cdot\text{m}^2/\text{s}^2$

$\tau = L\omega_p = (7.06\times10^{33}\text{ kg}\cdot\text{m}^2/\text{s})\left(\dfrac{2\pi\text{ rad}}{2.58\times10^4\text{ yr}}\right)\left(\dfrac{1\text{ yr}}{365.25\text{ d}}\right)\left(\dfrac{1\text{ d}}{86\,400\text{ s}}\right) = \boxed{5.45\times10^{22}\text{ N}\cdot\text{m}}$

Section 10.11 Rolling Motion of Rigid Objects

P10.53 (a) $K_{\text{trans}} = \dfrac{1}{2}mv^2 = \dfrac{1}{2}(10.0\text{ kg})(10.0\text{ m/s})^2 = \boxed{500\text{ J}}$

(b) $K_{\text{rot}} = \dfrac{1}{2}I\omega^2 = \dfrac{1}{2}\left(\dfrac{1}{2}mr^2\right)\left(\dfrac{v^2}{r^2}\right) = \dfrac{1}{4}(10.0\text{ kg})(10.0\text{ m/s})^2 = \boxed{250\text{ J}}$

(c) $K_{\text{total}} = K_{\text{trans}} + K_{\text{rot}} = \boxed{750\text{ J}}$

P10.54 $K = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}\left[m + \frac{I}{R^2}\right]v^2$ where $\omega = \frac{v}{R}$ since no slipping.

Also, $U_i = mgh,\ U_f = 0,$ and $v_i = 0$

Therefore, $\frac{1}{2}\left[m + \frac{I}{R^2}\right]v^2 = mgh$

Thus, $v^2 = \frac{2gh}{\left[1 + \left(\frac{I}{mR^2}\right)\right]}$

For a disk, $I = \frac{1}{2}mR^2$

So $v^2 = \frac{2gh}{1 + \frac{1}{2}}$ or $\boxed{v_{\text{disk}} = \sqrt{\frac{4gh}{3}}}$

For a ring, $I = mR^2$ so $v^2 = \frac{2gh}{2}$ or $\boxed{v_{\text{ring}} = \sqrt{gh}}$

Since $v_{\text{disk}} > v_{\text{ring}}$, $\boxed{\text{the disk}}$ reaches the bottom first.

P10.55 (a) Energy conservation for the system of the ball and the Earth between the horizontal section and top of loop:

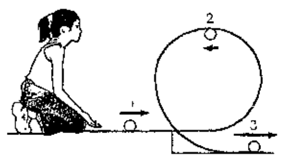

FIG. P10.55

$$\frac{1}{2}mv_2^2 + \frac{1}{2}I\omega_2^2 + mgy_2 = \frac{1}{2}mv_1^2 + \frac{1}{2}I\omega_1^2$$

$$\frac{1}{2}mv_2^2 + \frac{1}{2}\left(\frac{2}{3}mr^2\right)\left(\frac{v_2}{r}\right)^2 + mgy_2 = \frac{1}{2}mv_1^2 + \frac{1}{2}\left(\frac{2}{3}mr^2\right)\left(\frac{v_1}{r}\right)^2$$

$$\frac{5}{6}v_2^2 + gy_2 = \frac{5}{6}v_1^2$$

$$v_2 = \sqrt{v_1^2 - \frac{6}{5}gy_2} = \sqrt{(4.03\ \text{m/s})^2 - \frac{6}{5}(9.80\ \text{m/s}^2)(0.900\ \text{m})} = \boxed{2.38\ \text{m/s}}$$

The centripetal acceleration is $\frac{v_2^2}{r} = \frac{(2.38\ \text{m/s})^2}{0.450\ \text{m}} = 12.6\ \text{m/s}^2 > g$

Thus, the ball must be in contact with the track, with the track pushing downward on it.

(b) $$\frac{1}{2}mv_3^2 + \frac{1}{2}\left(\frac{2}{3}mr^2\right)\left(\frac{v_3}{r}\right)^2 + mgy_3 = \frac{1}{2}mv_1^2 + \frac{1}{2}\left(\frac{2}{3}mr^2\right)\left(\frac{v_1}{r}\right)^2$$

$$v_3 = \sqrt{v_1^2 - \frac{6}{5}gy_3} = \sqrt{(4.03\ \text{m/s})^2 - \frac{6}{5}(9.80\ \text{m/s}^2)(-0.200\ \text{m})} = \boxed{4.31\ \text{m/s}}$$

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(c) $$\frac{1}{2}mv_2^2 + mgy_2 = \frac{1}{2}mv_1^2$$

$$v_2 = \sqrt{v_1^2 - 2gy_2} = \sqrt{(4.03\ \text{m/s})^2 - 2(9.80\ \text{m/s}^2)(0.900\ \text{m})} = \sqrt{-1.40\ \text{m}^2/\text{s}^2}$$

This result is imaginary. In the case where the ball does not roll, the ball starts with less energy than in part (a) and $\boxed{\text{never makes it to the top}}$ of the loop.

P10.56 $\bar{v} = \frac{\Delta x}{\Delta t} = \frac{3.00\ \text{m}}{1.50\ \text{s}} = 2.00\ \text{m/s} = \frac{1}{2}(0 + v_f)$

$v_f = 4.00\ \text{m/s}$ and $\omega_f = \frac{v_f}{r} = \frac{4.00\ \text{m/s}}{(6.38\times10^{-2}\ \text{m})/2} = \frac{8.00}{6.38\times10^{-2}}\ \text{rad/s}$

We ignore internal friction and suppose the can rolls without slipping.

$$(K_{\text{trans}} + K_{\text{rot}} + U_g)_i + \Delta E_{\text{mech}} = (K_{\text{trans}} + K_{\text{rot}} + U_g)_f$$

$$(0 + 0 + mgy_i) + 0 = \left(\frac{1}{2}mv_f^2 + \frac{1}{2}I\omega_f^2 + 0\right)$$

$$0.215\ \text{kg}(9.80\ \text{m/s}^2)[(3.00\ \text{m})\sin 25.0°] = \frac{1}{2}(0.215\ \text{kg})(4.00\ \text{m/s})^2 + \frac{1}{2}I\left(\frac{8.00}{6.38\times10^{-2}}\ \text{rad/s}\right)^2$$

$$2.67\ \text{J} = 1.72\ \text{J} + (7\,860\ \text{s}^{-2})I$$

$$I = \frac{0.951\ \text{kg}\cdot\text{m}^2/\text{s}^2}{7\,860\ \text{s}^{-2}} = \boxed{1.21\times10^{-4}\ \text{kg}\cdot\text{m}^2}$$

The $\boxed{\text{height of the can}}$ is unnecessary data.

Section 10.12 Context Connection—Turning the Spacecraft

P10.57 Angular momentum of the system of the spacecraft and the gyroscope is conserved. The gyroscope and spacecraft turn in opposite directions.

$$0 = I_1\omega_1 + I_2\omega_2: \quad -I_1\omega_1 = I_2\frac{\theta}{t}$$

$$-20\ \text{kg}\cdot\text{m}^2(-100\ \text{rad/s}) = 5\times10^5\ \text{kg}\cdot\text{m}^2\left(\frac{30°}{t}\right)\left(\frac{\pi\ \text{rad}}{180°}\right)$$

$$t = \frac{2.62\times10^5\ \text{s}}{2\,000} = \boxed{131\ \text{s}}$$

Additional Problems

P10.58 The resistive force on each ball is $R = D\rho A v^2$. Here $v = r\omega$, where r is the radius of each ball's path. The resistive torque on each ball is $\tau = rR$, so the total resistive torque on the three ball system is $\tau_{\text{total}} = 3rR$.

The power required to maintain a constant rotation rate is $\mathcal{P} = \tau_{\text{total}}\omega = 3rR\omega$. This required power may be written as

$$\mathcal{P} = \tau_{\text{total}}\omega = 3r\left[D\rho A(r\omega)^2\right]\omega = \left(3r^3 DA\omega^3\right)\rho$$

With $\omega = \dfrac{2\pi \text{ rad}}{1 \text{ rev}}\left(\dfrac{10^3 \text{ rev}}{1 \text{ min}}\right)\left(\dfrac{1 \text{ min}}{60.0 \text{ s}}\right) = \dfrac{1\,000\pi}{30.0}$ rad/s

$$\mathcal{P} = 3(0.100 \text{ m})^3(0.600)\left(4.00\times10^{-4} \text{ m}^2\right)\left(\frac{1\,000\pi}{30.0 \text{ s}}\right)^3 \rho$$

or $\mathcal{P} = \left(0.827 \text{ m}^5/\text{s}^3\right)\rho$, where ρ is the density of the resisting medium.

(a) In air, $\rho = 1.20 \text{ kg/m}^3$,

and $\mathcal{P} = 0.827 \text{ m}^5/\text{s}^3\left(1.20 \text{ kg/m}^3\right) = 0.992 \text{ N}\cdot\text{m/s} = \boxed{0.992 \text{ W}}$

(b) In water, $\rho = 1\,000 \text{ kg/m}^3$ and $\mathcal{P} = \boxed{827 \text{ W}}$.

P10.59 (a) Since only conservative forces act within the system of the rod and the Earth,

$\Delta E = 0$ so $K_f + U_f = K_i + U_i$

$$\frac{1}{2}I\omega^2 + 0 = 0 + Mg\left(\frac{L}{2}\right)$$

where $I = \dfrac{1}{3}ML^2$

Therefore, $\omega = \boxed{\sqrt{\dfrac{3g}{L}}}$

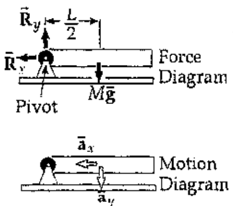

FIG. P10.59

(b) $\sum \tau = I\alpha$, so that in the horizontal orientation,

$$Mg\left(\frac{L}{2}\right) = \frac{ML^2}{3}\alpha$$

$$\alpha = \boxed{\frac{3g}{2L}}$$

(c) $a_x = a_r = -r\omega^2 = -\left(\dfrac{L}{2}\right)\omega^2 = \boxed{-\dfrac{3g}{2}}$ $\quad a_y = -a_t = -r\alpha = -\alpha\left(\dfrac{L}{2}\right) = \boxed{-\dfrac{3g}{4}}$

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(d) Using Newton's second law, we have

$$R_x = Ma_x = \boxed{-\frac{3Mg}{2}}$$

$$R_y - Mg = Ma_y = -\frac{3Mg}{4} \qquad R_y = \boxed{\frac{Mg}{4}}$$

P10.60 τ_f will oppose the torque due to the hanging object:

$$\sum \tau = I\alpha = TR - \tau_f \qquad \tau_f = TR - I\alpha \tag{1}$$

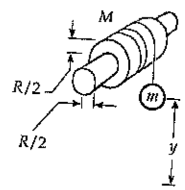


FIG. P10.60

Now find T, I and α in given or known terms and substitute into equation (1).

$$\sum F_y = T - mg = -ma: \qquad T = m(g-a) \tag{2}$$

also $\Delta y = v_i t + \dfrac{at^2}{2}$ $\qquad a = \dfrac{2y}{t^2}$ (3)

and $\qquad \alpha = \dfrac{a}{R} = \dfrac{2y}{Rt^2}$: (4)

$$I = \frac{1}{2}M\left[R^2 + \left(\frac{R}{2}\right)^2\right] = \frac{5}{8}MR^2 \tag{5}$$

Substituting (2), (3), (4), and (5) into (1),

we find $\tau_f = m\left(g - \dfrac{2y}{t^2}\right)R - \dfrac{5}{8}\dfrac{MR^2(2y)}{Rt^2} = \boxed{R\left[m\left(g - \dfrac{2y}{t^2}\right) - \dfrac{5}{4}\dfrac{My}{t^2}\right]}$

P10.61 (a) $W = \Delta K + \Delta U$

$W = K_f - K_i + U_f - U_i$

$0 = \dfrac{1}{2}mv^2 + \dfrac{1}{2}I\omega^2 - mgd\sin\theta - \dfrac{1}{2}kd^2$

$\dfrac{1}{2}\omega^2\left(I + mR^2\right) = mgd\sin\theta + \dfrac{1}{2}kd^2$

$\omega = \boxed{\sqrt{\dfrac{2mgd\sin\theta + kd^2}{I + mR^2}}}$

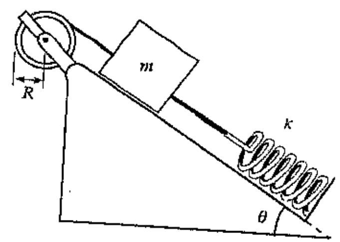


FIG. P10.61

(b) $\omega = \sqrt{\dfrac{2(0.500 \text{ kg})\left(9.80 \text{ m/s}^2\right)(0.200 \text{ m})(\sin 37.0°) + 50.0 \text{ N/m}(0.200 \text{ m})^2}{1.00 \text{ kg}\cdot\text{m}^2 + 0.500 \text{ kg}(0.300 \text{ m})^2}}$

$\omega = \sqrt{\dfrac{1.18 + 2.00}{1.05}} = \sqrt{3.04} = \boxed{1.74 \text{ rad/s}}$

P10.62 For m_1,

$\sum F_y = ma_y$: $+n - m_1 g = 0$

$n_1 = m_1 g = 19.6\text{ N}$

$f_{k1} = \mu_k n_1 = 7.06\text{ N}$

$\sum F_x = ma_x$: $-7.06\text{ N} + T_1 = (2.00\text{ kg})a$ (1)

For the pulley,

$\sum \tau = I\alpha$: $-T_1 R + T_2 R = \frac{1}{2}MR^2\left(\frac{a}{R}\right)$

$-T_1 + T_2 = \frac{1}{2}(10.0\text{ kg})a$

$-T_1 + T_2 = (5.00\text{ kg})a$ (2)

For m_2, $+n_2 - m_2 g\cos\theta = 0$

$n_2 = 6.00\text{ kg}(9.80\text{ m/s}^2)(\cos 30.0°)$
$= 50.9\text{ N}$

$f_{k2} = \mu_k n_2$
$= 18.3\text{ N}$: $-18.3\text{ N} - T_2 + m_2 g\sin\theta = m_2 a$

$-18.3\text{ N} - T_2 + 29.4\text{ N} = (6.00\text{ kg})a$ (3)

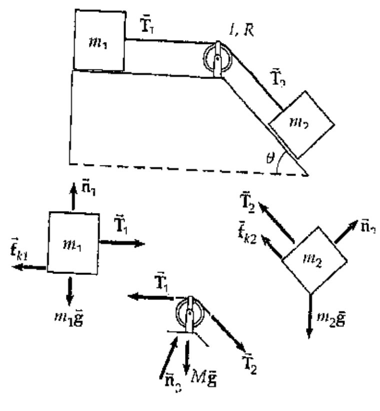


FIG. P10.62

(a) Add equations (1), (2), and (3):

$$-7.06\text{ N} - 18.3\text{ N} + 29.4\text{ N} = (13.0\text{ kg})a$$

$$a = \frac{4.01\text{ N}}{13.0\text{ kg}} = \boxed{0.309\text{ m/s}^2}$$

(b) $T_1 = 2.00\text{ kg}(0.309\text{ m/s}^2) + 7.06\text{ N} = \boxed{7.67\text{ N}}$

$T_2 = 7.67\text{ N} + 5.00\text{ kg}(0.309\text{ m/s}^2) = \boxed{9.22\text{ N}}$

P10.63 For the board just starting to move.

$\sum \tau = I\alpha$: $mg\left(\frac{\ell}{2}\right)\cos\theta = \left(\frac{1}{3}m\ell^2\right)\alpha$

$\alpha = \frac{3}{2}\left(\frac{g}{\ell}\right)\cos\theta$

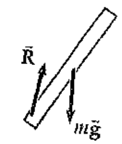


FIG. P10.63

The tangential acceleration of the end is $a_t = \ell\alpha = \frac{3}{2}g\cos\theta$

The vertical component is $a_y = a_t\cos\theta = \frac{3}{2}g\cos^2\theta$

If this is greater than g, the board will pull ahead of the ball falling:

(a) $\frac{3}{2}g\cos^2\theta \geq g$ gives $\cos^2\theta \geq \frac{2}{3}$ so $\cos\theta \geq \sqrt{\frac{2}{3}}$ and $\boxed{\theta \leq 35.3°}$

(b) When $\theta = 35.3°$, the cup will land underneath the release-point of the ball if $r_c = \ell\cos\theta$

When $\ell = 1.00$ m, and $\theta = 35.3°$ $r_c = 1.00\text{ m}\sqrt{\frac{2}{3}} = 0.816\text{ m}$

so the cup should be $(1.00\text{ m} - 0.816\text{ m}) = \boxed{0.184\text{ m from the moving end}}$

P10.64 Consider the total weight of each hand to act at the center of gravity (mid-point) of that hand. Then the total torque (taking CCW as positive) of these hands about the center of the clock is given by

$$\tau = -m_h g\left(\frac{L_h}{2}\right)\sin\theta_h - m_m g\left(\frac{L_m}{2}\right)\sin\theta_m = -\frac{g}{2}\left(m_h L_h\sin\theta_h + m_m L_m\sin\theta_m\right)$$

If we take $t = 0$ at 12 o'clock, then the angular positions of the hands at time t are

$\theta_h = \omega_h t$,

where $\omega_h = \frac{\pi}{6}$ rad/h

and $\theta_m = \omega_m t$,

where $\omega_m = 2\pi$ rad/h

Therefore, $\tau = -4.90\text{ m/s}^2\left[60.0\text{ kg}(2.70\text{ m})\sin\left(\frac{\pi t}{6}\right) + 100\text{ kg}(4.50\text{ m})\sin 2\pi t\right]$

or $\tau = -794\text{ N}\cdot\text{m}\left[\sin\left(\frac{\pi t}{6}\right) + 2.78\sin 2\pi t\right]$, where t is in hours.

continued on next page

(a) (i) At 3:00, $t = 3.00$ h,

so $$\tau = -794\text{ N}\cdot\text{m}\left[\sin\left(\frac{\pi}{2}\right) + 2.78\sin 6\pi\right] = \boxed{-794\text{ N}\cdot\text{m}}$$

(ii) At 5:15, $t = 5\text{ h} + \frac{15}{60}\text{ h} = 5.25\text{ h}$, and substitution gives:

$$\tau = \boxed{-2\,510\text{ N}\cdot\text{m}}$$

(iii) At 6:00, $\tau = \boxed{0\text{ N}\cdot\text{m}}$

(iv) At 8:20, $\tau = \boxed{-1\,160\text{ N}\cdot\text{m}}$

(v) At 9:45, $\tau = \boxed{-2\,940\text{ N}\cdot\text{m}}$

(b) The total torque is zero at those times when

$$\sin\left(\frac{\pi t}{6}\right) + 2.78\sin 2\pi t = 0$$

We proceed numerically, to find 0, 0.515 295 5, ..., corresponding to the times

12:00:00	12:30:55	12:58:19	1:32:31	1:57:01
2:33:25	2:56:29	3:33:22	3:56:55	4:32:24
4:58:14	5:30:52	6:00:00	6:29:08	7:01:46
7:27:36	8:03:05	8:26:38	9:03:31	9:26:35
10:02:59	10:27:29	11:01:41	11:29:05	

P10.65 $\sum F = T - Mg = -Ma$: $\sum \tau = TR = I\alpha = \frac{1}{2}MR^2\left(\frac{a}{R}\right)$

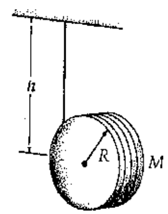


FIG. P10.65

(a) Combining the above two equations we find $T = M(g - a)$ and $a = \frac{2T}{M}$ thus $T = \boxed{\frac{Mg}{3}}$.

(b) $a = \frac{2T}{M} = \frac{2}{M}\left(\frac{Mg}{3}\right) = \boxed{\frac{2}{3}g}$

(c) $v_f^2 = v_i^2 + 2a(x_f - x_i)$ $\quad v_f^2 = 0 + 2\left(\frac{2}{3}g\right)(h - 0)$

$$v_f = \boxed{\sqrt{\frac{4gh}{3}}}$$

continued on next page

For comparison, from conservation of energy for the system of the disk and the Earth we have

$U_{gi} + K_{\text{rot }i} + K_{\text{trans }i} = U_{gf} + K_{\text{rot }f} + K_{\text{trans }f}$: $\quad Mgh + 0 + 0 = 0 + \frac{1}{2}\left(\frac{1}{2}MR^2\right)\left(\frac{v_f}{R}\right)^2 + \frac{1}{2}Mv_f^2$

$$v_f = \sqrt{\frac{4gh}{3}}$$

***P10.66** Model the stove as a uniform 68 kg box. Its center of mass is at its geometric center, $\frac{28}{2} = 14$ inches behind its feet at the front corners. Assume that the light oven door opens to be horizontal and that a person stands on its outer end, $46.375 - 28 = 18.375$ inches in front of the front feet. We find the weight F_g of a person standing on the oven door with the stove balanced on its front feet in equilibrium: $\sum \tau = 0$

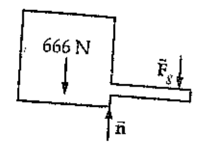


FIG. P10.66

$$(68\text{ kg})(9.8\text{ m/s}^2)(14\text{ in.}) + n(0) - F_g(18.375\text{ in.}) = 0$$
$$F_g = \boxed{508\text{ N}}$$

If the weight of the person is greater than this, the stove can tip forward. This weight corresponds to mass 51.8 kg, so the person could be a child. If the oven door is heavy (compared to the backsplash) or if the front feet are significantly far behind the front corners, the maximum weight will be significantly less than 508 N.

P10.67 (a) As the bicycle frame moves forward at speed v, the center of each wheel moves forward at the same speed and the wheels turn at angular speed $\omega = \frac{v}{R}$. The total kinetic energy of the bicycle is $K = K_{\text{trans}} + K_{\text{rot}}$ or

$$K = \frac{1}{2}(m_{\text{frame}} + 2m_{\text{wheel}})v^2 + 2\left(\frac{1}{2}I_{\text{wheel}}\omega^2\right) = \frac{1}{2}(m_{\text{frame}} + 2m_{\text{wheel}})v^2 + \left(\frac{1}{2}m_{\text{wheel}}R^2\right)\left(\frac{v^2}{R^2}\right).$$

This yields

$$K = \frac{1}{2}(m_{\text{frame}} + 3m_{\text{wheel}})v^2 = \frac{1}{2}[8.44\text{ kg} + 3(0.820\text{ kg})](3.35\text{ m/s})^2 = \boxed{61.2\text{ J}}.$$

(b) As the block moves forward with speed v, the top of each trunk moves forward at the same speed and the center of each trunk moves forward at speed $\frac{v}{2}$. The angular speed of each roller is $\omega = \frac{v}{2R}$. As in part (a), we have one object undergoing pure translation and two identical objects rolling without slipping. The total kinetic energy of the system of the stone and the trees is

$$K = K_{\text{trans}} + K_{\text{rot}}$$

continued on next page

or

$$K=\frac{1}{2}m_{stone}v^2+2\frac{1}{2}m_{tree}\left(\frac{v}{2}\right)^2+2\left(\frac{1}{2}I_{tree}\omega^2\right)=\frac{1}{2}\left(m_{stone}+\frac{1}{2}m_{tree}\right)v^2+\left(\frac{1}{2}m_{tree}R^2\right)\left(\frac{v^2}{4R^2}\right)$$

This gives

$$K=\frac{1}{2}\left(m_{stone}+\frac{3}{4}m_{tree}\right)v^2=\frac{1}{2}\left[844\text{ kg}+0.75(82.0\text{ kg})\right](0.335\text{ m/s})^2=\boxed{50.8\text{ J}}$$

*P10.68 (a) $(K+U_s)_A=(K+U_s)_B$

$$0+mgy_A=\frac{1}{2}mv_B^2+0$$

$$v_B=\sqrt{2gy_A}=\sqrt{2(9.8\text{ m/s}^2)6.30\text{ m}}=\boxed{11.1\text{ m/s}}$$

(b) $L=mvr=76\text{ kg }11.1\text{ m/s }6.3\text{ m}=\boxed{5.32\times10^3\text{ kg}\cdot\text{m}^2/\text{s}}$ toward you along the axis of the channel.

(c) The wheels on his skateboard prevent any tangential force from acting on him. Then no torque about the axis of the channel acts on him and his angular momentum is constant. His legs convert chemical into mechanical energy. They do work to increase his kinetic energy. The normal force acts forward on his body on its rising trajectory, to increase his linear momentum.

(d) $L=mvr$ $\qquad v=\dfrac{5.32\times10^3\text{ kg}\cdot\text{m}^2/\text{s}}{76\text{ kg }5.85\text{ m}}=\boxed{12.0\text{ m/s}}$

(e) For the skateboarder we may write conservation of energy as $(K+U_g+U_{chem})_B=(K+U_g)_C$ where U_{chem} is excess chemical energy converted by his muscles into mechanical energy between points B and C. This energy converted can also be counted as work, in

$$(K+U_g)_B+W=(K+U_g)_C$$

$$\frac{1}{2}76\text{ kg}(11.1\text{ m/s})^2+0+W=\frac{1}{2}76\text{ kg}(12.0\text{ m/s})^2+76\text{ kg }9.8\text{ m/s}^2\text{ }0.45\text{ m}$$

$$W=5.44\text{ kJ}-4.69\text{ kJ}+335\text{ J}=\boxed{1.08\text{ kJ}}$$

(f) $(K+U_g)_C=(K+U_g)_D$

$$\frac{1}{2}76\text{ kg}(12.0\text{ m/s})^2+0=\frac{1}{2}76\text{ kg}v_D^2+76\text{ kg }9.8\text{ m/s}^2\text{ }5.85\text{ m}$$

$$v_D=\boxed{5.34\text{ m/s}}$$

continued on next page

(g) Let point E be the apex of his flight:

$$(K+U_g)_D=(K+U_g)_E$$

$$\frac{1}{2}76\text{ kg}(5.34\text{ m/s})^2+0=0+76\text{ kg}(9.8\text{ m/s}^2)(y_E-y_D)$$

$$(y_E-y_D)=\boxed{1.46\text{ m}}$$

(h) For the motion between takeoff and touchdown

$$y_f=y_i+v_{yi}t+\frac{1}{2}a_yt^2$$

$$-2.34\text{ m}=0+5.34\text{ m/s}t-4.9\text{ m/s}^2t^2$$

$$t=\frac{-5.34\pm\sqrt{5.34^2+4(4.9)(2.34)}}{-9.8}=\boxed{1.43\text{ s}}$$

(i) This solution is more accurate. In chapter 7 we modeled the normal force as constant while the skateboarder stands up. Really it increases as the process goes on.

P10.69 (a) $L_i=2\left[Mv\left(\frac{d}{2}\right)\right]=\boxed{Mvd}$

(b) $K=2\left(\frac{1}{2}Mv^2\right)=\boxed{Mv^2}$

(c) $L_f=L_i=\boxed{Mvd}$

(d) $v_f=\dfrac{L_f}{2Mr_f}=\dfrac{Mvd}{2M\left(\frac{d}{4}\right)}=\boxed{2v}$

(e) $K_f=2\left(\frac{1}{2}Mv_f^2\right)=M(2v)^2=\boxed{4Mv^2}$

(f) $W=K_f-K_i=\boxed{3Mv^2}$

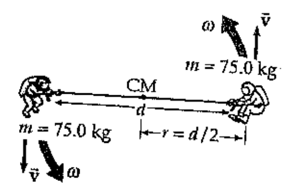


FIG. P10.69

*P10.70 Choosing the origin at $\vec{R}$

(1) $\sum F_x = +R\sin 15.0° - T\sin\theta = 0$

(2) $\sum F_y = 700 - R\cos 15.0° + T\cos\theta = 0$

(3) $\sum \tau = -700\cos\theta(0.180) + T(0.070\,0) = 0$

Solve the equations for θ

from (3), $T = 1\,800\cos\theta$ from (1), $R = \dfrac{1\,800\sin\theta\cos\theta}{\sin 15.0°}$

Then (2) gives $700 - \dfrac{1\,800\sin\theta\cos\theta\cos 15.0°}{\sin 15.0°} + 1\,800\cos^2\theta = 0$

or $\cos^2\theta + 0.388\,9 - 3.732\sin\theta\cos\theta = 0$

Squaring, $\cos^4\theta - 0.880\,9\cos^2\theta + 0.010\,13 = 0$

Let $u = \cos^2\theta$. Then using the quadratic formula

$u = 0.011\,65$ or $0.869\,3$

Only the second root is physically possible.

$\therefore \theta = \cos^{-1}\sqrt{0.869\,3} = \boxed{21.2°}$

$\therefore T = \boxed{1.68\times 10^3\ \text{N}}$ and $R = \boxed{2.34\times 10^3\ \text{N}}$

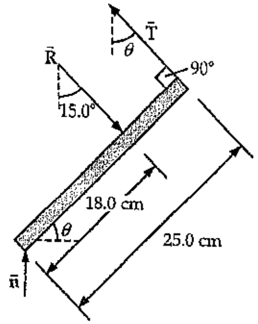


FIG. P10.70

P10.71 Choosing torques about $\vec{R}$, with $\sum\tau = 0$

$$-\frac{L}{2}(350\ \text{N}) + (T\sin 12.0°)\left(\frac{2L}{3}\right) - (200\ \text{N})L = 0.$$

From which, $T = \boxed{2.71\ \text{kN}}$.

Let R_y = compression force along spine, and from $\sum F_x = 0$

$$R_y = T_x = T\cos 12.0° = \boxed{2.65\ \text{kN}}.$$

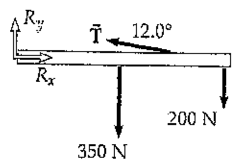


FIG. P10.71

P10.72 (a) Consider the system to consist of the wad of clay and the cylinder. No external forces acting on this system have a torque about the center of the cylinder. Thus, angular momentum of the system is conserved about the axis of the cylinder.

$L_f = L_i$: $I\omega = mv_i d$

or $\left[\frac{1}{2}MR^2 + mR^2\right]\omega = mv_i d$

Thus, $\omega = \boxed{\dfrac{2mv_i d}{(M+2m)R^2}}$.

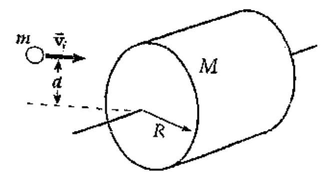


FIG. P10.72

(b) $\boxed{\text{No}}$. Some mechanical energy changes to internal energy in this perfectly inelastic collision.

P10.73 (a) Locate the origin at the bottom left corner of the cabinet and let x = distance between the *resultant normal force* and the *front of the cabinet*. Then we have

$\sum F_x = 200\cos 37.0° - \mu n = 0$ (1)

$\sum F_y = 200\sin 37.0° + n - 400 = 0$ (2)

$\sum \tau = n(0.600 - x) - 400(0.300) + 200\sin 37.0°(0.600)$

$-200\cos 37.0°(0.400) = 0$ (3)

From (2), $n = 400 - 200\sin 37.0° = 280\ \text{N}$

From (3), $x = \dfrac{72.2 - 120 + 280(0.600) - 64.0}{280}$

$x = \boxed{20.1\ \text{cm}}$ to the left of the front edge

From (1), $\mu_k = \dfrac{200\cos 37.0°}{280} = \boxed{0.571}$

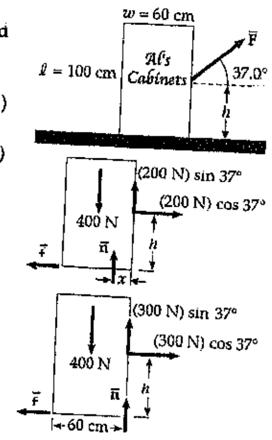


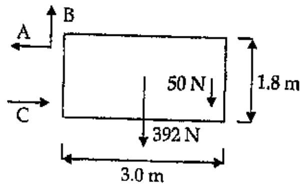
FIG. P10.73

(b) In this case, locate the origin $x = 0$ at the bottom right corner of the cabinet. Since the cabinet is about to tip, we can use $\sum\tau = 0$ to find h:

$\sum\tau = 400(0.300) - (300\cos 37.0°)h = 0$ $h = \dfrac{120}{300\cos 37.0°} = \boxed{0.501\ \text{m}}$

***P10.74** The 392 N is the weight of the uniform gate, which is 3 m wide. The hinges are 1.8 m apart. They exert horizontal forces A and C. Only one hinge exerts a vertical force. We assume it is the upper hinge.

(a) Free body diagram:



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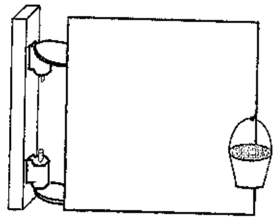

FIG. P10.74

A uniform 40.0-kg farm gate, 3.00 m wide and 1.80 m high, supports a 50.0-N bucket of grain hanging from its latch as shown. The gate is supported by two hinges. Find the force each hinge exerts on the gate.

(b) From the torque equation,

$$C=\frac{738\text{ N}\cdot\text{m}}{1.8\text{ m}}=410\text{ N}.$$

Then $A=410$ N. Also $B=442$ N.

The upper hinge exerts 410 N to the left and 442 N up.
The lower hinge exerts 410 N to the right.

P10.75 From geometry, observe that

$$\cos\theta=\frac{1}{4} \quad \text{and} \quad \theta=75.5°$$

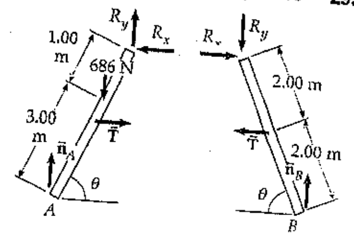

FIG. P10.75

For the left half of the ladder, we have

$$\sum F_x=T-R_x=0 \quad (1)$$

$$\sum F_y=R_y+n_A-686\text{ N}=0 \quad (2)$$

$$\sum\tau_{top}=686\text{ N}(1.00\cos 75.5°)+T(2.00\sin 75.5°)$$
$$-n_A(4.00\cos 75.5°)=0 \quad (3)$$

For the right half of the ladder we have

$$\sum F_x=R_x-T=0$$

$$\sum F_y=n_B-R_y=0 \quad (4)$$

$$\sum\tau_{top}=n_B(4.00\cos 75.5°)-T(2.00\sin 75.5°)=0 \quad (5)$$

Solving equations 1 through 5 simultaneously yields:

(a) $\boxed{T=133\text{ N}}$

(b) $\boxed{n_A=429\text{ N}}$ and $\boxed{n_B=257\text{ N}}$

(c) $\boxed{R_x=133\text{ N}}$ and $\boxed{R_y=257\text{ N}}$

The force exerted by the left half of the ladder on the right half is to the right and downward.

***P10.76** (a) $\Delta K_{rot}+\Delta K_{trans}+\Delta U=0$

Note that initially the center of mass of the sphere is a distance $h+r$ above the bottom of the loop; and as the mass reaches the top of the loop, this distance above the reference level is $2R-r$. The conservation of energy requirement gives

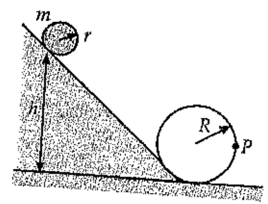

FIG. P10.76

$$mg(h+r)=mg(2R-r)+\frac{1}{2}mv^2+\frac{1}{2}I\omega^2$$

For the sphere $I=\frac{2}{5}mr^2$ and $v=r\omega$ so that the expression becomes

continued on next page

$$gh + 2gr = 2gR + \frac{7}{10}v^2 \qquad (1)$$

Note that $h = h_{\min}$ when the speed of the sphere at the top of the loop satisfies the condition

$$\sum F = mg = \frac{mv^2}{(R-r)} \text{ or } v^2 = g(R-r)$$

Substituting this into Equation (1) gives

$$h_{\min} = 2(R-r) + 0.700(R-r) \text{ or } \boxed{h_{\min} = 2.70(R-r) = 2.70R}$$

(b) When the sphere is initially at $h = 3R$ and finally at point P, the conservation of energy equation gives

$$mg(3R+r) = mgR + \frac{1}{2}mv^2 + \frac{1}{5}mv^2, \text{ or } v^2 = \frac{10}{7}(2R+r)g$$

Turning clockwise as it rolls without slipping past point P, the sphere is slowing down with counterclockwise angular acceleration caused by the torque of an upward force f of static friction. We have $\sum F_y = ma_y$ and $\sum \tau = I\alpha$ becoming $f - mg = -m\alpha r$ and $fr = \left(\frac{2}{5}\right)mr^2\alpha$.

Eliminating f by substitution yields $\alpha = \frac{5g}{7r}$ so that $\sum F_y = \boxed{\frac{-5mg}{7}}$

$$\sum F_x = -n = -\frac{mv^2}{R-r} = -\frac{\left(\frac{10}{7}\right)(2R+r)}{R-r}mg = \boxed{\frac{-20mg}{7}} \quad (\text{since } R >> r)$$

P10.77 When it is on the verge of slipping, the cylinder is in equilibrium.

$\sum F_x = 0$: $\quad f_1 = n_2 = \mu_s n_1 \quad$ and $\quad f_2 = \mu_s n_2$

$\sum F_y = 0$: $\quad P + n_1 + f_2 = F_g$

$\sum \tau = 0$: $\quad P = f_1 + f_2$

As P grows so do f_1 and f_2

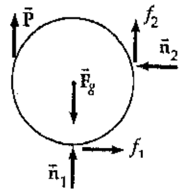

FIG. P10.77

Therefore, since $\mu_s = \frac{1}{2}$, $\quad f_1 = \frac{n_1}{2} \quad$ and $\quad f_2 = \frac{n_2}{2} = \frac{n_1}{4}$

then $\quad P + n_1 + \frac{n_1}{4} = F_g \quad (1) \quad$ and $\quad P = \frac{n_1}{2} + \frac{n_1}{4} = \frac{3}{4}n_1 \quad (2)$

So $\quad P + \frac{5}{4}n_1 = F_g \quad$ becomes $\quad P + \frac{5}{4}\left(\frac{4}{3}P\right) = F_g \quad$ or $\quad \frac{8}{3}P = F_g$

Therefore, $P = \boxed{\frac{3}{8}F_g}$.

ANSWERS TO EVEN PROBLEMS

P10.2 (a) 8.22×10^2 rad/s^2; (b) 4.21×10^4 rad

P10.4 -226 rad/s^2

P10.6 13.7 rad/s^2

P10.8 $\sim 10^7$ rev/yr

P10.10 (a) 8.00 rad/s;
(b) 8.00 m/s, 64.1 m/s^2 inward at 3.58° ahead of the radius;
(c) 9.00 rad

P10.12 (a) 56.5 rad/s; (b) 22.4 rad/s;
(c) -7.63×10^{-3} rad/s^2; (d) 1.77×10^3 rad;
(e) 5.81×10^3 m

P10.14 (a) 92.0 kg·m^2, 184 J;
(b) 6.00 m/s, 4.00 m/s, 8.00 m/s, 184 J

P10.16 1.03×10^{-3} J.

P10.18 see the solution

P10.20 168 N·m clockwise

P10.22 $-7.00\hat{i} + 16.0\hat{j} - 10.0\hat{k}$

P10.24 (a) 168°; (b) principal value 11.9°;
(c) the first method is unambiguous

P10.26 1.09 m

P10.28 see the solution, 0.643 m

P10.30 (a) $\left[\frac{m_1 g}{2} + \frac{x m_2 g}{L}\right]\cot\theta$, $(m_1 + m_2)g$;
(b) $\frac{\left(\frac{m_1}{2} + \frac{m_2 d}{L}\right)\cot\theta}{m_1 + m_2}$

P10.32 $F_{By} = 0$, $F_{Ax} = 6.47 \times 10^5$ N left,
$F_{Bx} = 6.47 \times 10^5$ N right,
$F_{Ay} = 1.27 \times 10^3$ N up

P10.34 0.312

P10.36 (a) 1.59 m/s; (b) 53.1 rad/s

P10.38 $\frac{3g}{2L}$; $-\frac{3}{2}g\hat{j}$

P10.40 (a) $\left(-9.03 \times 10^9 \text{ kg}\cdot\text{m}^2/\text{s}\right)\hat{j}$; (b) no;
(c) zero

P10.42 $L = 1.20$ kg·m^2/s perpendicularly into the clock face

P10.44 (a) 1.57×10^8 kg·m^2/s; (b) 6.26×10^3 s;
(c) see the solution

P10.46 7.14 rev/min

P10.48 (a) 1.91 rad/s; (b) 2.53 J, 6.44 J

P10.50 12.3 m/s^2

P10.52 5.45×10^{22} N·m

P10.54 $v_{\text{disk}} = \sqrt{\frac{4gh}{3}}$, $v_{\text{ring}} = \sqrt{gh}$, the disk

P10.56 1.21×10^{-4} kg·m^2, height of the can

P10.58 (a) 0.992 W; (b) 827 W

P10.60 see the solution

P10.62 (a) 0.309 m/s^2; (b) 7.67 N, 9.22 N

P10.64 (a) -794 N·m; $-2\,510$ N·m; 0;
$-1\,160$ N·m; $-2\,940$ N·m;
(b) At the following times: 12:00:00
12:30:55 12:58:19 1:32:31 1:57:01 2:33:25
2:56:29 3:33:22 3:56:55 4:32:24 4:58:14
5:30:52 6:00:00 6:29:08 7:01:46 7:27:36
8:03:05 8:26:38 9:03:31 9:26:35 10:02:59
10:27:29 11:01:41 11:29:05

P10.66 The weight must be 508 N or more. The person could be a child, see the solution