ANSWERS TO EVEN PROBLEMS

P13.2 see the solution

P13.4 0.800 m/s

P13.6 2.00 cm, 2.98 m, 0.576 Hz, 1.72 m/s

P13.8 ±6.67 cm

P13.10 (a) −1.51 m/s, 0 m/s^2; (b) 16.0 m, 0.500 s, 32.0 m/s

P13.12 see the solution

P13.14 1.64 m/s^2

P13.16 (a) $v=\left(30.4\ \frac{\text{m/s}}{\sqrt{\text{kg}}}\right)\sqrt{m}$; (b) 3.89 kg

P13.18 (a) zero; (b) 0.300 m

P13.20 (a) $y=\left(7.50\times10^{-2}\right)\sin(4.19x-314t)$; (b) 625 W

P13.22 $\sqrt{2}\%$

P13.24 0.196 s

P13.26 5.67 mm

P13.28 (a) 0.625 mm; (b) 1.50 mm to 75.0 μm

P13.30 (a) 2.00 μm, 40.0 cm, 54.6 m/s; (b) −0.433 μm; (c) 1.72 mm/s

P13.32 0.103 Pa

P13.34 (a) 0.021 7 m/s; (b) 2 000 028.9 Hz; (c) 2 000 057.8 Hz

P13.36 439 Hz and 441 Hz

P13.38 (a) 325 m/s; (b) 29.5 m/s

P13.40 (a) longitudinal; (b) 665 s

P13.42 ~1 min

P13.44 (a) $2Mg$; (b) $L_0+\frac{2Mg}{k}$; (c) $\sqrt{\frac{2Mg}{m}\left(L_0+\frac{2Mg}{k}\right)}$

P13.46 (a) $v=\sqrt{\frac{kL}{\mu}}$; (b) 31.6 m/s

P13.48 see the solution

P13.50 130 m/s, 1 730 m

P13.52 7.82 m

P13.54 (a) 0.515/min; (b) 0.614/min

P13.56 (a) $\frac{2\frac{u}{v}}{1-\frac{u^2}{v^2}}f$; (b) 85.9 Hz

P13.58 (a) see the solution; (b) 0.343 m; (c) 0.303 m; (d) 0.383 m; (e) 1.03 kHz

P13.60 (a) see the solution; (b) 531 Hz

Chapter 14

Superposition and Standing Waves

CHAPTER OUTLINE

ANSWERS TO QUESTIONS

Q14.1 No. Waves with other waveforms are also trains of disturbance that add together when waves from different sources move through the same medium at the same time.

Q14.2 No. A wave is not a solid object, but a chain of disturbance. As described by the principle of superposition, the waves move through each other.

Q14.3 They can, wherever the two waves are nearly enough in phase that their displacements will add to create a total displacement greater than the amplitude of either of the two original waves.

When two one-dimensional sinusoidal waves of the same amplitude interfere, this condition is satisfied whenever the absolute value of the phase difference between the two waves is less than 120°.

Q14.4 When the two tubes together are not an efficient transmitter of sound from source to receiver, they are an efficient reflector. The incoming sound is reflected back to the source. The waves reflected by the two tubes separately at the junction interfere constructively.

Q14.5 No. The total energy of the pair of waves remains the same. Energy missing from zones of destructive interference appears in zones of constructive interference.

Q14.6 Damping, and non-linear effects in the vibration turn the energy of vibration into internal energy.

Q14.7 The air in the shower stall can vibrate in standing wave patterns to intensify those frequencies in your voice which correspond to its free vibrations. The hard walls of the bathroom reflect sound very well to make your voice more intense at all frequencies, and giving the room a longer reverberation time. The reverberant sound may help you to stay on key.

Q14.8 The trombone slide and trumpet valves change the length of the air column inside the instrument, to change its resonant frequencies.

Q14.9 In a classical guitar, vibrations of the strings are transferred to the wooden body through the bridge. Because of its large area, the guitar body is a much more efficient radiator of sound than an individual guitar string. Thus, energy associated with the vibration is transferred to the air relatively rapidly by the guitar body, resulting in a more intense sound.

Q14.10 The vibration of the air must have zero amplitude at the closed end. For air in a pipe closed at one end, the diagrams show how resonance vibrations have NA distances that are odd integer submultiples of the NA distance in the fundamental vibration. If the pipe is open, resonance vibrations have NA distances that are all integer submultiples of the NA distance in the fundamental.

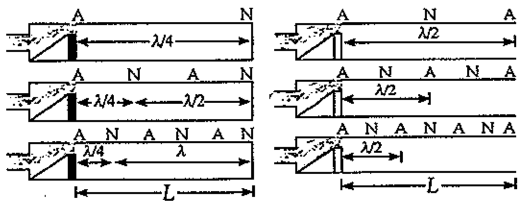

FIG. Q14.10

Q14.11 The bow string is pulled away from equilibrium and released, similar to the way that a guitar string is pulled and released when it is plucked. Thus, standing waves will be excited in the bow string. If the arrow leaves from the exact center of the string, then a series of odd harmonics will be excited. Even harmonies will not be excited because they have a node at the point where the string exhibits its maximum displacement.

Q14.12 What is needed is a tuning fork—or other pure-tone generator—of the desired frequency. Strike the tuning fork and pluck the corresponding string on the piano at the same time. If they are precisely in tune, you will hear a single pitch with no amplitude modulation. If the two pitches are a bit off, you will hear beats. As they vibrate, retune the piano string until the beat frequency goes to zero.

Q14.13 Beats. The propellers are rotating at slightly different frequencies.

Q14.14 Walking makes the person's hand vibrate a little. If the frequency of this motion is equal to the natural frequency of coffee sloshing from side to side in the cup, then a large–amplitude vibration of the coffee will build up in resonance. To get off resonance and back to the normal case of a small-amplitude disturbance producing a small–amplitude result, the person can walk faster, walk slower, or get a larger or smaller cup. Alternatively, even at resonance he can reduce the amplitude by adding damping, as by stirring high–fiber quick–cooking oatmeal into the hot coffee.

Q14.15 Stick a bit of chewing gum to one tine of the second fork. If the beat frequency is then faster than 4 beats per second, the second has a lower frequency than the standard fork. If the beats have slowed down, the second fork has a higher frequency than the standard. Remove the gum, clean the fork, add or subtract 4 Hz according to what you found, and your answer will be the frequency of the second fork.

Q14.16 Instead of just radiating sound very softly into the surrounding air, the tuning fork makes the chalkboard vibrate. With its large area this stiff sounding board radiates sound into the air with higher power. So it drains away the fork's energy of vibration faster and the fork stops vibrating sooner. This process exemplifies conservation of energy, as the energy of vibration of the fork is transferred through the blackboard into energy of vibration of the air.

SOLUTIONS TO PROBLEMS

Section 14.1 The Principle of Superposition

P14.1 $y = y_1 + y_2 = 3.00\cos(4.00x - 1.60t) + 4.00\sin(5.0x - 2.00t)$ evaluated at the given x values.

(a) $x = 1.00,\ t = 1.00$ $\quad y = 3.00\cos(2.40 \text{ rad}) + 4.00\sin(+3.00 \text{ rad}) = \boxed{-1.65 \text{ cm}}$

(b) $x = 1.00,\ t = 0.500$ $\quad y = 3.00\cos(+3.20 \text{ rad}) + 4.00\sin(+4.00 \text{ rad}) = \boxed{-6.02 \text{ cm}}$

(c) $x = 0.500,\ t = 0$ $\quad y = 3.00\cos(+2.00 \text{ rad}) + 4.00\sin(+2.50 \text{ rad}) = \boxed{1.15 \text{ cm}}$

P14.2

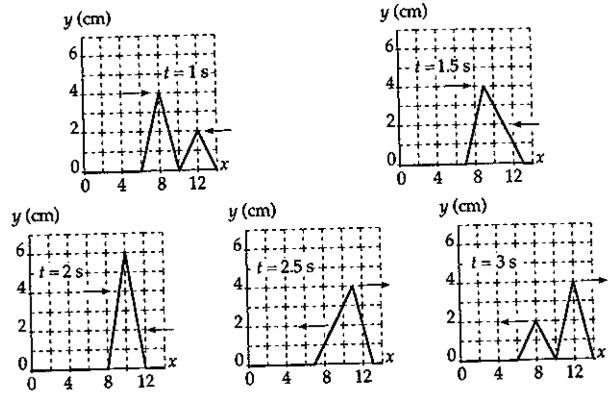

FIG. P14.2

P14.3 (a) $y_1 = f(x - vt)$, so wave 1 travels in the $\boxed{+x \text{ direction}}$

$y_2 = f(x + vt)$, so wave 2 travels in the $\boxed{-x \text{ direction}}$

(b) To cancel, $y_1 + y_2 = 0$: $\quad \dfrac{5}{(3x-4t)^2+2} = \dfrac{+5}{(3x+4t-6)^2+2}$

$$(3x-4t)^2 = (3x+4t-6)^2$$
$$3x - 4t = \pm(3x+4t-6)$$

for the positive root, $\quad 8t = 6 \quad \boxed{t = 0.750 \text{ s}}$

(at $t = 0.750$ s, the waves cancel everywhere)

(c) for the negative root, $\quad 6x = 6 \quad \boxed{x = 1.00 \text{ m}}$

(at $x = 1.00$ m, the waves cancel always)

Section 14.2 Interference of Waves

P14.4 Suppose the waves are sinusoidal.

The sum is $(4.00\text{ cm})\sin(kx-\omega t)+(4.00\text{ cm})\sin(kx-\omega t+90.0°)$

$$2(4.00\text{ cm})\sin(kx-\omega t+45.0°)\cos 45.0°$$

So the amplitude is $(8.00\text{ cm})\cos 45.0° = \boxed{5.66\text{ cm}}$.

P14.5 The resultant wave function has the form

$$y = 2A_0\cos\left(\frac{\phi}{2}\right)\sin\left(kx-\omega t+\frac{\phi}{2}\right)$$

(a) $A = 2A_0\cos\left(\frac{\phi}{2}\right) = 2(5.00)\cos\left[\frac{-\pi/4}{2}\right] = \boxed{9.24\text{ m}}$

(b) $f = \frac{\omega}{2\pi} = \frac{1\,200\pi}{2\pi} = \boxed{600\text{ Hz}}$

P14.6 $2A_0\cos\left(\frac{\phi}{2}\right) = A_0$ so $\quad \frac{\phi}{2} = \cos^{-1}\left(\frac{1}{2}\right) = 60.0° = \frac{\pi}{3}$

Thus, the phase difference is $\quad \phi = 120° = \frac{2\pi}{3}$

This phase difference results if the time delay is $\frac{T}{3} = \frac{1}{3f} = \frac{\lambda}{3v}$

$$\text{Time delay} = \frac{3.00\text{ m}}{3(2.00\text{ m/s})} = \boxed{0.500\text{ s}}$$

P14.7 Waves reflecting from the near end travel 28.0 m (14.0 m down and 14.0 m back), while waves reflecting from the far end travel 66.0 m. The path difference for the two waves is:

$$\Delta r = 66.0\text{ m} - 28.0\text{ m} = 38.0\text{ m}$$

Since $\quad \lambda = \frac{v}{f}$

Then $\quad \frac{\Delta r}{\lambda} = \frac{(\Delta r)f}{v} = \frac{(38.0\text{ m})(246\text{ Hz})}{343\text{ m/s}} = 27.254$

or, $\quad \Delta r = 27.254\lambda$

The phase difference between the two reflected waves is then

$$\phi = 0.254(1\text{ cycle}) = 0.254(2\pi\text{ rad}) = \boxed{91.3°}$$

P14.8 (a) $\Delta x = \sqrt{9.00+4.00} - 3.00 = \sqrt{13} - 3.00 = 0.606\text{ m}$

The wavelength is $\quad \lambda = \frac{v}{f} = \frac{343\text{ m/s}}{300\text{ Hz}} = 1.14\text{ m}$

Thus, $\frac{\Delta x}{\lambda} = \frac{0.606}{1.14} = 0.530$ of a wave,

or $\quad \Delta\phi = 2\pi(0.530) = \boxed{3.33\text{ rad}}$

(b) For destructive interference, we want $\frac{\Delta x}{\lambda} = 0.500 = f\frac{\Delta x}{v}$

where Δx is a constant in this set up. $\quad f = \frac{v}{2\Delta x} = \frac{343}{2(0.606)} = \boxed{283\text{ Hz}}$

***P14.9** (a) $\phi_1 = (20.0\text{ rad/cm})(5.00\text{ cm}) - (32.0\text{ rad/s})(2.00\text{ s}) = 36.0\text{ rad}$

$\phi_1 = (25.0\text{ rad/cm})(5.00\text{ cm}) - (40.0\text{ rad/s})(2.00\text{ s}) = 45.0\text{ rad}$

$\Delta\phi = 9.00\text{ radians} = 516° = \boxed{156°}$

(b) $\Delta\phi = \left|20.0x - 32.0t - [25.0x - 40.0t]\right| = \left|-5.00x + 8.00t\right|$

At $t = 2.00$ s, the requirement is

$$\Delta\phi = \left|-5.00x + 8.00(2.00)\right| = (2n+1)\pi \text{ for any integer } n.$$

For $x < 3.20$, $-5.00x + 16.0$ is positive, so we have

$$-5.00x + 16.0 = (2n+1)\pi, \text{ or}$$

$$x = 3.20 - \frac{(2n+1)\pi}{5.00}$$

The smallest positive value of x occurs for $n = 2$ and is

$$x = 3.20 - \frac{(4+1)\pi}{5.00} = 3.20 - \pi = \boxed{0.058\,4\text{ cm}}.$$

P14.10 Suppose the man's ears are at the same level as the lower speaker. Sound from the upper speaker is delayed by traveling the extra distance $\Delta r = \sqrt{L^2+d^2} - L$.

He hears a minimum when $\Delta r = (2n-1)\left(\frac{\lambda}{2}\right)$ with $n = 1, 2, 3, \ldots$

Then, $\quad \sqrt{L^2+d^2} - L = \left(n-\frac{1}{2}\right)\left(\frac{v}{f}\right)$

$$\sqrt{L^2+d^2} = \left(n-\frac{1}{2}\right)\left(\frac{v}{f}\right) + L$$

$$L^2+d^2 = \left(n-\frac{1}{2}\right)^2\left(\frac{v}{f}\right)^2 + 2\left(n-\frac{1}{2}\right)\left(\frac{v}{f}\right)L + L^2$$

continued on next page

$$d^2-\left(n-\frac{1}{2}\right)^2\left(\frac{v}{f}\right)^2=2\left(n-\frac{1}{2}\right)\left(\frac{v}{f}\right)L \quad (1)$$

Equation 1 gives the distances from the lower speaker at which the man will hear a minimum. The path difference Δr starts from nearly zero when the man is very far away and increases to d when $L=0$.

(a) The number of minima he hears is the greatest integer value for which $L \geq 0$. This is the same as the greatest integer solution to $d \geq \left(n-\frac{1}{2}\right)\left(\frac{v}{f}\right)$, or

$$\boxed{\text{number of minima heard } = n_{max} = \text{greatest integer } \leq d\left(\frac{f}{v}\right)+\frac{1}{2}}.$$

(b) From equation 1, the distances at which minima occur are given by

$$\boxed{L_n=\frac{d^2-\left(n-\frac{1}{2}\right)^2\left(\frac{v}{f}\right)^2}{2\left(n-\frac{1}{2}\right)\left(\frac{v}{f}\right)} \quad \text{where } n=1, 2, \ldots, n_{max}}.$$

P14.11 (a) First we calculate the wavelength: $\lambda=\frac{v}{f}=\frac{344 \text{ m/s}}{21.5 \text{ Hz}}=16.0 \text{ m}$

Then we note that the path difference equals $9.00 \text{ m}-1.00 \text{ m}=\boxed{\frac{1}{2}\lambda}$

Therefore, the receiver will record a minimum in sound intensity.

(b) We choose the origin at the midpoint between the speakers. If the receiver is located at point (x, y), then we must solve:

$$\sqrt{(x+5.00)^2+y^2}-\sqrt{(x-5.00)^2+y^2}=\frac{1}{2}\lambda$$

Then, $\sqrt{(x+5.00)^2+y^2}=\sqrt{(x-5.00)^2+y^2}+\frac{1}{2}\lambda$

Square both sides and simplify to get: $20.0x-\frac{\lambda^2}{4}=\lambda\sqrt{(x-5.00)^2+y^2}$

Upon squaring again, this reduces to: $400x^2-10.0\lambda^2x+\frac{\lambda^4}{16.0}=\lambda^2(x-5.00)^2+\lambda^2y^2$

Substituting $\lambda=16.0$ m, and reducing, $\boxed{9.00x^2-16.0y^2=144}$

or $\frac{x^2}{16.0}-\frac{y^2}{9.00}=1$

(When plotted this yields a curve called a hyperbola.)

Section 14.3 Standing Waves

P14.12 $y=(1.50 \text{ m})\sin(0.400x)\cos(200t)=2A_0 \sin kx \cos \omega t$

Therefore, $k=\frac{2\pi}{\lambda}=0.400 \text{ rad/m}$ $\quad \lambda=\frac{2\pi}{0.400 \text{ rad/m}}=\boxed{15.7 \text{ m}}$

and $\omega=2\pi f$ so $\quad f=\frac{\omega}{2\pi}=\frac{200 \text{ rad/s}}{2\pi \text{ rad}}=\boxed{31.8 \text{ Hz}}$

The speed of waves in the medium is $v=\lambda f=\frac{\lambda}{2\pi}2\pi f=\frac{\omega}{k}=\frac{200 \text{ rad/s}}{0.400 \text{ rad/m}}=\boxed{500 \text{ m/s}}$

P14.13 The facing speakers produce a standing wave in the space between them, with the spacing between nodes being

$$d_{NN}=\frac{\lambda}{2}=\frac{v}{2f}=\frac{343 \text{ m/s}}{2\left(800 \text{ s}^{-1}\right)}=0.214 \text{ m}$$

If the speakers vibrate in phase, the point halfway between them is an antinode of pressure at a distance from either speaker of

$$\frac{1.25 \text{ m}}{2}=0.625.$$

Then there isa node at $0.625-\frac{0.214}{2}=\boxed{0.518 \text{ m}}$

a node at $0.518 \text{ m}-0.214 \text{ m}=\boxed{0.303 \text{ m}}$

a node at $0.303 \text{ m}-0.214 \text{ m}=\boxed{0.0891 \text{ m}}$

a node at $0.518 \text{ m}+0.214 \text{ m}=\boxed{0.732 \text{ m}}$

a node at $0.732 \text{ m}+0.214 \text{ m}=\boxed{0.947 \text{ m}}$

and a node at $0.947 \text{ m}+0.214 \text{ m}=\boxed{1.16 \text{ m}}$ from either speaker.

P14.14 $y=2A_0 \sin kx \cos \omega t$

$$\frac{\partial^2 y}{\partial x^2}=-2A_0k^2 \sin kx \cos \omega t \qquad \frac{\partial^2 y}{\partial t^2}=-2A_0\omega^2 \sin kx \cos \omega t$$

Substitution into the wave equation gives $-2A_0k^2 \sin kx \cos \omega t=\left(\frac{1}{v^2}\right)\left(-2A_0\omega^2 \sin kx \cos \omega t\right)$

This is satisfied, provided that $v=\frac{\omega}{k}$

P14.15 $y_1 = 3.00\sin[\pi(x+0.600t)]$ cm; $y_2 = 3.00\sin[\pi(x-0.600t)]$ cm

$y = y_1 + y_2 = [3.00\sin(\pi x)\cos(0.600\pi t) + 3.00\sin(\pi x)\cos(0.600\pi t)]$ cm

$y = (6.00 \text{ cm})\sin(\pi x)\cos(0.600\pi t)$

(a) We can take $\cos(0.600\pi t) = 1$ to get the maximum y.

At $x = 0.250$ cm, $\quad y_{max} = (6.00 \text{ cm})\sin(0.250\pi) = \boxed{4.24 \text{ cm}}$

(b) At $x = 0.500$ cm, $\quad y_{max} = (6.00 \text{ cm})\sin(0.500\pi) = \boxed{6.00 \text{ cm}}$

(c) Now take $\cos(0.600\pi t) = -1$ to get y_{max}:

At $x = 1.50$ cm, $\quad y_{max} = (6.00 \text{ cm})\sin(1.50\pi)(-1) = \boxed{6.00 \text{ cm}}$

(d) The antinodes occur when $\quad x = \frac{n\lambda}{4}$ $(n = 1, 3, 5, \ldots)$

But $k = \frac{2\pi}{\lambda} = \pi$, so $\quad \lambda = 2.00$ cm

and $\quad x_1 = \frac{\lambda}{4} = \boxed{0.500 \text{ cm}}$ as in (b)

$x_2 = \frac{3\lambda}{4} = \boxed{1.50 \text{ cm}}$ as in (c)

$x_3 = \frac{5\lambda}{4} = \boxed{2.50 \text{ cm}}$

P14.16 (a) The resultant wave is $\quad y = 2A\sin\left(kx + \frac{\phi}{2}\right)\cos\left(\omega t - \frac{\phi}{2}\right)$

The nodes are located at $\quad kx + \frac{\phi}{2} = n\pi$

so $\quad x = \frac{n\pi}{k} - \frac{\phi}{2k}$

which means that each node is shifted $\frac{\phi}{2k}$ to the left.

(b) The separation of nodes is $\quad \Delta x = \left[(n+1)\frac{\pi}{k} - \frac{\phi}{2k}\right] - \left[\frac{n\pi}{k} - \frac{\phi}{2k}\right] \qquad \Delta x = \frac{\pi}{k} = \frac{\lambda}{2}$

The nodes are still separated by half a wavelength.

Section 14.4 Standing Waves in Strings

P14.17 $L = 30.0$ m; $\mu = 9.00 \times 10^{-3}$ kg/m; $T = 20.0$ N; $f_1 = \frac{v}{2L}$

where $\quad v = \left(\frac{T}{\mu}\right)^{1/2} = 47.1$ m/s

so $\quad f_1 = \frac{47.1}{60.0} = \boxed{0.786 \text{ Hz}} \qquad f_2 = 2f_1 = \boxed{1.57 \text{ Hz}}$

$f_3 = 3f_1 = \boxed{2.36 \text{ Hz}} \qquad f_4 = 4f_1 = \boxed{3.14 \text{ Hz}}$

P14.18 $L = 120$ cm, $f = 120$ Hz

(a) For four segments, $L = 2\lambda$ or $\lambda = 60.0 \text{ cm} = \boxed{0.600 \text{ m}}$

(b) $v = \lambda f = 72.0$ m/s $\quad f_1 = \frac{v}{2L} = \frac{72.0}{2(1.20)} = \boxed{30.0 \text{ Hz}}$

P14.19 The tension in the string is $\quad T = (4 \text{ kg})(9.8 \text{ m/s}^2) = 39.2$ N

Its linear density is $\quad \mu = \frac{m}{L} = \frac{8 \times 10^{-3} \text{ kg}}{5 \text{ m}} = 1.6 \times 10^{-3}$ kg/m

and the wave speed on the string is $\quad v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{39.2 \text{ N}}{1.6 \times 10^{-3} \text{ kg/m}}} = 156.5$ m/s

In its fundamental mode of vibration, we have $\quad \lambda = 2L = 2(5 \text{ m}) = 10$ m

Thus, $\quad f = \frac{v}{\lambda} = \frac{156.5 \text{ m/s}}{10 \text{ m}} = \boxed{15.7 \text{ Hz}}$

P14.20 (a) Let n be the number of nodes in the standing wave resulting from the 25.0-kg mass. Then $n+1$ is the number of nodes for the standing wave resulting from the 16.0-kg mass. For standing waves, $\lambda = \frac{2L}{n}$, and the frequency is $f = \frac{v}{\lambda}$.

Thus, $\quad f = \frac{n}{2L}\sqrt{\frac{T_n}{\mu}}$

and also $\quad f = \frac{n+1}{2L}\sqrt{\frac{T_{n+1}}{\mu}}$

Thus, $\quad \frac{n+1}{n} = \sqrt{\frac{T_n}{T_{n+1}}} = \sqrt{\frac{(25.0 \text{ kg})g}{(16.0 \text{ kg})g}} = \frac{5}{4}$

Therefore, $\quad 4n + 4 = 5n$, or $n = 4$

Then, $\quad f = \frac{4}{2(2.00 \text{ m})}\sqrt{\frac{(25.0 \text{ kg})(9.80 \text{ m/s}^2)}{0.002\,00 \text{ kg/m}}} = \boxed{350 \text{ Hz}}$

continued on next page

(b) The largest mass will correspond to a standing wave of 1 loop

$(n=1)$ so $\quad 350\text{ Hz}=\dfrac{1}{2(2.00\text{ m})}\sqrt{\dfrac{m(9.80\text{ m/s}^2)}{0.002\,00\text{ kg/m}}}$

yielding $\quad m=\boxed{400\text{ kg}}$

*P14.21 $f_1=\dfrac{v}{2L}$, where $v=\left(\dfrac{T}{\mu}\right)^{1/2}$

(a) If L is doubled, then $f_1\propto L^{-1}$ will be reduced by a factor $\dfrac{1}{2}$.

(b) If μ is doubled, then $f_1\propto\mu^{-1/2}$ will be reduced by a factor $\dfrac{1}{\sqrt{2}}$.

(c) If T is doubled, then $f_1\propto\sqrt{T}$ will increase by a factor of $\sqrt{2}$.

*P14.22 For the whole string vibrating, $d_{NN}=0.64\text{ m}=\dfrac{\lambda}{2}$; $\lambda=1.28\text{ m}$. The speed of a pulse on the string is $v=f\lambda=330\dfrac{1}{s}1.28\text{ m}=422\text{ m/s}$.

(a) When the string is stopped at the fret, $d_{NN}=\dfrac{2}{3}0.64\text{ m}=\dfrac{\lambda}{2}$;

$\lambda=0.853\text{ m}$

$$f=\frac{v}{\lambda}=\frac{422\text{ m/s}}{0.853\text{ m}}=\boxed{495\text{ Hz}}.$$

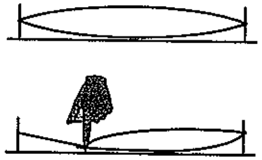

FIG. P14.22(a)

(b) The light touch at a point one third of the way along the string damps out vibration in the two lowest vibration states of the string as a whole. The whole string vibrates in its third resonance possibility: $3d_{NN}=0.64\text{ m}=3\dfrac{\lambda}{2}$;

$\lambda=0.427\text{ m}$

FIG. P14.22(b)

$$f=\frac{v}{\lambda}=\frac{422\text{ m/s}}{0.427\text{ m}}=\boxed{990\text{ Hz}}.$$

P14.23 $d_{NN}=0.700\text{ m}$

$\lambda=1.40\text{ m}$

$f\lambda=v=308\text{ m/s}=\sqrt{\dfrac{T}{(1.20\times10^{-3})/(0.700)}}$

(a) $T=\boxed{163\text{ N}}$

(b) $f_3=\boxed{660\text{ Hz}}$

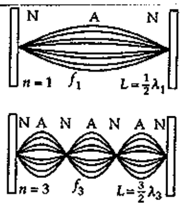

FIG. P14.23

P14.24 $\lambda_G=2(0.350\text{ m})=\dfrac{v}{f_G}$; $\lambda_A=2L_A=\dfrac{v}{f_A}$

$L_G-L_A=L_G-\left(\dfrac{f_G}{f_A}\right)L_G=L_G\left(1-\dfrac{f_G}{f_A}\right)=(0.350\text{ m})\left(1-\dfrac{392}{440}\right)=0.038\,2\text{ m}$

Thus, $L_A=L_G-0.038\,2\text{ m}=0.350\text{ m}-0.038\,2\text{ m}=0.312\text{ m}$,

or the finger should be placed $\boxed{31.2\text{ cm from the bridge}}$.

$L_A=\dfrac{v}{2f_A}=\dfrac{1}{2f_A}\sqrt{\dfrac{T}{\mu}}$; $dL_A=\dfrac{dT}{4f_A\sqrt{T\mu}}$; $\dfrac{dL_A}{L_A}=\dfrac{1}{2}\dfrac{dT}{T}$

$\dfrac{dT}{T}=2\dfrac{dL_A}{L_A}=2\dfrac{0.600\text{ cm}}{(35.0-3.82)\text{ cm}}=\boxed{3.84\%}$

*P14.25 In the fundamental mode, the string above the rod has only two nodes, at A and B, with an anti-node halfway between A and B. Thus,

$$\frac{\lambda}{2}=\overline{AB}=\frac{L}{\cos\theta}\text{ or }\lambda=\frac{2L}{\cos\theta}.$$

Since the fundamental frequency is f, the wave speed in this segment of string is

$$v=\lambda f=\frac{2Lf}{\cos\theta}.$$

Also, $v=\sqrt{\dfrac{T}{\mu}}=\sqrt{\dfrac{T}{m/\overline{AB}}}=\sqrt{\dfrac{TL}{m\cos\theta}}$ where T is the tension in this part of the string. Thus,

$$\frac{2Lf}{\cos\theta}=\sqrt{\frac{TL}{m\cos\theta}}\text{ or }\frac{4L^2f^2}{\cos^2\theta}=\frac{TL}{m\cos\theta}$$

and the mass of string above the rod is:

$$m=\frac{T\cos\theta}{4Lf^2}\qquad\text{[Equation 1]}$$

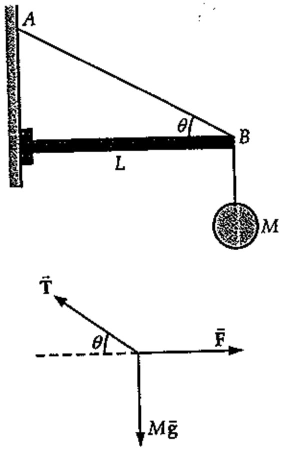

FIG. P14.25

Now, consider the tension in the string. The light rod would rotate about point P if the string exerted any vertical force on it. Therefore, recalling Newton's third law, the rod must exert only a horizontal force on the string. Consider a free-body diagram of the string segment in contact with the end of the rod.

$$\sum F_y=T\sin\theta-Mg=0\Rightarrow T=\frac{Mg}{\sin\theta}$$

Then, from Equation 1, the mass of string above the rod is

$$m=\left(\frac{Mg}{\sin\theta}\right)\frac{\cos\theta}{4Lf^2}=\boxed{\frac{Mg}{4Lf^2\tan\theta}}.$$

P14.26 Comparing $y = (0.002\text{ m})\sin((\pi\text{ rad/m})x)\cos((100\pi\text{ rad/s})t)$

with $y = 2A\sin kx\cos\omega t$

we find $k = \frac{2\pi}{\lambda} = \pi\text{ m}^{-1}$, $\lambda = 2.00\text{ m}$, and $\omega = 2\pi f = 100\pi\text{ s}^{-1}$: $f = 50.0\text{ Hz}$

(a) Then the distance between adjacent nodes is $d_{NN} = \frac{\lambda}{2} = 1.00\text{ m}$

and on the string are $\frac{L}{d_{NN}} = \frac{3.00\text{ m}}{1.00\text{ m}} = \boxed{3\text{ loops}}$

For the speed we have $v = f\lambda = (50\text{ s}^{-1})2\text{ m} = 100\text{ m/s}$

(b) In the simplest standing wave vibration, $d_{NN} = 3.00\text{ m} = \frac{\lambda_b}{2}$, $\lambda_b = 6.00\text{ m}$

and $f_b = \frac{v_a}{\lambda_b} = \frac{100\text{ m/s}}{6.00\text{ m}} = \boxed{16.7\text{ Hz}}$

(c) In $v_0 = \sqrt{\frac{T_0}{\mu}}$, if the tension increases to $T_c = 9T_0$ and the string does not stretch, the speed increases to

$$v_c = \sqrt{\frac{9T_0}{\mu}} = 3\sqrt{\frac{T_0}{\mu}} = 3v_0 = 3(100\text{ m/s}) = 300\text{ m/s}$$

Then $\lambda_c = \frac{v_c}{f_a} = \frac{300\text{ m/s}}{50\text{ s}^{-1}} = 6.00\text{ m}$ $\quad d_{NN} = \frac{\lambda_c}{2} = 3.00\text{ m}$

and $\boxed{\text{one}}$ loop fits onto the string.

Section 14.5 Standing Waves in Air Columns

P14.27 (a) For the fundamental mode in a closed pipe, $\lambda = 4L$, as in the diagram.

But $v = f\lambda$, therefore $L = \frac{v}{4f}$

So, $L = \frac{343\text{ m/s}}{4(240\text{ s}^{-1})} = \boxed{0.357\text{ m}}$

(b) For an open pipe, $\lambda = 2L$, as in the diagram.

So, $L = \frac{v}{2f} = \frac{343\text{ m/s}}{2(240\text{ s}^{-1})} = \boxed{0.715\text{ m}}$

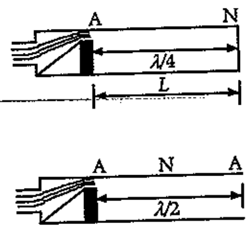

FIG. P14.27

P14.28 $d_{AA} = 0.320\text{ m}$; $\lambda = 0.640\text{ m}$

(a) $f = \frac{v}{\lambda} = \boxed{531\text{ Hz}}$

(b) $\lambda = 0.0850\text{ m}$; $d_{AA} = \boxed{42.5\text{ mm}}$

***P14.29** Assuming an air temperature of $T = 37°\text{C} = 310\text{ K}$, the speed of sound inside the pipe is

$$v = 331\text{ m/s} + 0.6\text{ m/s}\cdot\text{C}°(37°\text{C}) = 353\text{ m/s}.$$

In the fundamental resonant mode, the wavelength of sound waves in a pipe closed at one end is $\lambda = 4L$. Thus, for the whooping crane

$$\lambda = 4(5.0\text{ ft}) = 2.0\times10^1\text{ ft} \quad\text{and}\quad f = \frac{v}{\lambda} = \frac{(353\text{ m/s})}{2.0\times10^1\text{ ft}}\left(\frac{3.281\text{ ft}}{1\text{ m}}\right) = \boxed{57.9\text{ Hz}}.$$

P14.30 The wavelength is $\lambda = \frac{v}{f} = \frac{343\text{ m/s}}{261.6/\text{s}} = 1.31\text{ m}$

so the length of the open pipe vibrating in its simplest (A-N-A) mode is

$$d_{A\text{ to }A} = \frac{1}{2}\lambda = \boxed{0.656\text{ m}}$$

A closed pipe has (N-A) for its simplest resonance,

(N-A-N-A) for the second,

and (N-A-N-A-N-A) for the third.

Here, the pipe length is $5d_{N\text{ to }A} = \frac{5\lambda}{4} = \frac{5}{4}(1.31\text{ m}) = \boxed{1.64\text{ m}}$

P14.31 For a closed box, the resonant frequencies will have nodes at both sides, so the permitted wavelengths will be $L = \frac{1}{2}n\lambda$, $(n = 1, 2, 3, \ldots)$.

i.e., $L = \frac{n\lambda}{2} = \frac{nv}{2f}$ and $f = \frac{nv}{2L}$.

Therefore, with $L = 0.860\text{ m}$ and $L' = 2.10\text{ m}$, the resonant frequencies are

$f_n = \boxed{n(206\text{ Hz})}$ for $L = 0.860\text{ m}$ for each n from 1 to 9

and $f_n' = \boxed{n(84.5\text{ Hz})}$ for $L' = 2.10\text{ m}$ for each n from 2 to 23.

P14.32 The air in the auditory canal, about 3 cm long, can vibrate with a node at the closed end and antinode at the open end,

with $d_{N\text{ to }A} = 3\text{ cm} = \frac{\lambda}{4}$

so $\lambda = 0.12\text{ m}$

and $f = \frac{v}{\lambda} = \frac{343\text{ m/s}}{0.12\text{ m}} \approx \boxed{3\text{ kHz}}$

A small-amplitude external excitation at this frequency can, over time, feed energy into a larger-amplitude resonance vibration of the air in the canal, making it audible.

P14.33 For both open and closed pipes, resonant frequencies are equally spaced as numbers. The set of resonant frequencies then must be 650 Hz, 550 Hz, 450 Hz, 350 Hz, 250 Hz, 150 Hz, 50 Hz. These are odd-integer multipliers of the fundamental frequency of $\boxed{50.0\text{ Hz}}$. Then the pipe length is

$d_{NA} = \frac{\lambda}{4} = \frac{v}{4f} = \frac{340\text{ m/s}}{4(50/\text{s})} = \boxed{1.70\text{ m}}$.

P14.34 The wavelength of sound is $\lambda = \frac{v}{f}$

The distance between water levels at resonance is $d = \frac{v}{2f}$ $\quad\therefore Rt = \pi r^2 d = \frac{\pi r^2 v}{2f}$

and $t = \boxed{\frac{\pi r^2 v}{2Rf}}$.

P14.35 $\frac{\lambda}{2} = d_{AA} = \frac{L}{n}$ or $L = \frac{n\lambda}{2}$ for $n = 1, 2, 3, \ldots$

Since $\lambda = \frac{v}{f}$ $\quad L = n\left(\frac{v}{2f}\right)$ for $n = 1, 2, 3, \ldots$

With $v = 343$ m/s and $f = 680$ Hz,

$$L = n\left(\frac{343\text{ m/s}}{2(680\text{ Hz})}\right) = n(0.252\text{ m}) \quad \text{for } n = 1, 2, 3, \ldots$$

Possible lengths for resonance are: $L = \boxed{0.252\text{ m}, 0.504\text{ m}, 0.757\text{ m}, \ldots, n(0.252)\text{ m}}$.

P14.36 The length corresponding to the fundamental satisfies $f = \frac{v}{4L}$: $L_1 = \frac{v}{4f} = \frac{343}{4(512)} = 0.167\text{ m}$.

Since $L > 20.0$ cm, the *next* two modes will be observed, corresponding to $f = \frac{3v}{4L_2}$ and $f = \frac{5v}{4L_3}$.

or $L_2 = \frac{3v}{4f} = \boxed{0.502\text{ m}}$ and $L_3 = \frac{5v}{4f} = \boxed{0.837\text{ m}}$.

P14.37 For resonance in a narrow tube open at one end,

$$f = n\frac{v}{4L}\ (n = 1, 3, 5, \ldots).$$

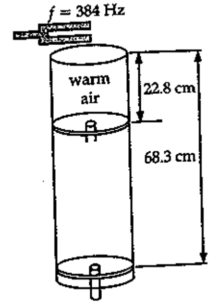

FIG. P14.37

(a) Assuming $n = 1$ and $n = 3$,

$$384 = \frac{v}{4(0.228)} \text{ and } 384 = \frac{3v}{4(0.683)}.$$

In either case, $v = \boxed{350\text{ m/s}}$.

(b) For the next resonance $n = 5$, and $L = \frac{5v}{4f} = \frac{5(350\text{ m/s})}{4(384\text{ s}^{-1})} = \boxed{1.14\text{ m}}$.

***P14.38** (a) For the fundamental mode of an open tube,

$$L = \frac{\lambda}{2} = \frac{v}{2f} = \frac{343\text{ m/s}}{2(880\text{ s}^{-1})} = \boxed{0.195\text{ m}}.$$

(b) $v = 331\text{ m/s} + 0.6\text{ m/s}\cdot°\text{C}(-5°\text{C}) = 328\text{ m/s}$

We ignore the thermal expansion of the metal.

$$f = \frac{v}{\lambda} = \frac{v}{2L} = \frac{328\text{ m/s}}{2(0.195\text{ m})} = \boxed{842\text{ Hz}}$$

The flute is flat by a semitone.

Section 14.6 Beats: Interference in Time

P14.39 $f \propto v \propto \sqrt{T}$ $\qquad f_{new} = 110\sqrt{\frac{540}{600}} = 104.4\text{ Hz}$

$\Delta f = \boxed{5.64\text{ beats/s}}$

P14.40 (a) The string could be tuned to either $\boxed{521\text{ Hz or }525\text{ Hz}}$ from this evidence.

(b) Tightening the string raises the wave speed and frequency. If the frequency were originally 521 Hz, the beats would slow down.

Instead, the frequency must have started at 525 Hz to become $\boxed{526\text{ Hz}}$.

continued on next page

(c) From $f = \frac{v}{\lambda} = \frac{\sqrt{T/\mu}}{2L} = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$

$$\frac{f_2}{f_1} = \sqrt{\frac{T_2}{T_1}} \text{ and } T_2 = \left(\frac{f_2}{f_1}\right)^2 T_1 = \left(\frac{523 \text{ Hz}}{526 \text{ Hz}}\right)^2 T_1 = 0.989T_1.$$

The fractional change that should be made in the tension is then

$$\text{fractional change} = \frac{T_1 - T_2}{T_1} = 1 - 0.989 = 0.011\,4 = 1.14\% \text{ lower.}$$

The tension should be $\boxed{\text{reduced by } 1.14\%}$.

P14.41 For an echo $f' = f\frac{(v+v_s)}{(v-v_s)}$ the beat frequency is $f_b = |f' - f|$.

Solving for f_b.

gives $f_b = f\frac{(2v_s)}{(v-v_s)}$ when approaching wall.

(a) $f_b = (256)\frac{2(1.33)}{(343-1.33)} = \boxed{1.99 \text{ Hz}}$ beat frequency

(b) When he is moving away from the wall, v_s changes sign. Solving for v_s gives

$$v_s = \frac{f_b v}{2f - f_b} = \frac{(5)(343)}{(2)(256)-5} = \boxed{3.38 \text{ m/s}}.$$

Section 14.7 Nonsinusoidal Wave Patterns

P14.42 We evaluate

$$s = 100\sin\theta + 157\sin 2\theta + 62.9\sin 3\theta + 105\sin 4\theta$$
$$+51.9\sin 5\theta + 29.5\sin 6\theta + 25.3\sin 7\theta$$

where s represents particle displacement in nanometers and θ represents the phase of the wave in radians. As θ advances by 2π, time advances by (1/523) s. Here is the result:

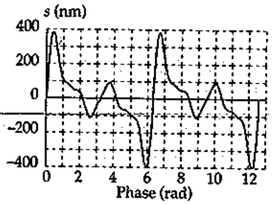

FIG. P14.42

***P14.43** We list the frequencies of the harmonics of each note in Hz:

	Harmonic				
Note	1	2	3	4	5
A	440.00	880.00	1 320.0	1 760.0	2 200.0
C#	554.37	1 108.7	1 663.1	2 217.5	2 771.9
E	659.26	1 318.5	1 977.8	2 637.0	3 296.3

The second harmonic of E is close the the third harmonic of A, and the fourth harmonic of C# is close to the fifth harmonic of A.

Section 14.8 Context Connection—Building on Antinodes

P14.44 (a) The wave speed is $v = \frac{9.15 \text{ m}}{2.50 \text{ s}} = \boxed{3.66 \text{ m/s}}$

(b) From the figure, there are antinodes at both ends of the pond, so the distance between adjacent antinodes

is $d_{AA} = \frac{\lambda}{2} = 9.15 \text{ m}$,

and the wavelength is $\lambda = 18.3 \text{ m}$

The frequency is then $f = \frac{v}{\lambda} = \frac{3.66 \text{ m/s}}{18.3 \text{ m}} = \boxed{0.200 \text{ Hz}}$

We have assumed the wave speed is the same for all wavelengths.

P14.45 The wave speed is $v = \sqrt{gd} = \sqrt{(9.80 \text{ m/s}^2)(36.1 \text{ m})} = 18.8 \text{ m/s}$

The bay has one end open and one closed. Its simplest resonance is with a node of horizontal velocity, which is also an antinode of vertical displacement, at the head of the bay and an antinode of velocity, which is a node of displacement, at the mouth. The vibration of the water in the bay is like that in one half of the pond shown in Figure P14.44.

Then, $d_{NA} = 210 \times 10^3 \text{ m} = \frac{\lambda}{4}$

and $\lambda = 840 \times 10^3 \text{ m}$

Therefore, the period is $T = \frac{1}{f} = \frac{\lambda}{v} = \frac{840 \times 10^3 \text{ m}}{18.8 \text{ m/s}} = 4.47 \times 10^4 \text{ s} = \boxed{12 \text{ h } 24 \text{ min}}$

This agrees precisely with the period of the lunar excitation, so we identify the extra-high tides as amplified by resonance.

Additional Problems

P14.46 The distance between adjacent nodes is one-quarter of the circumference.

$$d_{NN} = d_{AA} = \frac{\lambda}{2} = \frac{20.0 \text{ cm}}{4} = 5.00 \text{ cm}$$

so $\lambda = 10.0$ cm and $f = \frac{v}{\lambda} = \frac{900 \text{ m/s}}{0.100 \text{ m}} = 9\,000 \text{ Hz} = \boxed{9.00 \text{ kHz}}$.

The singer must match this frequency quite precisely for some interval of time to feed enough energy into the glass to crack it.

P14.47 $f = 87.0$ Hz

speed of sound in air: $v_a = 340$ m/s

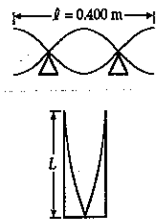

FIG. P14.47

(a) $\lambda_b = \ell$ $\quad v = f\lambda_b = (87.0 \text{ s}^{-1})(0.400 \text{ m})$

$v = \boxed{34.8 \text{ m/s}}$

(b) $\left.\begin{array}{l}\lambda_a = 4L\\ v_a = \lambda_a f\end{array}\right\}$ $\quad L = \frac{v_a}{4f} = \frac{340 \text{ m/s}}{4(87.0 \text{ s}^{-1})} = \boxed{0.977 \text{ m}}$

***P14.48**

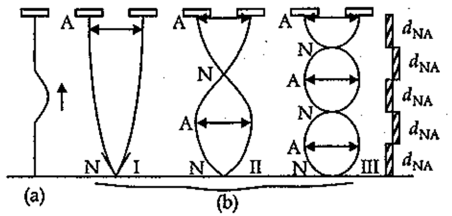

FIG. P14.48

(a) $\mu = \frac{5.5 \times 10^{-3} \text{ kg}}{0.86 \text{ m}} = 6.40 \times 10^{-3}$ kg/m

$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{1.30 \text{ kg}\cdot\text{m/s}^2}{6.40 \times 10^{-3} \text{ kg/m}}} = \boxed{14.3 \text{ m/s}}$

(b) In state I, $d_{NA} = \boxed{0.860 \text{ m}} = \frac{\lambda}{4}$

(c) $\lambda = 3.44$ m $\quad f = \frac{v}{\lambda} = \frac{14.3 \text{ m/s}}{3.44 \text{ m}} = \boxed{4.14 \text{ Hz}}$

In state II, $d_{NA} = \frac{1}{3}(0.86 \text{ m}) = \boxed{0.287 \text{ m}}$

$\lambda = 4(0.287 \text{ m}) = 1.15$ m $\quad f = \frac{v}{\lambda} = \frac{14.3 \text{ m/s}}{1.15 \text{ m}} = \boxed{12.4 \text{ Hz}}$

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In state III, $d_{NA} = \frac{1}{5}(0.86 \text{ m}) = \boxed{0.172 \text{ m}}$

$f = \frac{v}{\lambda} = \frac{14.3 \text{ m/s}}{4(0.172 \text{ m})} = \boxed{20.7 \text{ Hz}}$

P14.49 Moving away from station, frequency is depressed:

$f' = 180 - 2.00 = 178$ Hz: $\quad 178 = 180\frac{343}{343-(-v)}$

Solving for v gives $\quad v = \frac{(2.00)(343)}{178}$

Therefore, $\quad v = \boxed{3.85 \text{ m/s away from station}}$

Moving toward the station, the frequency is enhanced:

$f' = 180 + 2.00 = 182$ Hz: $\quad 182 = 180\frac{343}{343 - v}$

Solving for v gives $\quad 4 = \frac{(2.00)(343)}{182}$

Therefore, $\quad v = \boxed{3.77 \text{ m/s toward the station}}$

***P14.50** (a) Use the Doppler formula

$$f' = f\frac{(v \pm v_0)}{(v \mp v_s)}.$$

With f_1' = frequency of the speaker in front of student and

f_2' = frequency of the speaker behind the student.

$$f_1' = (456 \text{ Hz})\frac{(343 \text{ m/s} + 1.50 \text{ m/s})}{(343 \text{ m/s} - 0)} = 458 \text{ Hz}$$

$$f_2' = (456 \text{ Hz})\frac{(343 \text{ m/s} - 1.50 \text{ m/s})}{(343 \text{ m/s} + 0)} = 454 \text{ Hz}$$

Therefore, $f_b = f_1' - f_2' = \boxed{3.99 \text{ Hz}}$.

(b) The waves broadcast by both speakers have $\lambda = \frac{v}{f} = \frac{343 \text{ m/s}}{456/\text{s}} = 0.752$ m. The standing wave between them has $d_{AA} = \frac{\lambda}{2} = 0.376$ m. The student walks from one maximum to the next in time $\Delta t = \frac{0.376 \text{ m}}{1.50 \text{ m/s}} = 0.251$ s, so the frequency at which she hears maxima is $f = \frac{1}{T} = \boxed{3.99 \text{ Hz}}$.

P14.51 Call L the depth of the well and v the speed of sound.

Then for some integer n $\quad L=(2n-1)\frac{\lambda_1}{4}=(2n-1)\frac{v}{4f_1}=\frac{(2n-1)(343\text{ m/s})}{4(51.5\text{ s}^{-1})}$

and for the next resonance $\quad L=[2(n+1)-1]\frac{\lambda_2}{4}=(2n+1)\frac{v}{4f_2}=\frac{(2n+1)(343\text{ m/s})}{4(60.0\text{ s}^{-1})}$

Thus, $\quad \frac{(2n-1)(343\text{ m/s})}{4(51.5\text{ s}^{-1})}=\frac{(2n+1)(343\text{ m/s})}{4(60.0\text{ s}^{-1})}$

and we require an *integer* solution to $\quad \frac{2n+1}{60.0}=\frac{2n-1}{51.5}$

The equation gives $n=\frac{111.5}{17}=6.56$, so the best fitting integer is $n=7$.

Then $\quad L=\frac{[2(7)-1](343\text{ m/s})}{4(51.5\text{ s}^{-1})}=21.6\text{ m}$

and $\quad L=\frac{[2(7)+1](343\text{ m/s})}{4(60.0\text{ s}^{-1})}=21.4\text{ m}$

suggest the best value for the depth of the well is $\boxed{21.5\text{ m}}$.

P14.52 $v=\sqrt{\frac{(48.0)(2.00)}{4.80\times10^{-3}}}=141\text{ m/s}$

$d_{NN}=1.00\text{ m};\ \lambda=2.00\text{ m};\ f=\frac{v}{\lambda}=70.7\text{ Hz}$

$\lambda_a=\frac{v_a}{f}=\frac{343\text{ m/s}}{70.7\text{ Hz}}=\boxed{4.85\text{ m}}$

P14.53 (a) Since the first node is at the weld, the wavelength in the thin wire is $2L$ or 80.0 cm. The frequency and tension are the same in both sections, so

$$f=\frac{1}{2L}\sqrt{\frac{T}{\mu}}=\frac{1}{2(0.400)}\sqrt{\frac{4.60}{2.00\times10^{-3}}}=\boxed{59.9\text{ Hz}}.$$

(b) As the thick wire is twice the diameter, the linear density is 4 times that of the thin wire.

$\mu'=8.00\text{ g/m}$

so $L'=\frac{1}{2f}\sqrt{\frac{T}{\mu'}}$ $\quad L'=\left[\frac{1}{(2)(59.9)}\right]\sqrt{\frac{4.60}{8.00\times10^{-3}}}=\boxed{20.0\text{ cm}}$ half the length of the thin wire.

P14.54 The second standing wave mode of the air in the pipe reads ANAN, with $d_{NA}=\frac{\lambda}{4}=\frac{1.75\text{ m}}{3}$

so $\quad \lambda=2.33\text{ m}$

and $\quad f=\frac{v}{\lambda}=\frac{343\text{ m/s}}{2.33\text{ m}}=147\text{ Hz}$

For the string, λ and v are different but f is the same.

$$\frac{\lambda}{2}=d_{NN}=\frac{0.400\text{ m}}{2}$$

so $\quad \lambda=0.400\text{ m}$

$v=\lambda f=(0.400\text{ m})(147\text{ Hz})=58.8\text{ m/s}=\sqrt{\frac{T}{\mu}}$

$T=\mu v^2=(9.00\times10^{-3}\text{ kg/m})(58.8\text{ m/s})^2=\boxed{31.1\text{ N}}$

P14.55 (a) $f=\frac{n}{2L}\sqrt{\frac{T}{\mu}}$

so $\frac{f'}{f}=\frac{L}{L'}=\frac{L}{2L}=\frac{1}{2}$

The $\boxed{\text{frequency should be halved}}$ to get the same number of antinodes for twice the length.

(b) $\frac{n'}{n}=\sqrt{\frac{T}{T'}}$ $\quad$ so $\quad \frac{T'}{T}=\left(\frac{n}{n'}\right)^2=\left[\frac{n}{n+1}\right]^2$

The tension must be $\quad T'=\boxed{\left[\frac{n}{n+1}\right]^2 T}$

(c) $\frac{f'}{f}=\frac{n'L}{nL'}\sqrt{\frac{T'}{T}}$ $\quad$ so $\quad \frac{T'}{T}=\left(\frac{nf'L'}{n'fL}\right)^2$

$\frac{T'}{T}=\left(\frac{3}{2\cdot2}\right)^2$ $\quad \boxed{\frac{T'}{T}=\frac{9}{16}}$ to get twice as many antinodes.

P14.56 (a) For the block:

$\sum F_x=T-Mg\sin30.0°=0$

so $T=Mg\sin30.0°=\boxed{\frac{1}{2}Mg}$.

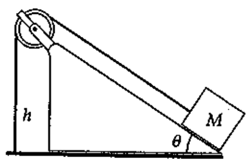

FIG. P14.56

(b) The length of the section of string parallel to the incline is $\frac{h}{\sin30.0°}=2h$. The total length of the string is then $\boxed{3h}$.

(c) The mass per unit length of the string is $\quad \mu=\boxed{\frac{m}{3h}}$

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The speed of waves in the string is $v=\sqrt{\frac{T}{\mu}}=\sqrt{\left(\frac{Mg}{2}\right)\left(\frac{3h}{m}\right)}=\boxed{\sqrt{\frac{3Mgh}{2m}}}$

In the fundamental mode, the segment of length h vibrates as one loop. The distance between adjacent nodes is then $d_{NN}=\frac{\lambda}{2}=h$, so the wavelength is $\lambda=2h$.

The frequency is $f=\frac{v}{\lambda}=\frac{1}{2h}\sqrt{\frac{3Mgh}{2m}}=\boxed{\sqrt{\frac{3Mg}{8mh}}}$

(g) When the vertical segment of string vibrates with 2 loops (i.e., 3 nodes), then $h=2\left(\frac{\lambda}{2}\right)$ and the wavelength is $\lambda=\boxed{h}$.

(f) The period of the standing wave of 3 nodes (or two loops) is

$$T=\frac{1}{f}=\frac{\lambda}{v}=h\sqrt{\frac{2m}{3Mgh}}=\boxed{\sqrt{\frac{2mh}{3Mg}}}$$

(h) $f_b=1.02f-f=\left(2.00\times10^{-2}\right)f=\boxed{\left(2.00\times10^{-2}\right)\sqrt{\frac{3Mg}{8mh}}}$

P14.57 We look for a solution of the form

$$5.00\sin(2.00x-10.0t)+10.0\cos(2.00x-10.0t)=A\sin(2.00x-10.0t+\phi)$$
$$=A\sin(2.00x-10.0t)\cos\phi+A\cos(2.00x-10.0t)\sin\phi$$

This will be true if both $5.00=A\cos\phi$ and $10.0=A\sin\phi$,

requiring $(5.00)^2+(10.0)^2=A^2$

$A=11.2$ and $\phi=63.4°$

The resultant wave $\boxed{11.2\sin(2.00x-10.0t+63.4°)}$ is sinusoidal.

P14.58 For the wire, $\mu=\frac{0.0100\text{ kg}}{2.00\text{ m}}=5.00\times10^{-3}\text{ kg/m}$: $v=\sqrt{\frac{T}{\mu}}=\sqrt{\frac{\left(200\text{ kg}\cdot\text{m/s}^2\right)}{5.00\times10^{-3}\text{ kg/m}}}$

$v=200\text{ m/s}$

If it vibrates in its simplest state, $d_{NN}=2.00\text{ m}=\frac{\lambda}{2}$: $f=\frac{v}{\lambda}=\frac{(200\text{ m/s})}{4.00\text{ m}}=50.0\text{ Hz}$

(a) The tuning fork can have frequencies $\boxed{45.0\text{ Hz or }55.0\text{ Hz}}$.

continued on next page

(b) If $f=45.0\text{ Hz}$, $v=f\lambda=(45.0/\text{s})4.00\text{ m}=180\text{ m/s}$.

Then, $T=v^2\mu=(180\text{ m/s})^2\left(5.00\times10^{-3}\text{ kg/m}\right)=\boxed{162\text{ N}}$

$\boxed{\text{or}}$ if $f=55.0\text{ Hz}$, $T=v^2\mu=f^2\lambda^2\mu=(55.0/\text{s})^2(4.00\text{ m})^2\left(5.00\times10^{-3}\text{ kg/m}\right)=\boxed{242\text{ N}}$.

P14.59 (a) Let θ represent the angle each slanted rope makes with the vertical.

In the diagram, observe that:

$$\sin\theta=\frac{1.00\text{ m}}{1.50\text{ m}}=\frac{2}{3}$$

or $\theta=41.8°$.

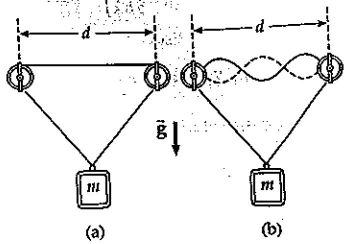

FIG. P14.59

Considering the mass,

$$\sum F_y=0:\ 2T\cos\theta=mg$$

or $T=\frac{(12.0\text{ kg})\left(9.80\text{ m/s}^2\right)}{2\cos41.8°}=\boxed{78.9\text{ N}}$

(b) The speed of transverse waves in the string is $v=\sqrt{\frac{T}{\mu}}=\sqrt{\frac{78.9\text{ N}}{0.00100\text{ kg/m}}}=281\text{ m/s}$.

For the standing wave pattern shown (3 loops), $d=\frac{3}{2}\lambda$

or $\lambda=\frac{2(2.00\text{ m})}{3}=1.33\text{ m}$.

Thus, the required frequency is $f=\frac{v}{\lambda}=\frac{281\text{ m/s}}{1.33\text{ m}}=\boxed{211\text{ Hz}}$.

P14.60 $d_{AA}=\frac{\lambda}{2}=7.05\times10^{-3}\text{ m}$ is the distance between antinodes.

Then $\lambda=14.1\times10^{-3}\text{ m}$

and $f=\frac{v}{\lambda}=\frac{3.70\times10^3\text{ m/s}}{14.1\times10^{-3}\text{ m}}=\boxed{2.62\times10^5\text{ Hz}}$.

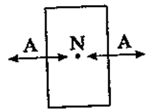

FIG. P14.60

The crystal can be tuned to vibrate at 2^{18} Hz, so that binary counters can derive from it a signal at precisely 1 Hz.

ANSWERS TO EVEN PROBLEMS

P14.2 see the solution

P14.4 5.66 cm

P14.6 0.500 s

P14.8 (a) 3.33 rad; (b) 283 Hz

P14.10 (a) number of minima heard $= n_{max} =$ greatest integer $\le d\left(\frac{f}{v}\right)+\frac{1}{2}$; (b) $L_n = \frac{d^2-\left(n-\frac{1}{2}\right)^2\left(\frac{v}{f}\right)^2}{2\left(n-\frac{1}{2}\right)\left(\frac{v}{f}\right)}$ where $n = 1, 2, \ldots, n_{max}$

P14.12 15.7 m, 31.8 Hz, 500 m/s

P14.14 see the solution

P14.16 (a) see the solution; (b) see the solution

P14.18 (a) 0.600 m; (b) 30.0 Hz

P14.20 (a) 350 Hz; (b) 400 kg

P14.22 (a) 495 Hz; (b) 990 Hz

P14.24 31.2 cm from the bridge, 3.84%

P14.26 (a) 3 loops; (b) 16.7 Hz; (c) 1 loop

P14.28 (a) 531 Hz; (b) 42.5 mm

P14.30 0.656 m, 1.64 m

P14.32 around 3 kHz. A small-amplitude external excitation at this frequency can, over time, feed energy into a larger-amplitude resonance vibration of the air in the canal, making it audible.

P14.34 $\frac{\pi r^2 v}{2Rf}$

P14.36 0.502 m, 0.837 m

P14.38 (a) 0.195 m; (b) 842 Hz

P14.40 (a) 521 Hz or 525 Hz; (b) 526 Hz; (c) reduced by 1.14%

P14.42 see the solution

P14.44 (a) 3.66 m/s; (b) 0.200 Hz

P14.46 9.00 kHz

P14.48 (a) 14.3 m/s; (b) 0.860 m, 0.287 m, 0.172 m; (c) 4.14 Hz, 12.4 Hz, 20.7 Hz

P14.50 (a) 3.99 Hz; (b) 3.99 Hz

P14.52 4.85 m

P14.54 31.1 N

P14.56 (a) $\frac{1}{2}Mg$; (b) $3h$; (c) $\frac{m}{3h}$; (d) $\sqrt{\frac{3Mgh}{2m}}$; (e) $\sqrt{\frac{3Mg}{8mh}}$; (f) $\sqrt{\frac{2mh}{3Mg}}$; (g) h; (h) $\left(2.00\times10^{-2}\right)\sqrt{\frac{3Mg}{8mh}}$

P14.58 (a) 45.0 or 55.0 Hz; (b) 162 or 242 N

P14.60 2.62×10^5 Hz

CONTEXT 3 CONCLUSION SOLUTIONS TO PROBLEMS

CC3.1 Let point 1 be $r = 10$ km from the epicenter and point 2 be at 20 km. The intensity is proportional to $\frac{1}{r}$ according to $I = \frac{C}{r}$, where C is some constant. Intensity is defined as the energy a wave carries each second through a unit area of wavefront, so it is proportional to the amplitude squared according to $I = DA^2$, where D is another constant. Then the factors of change are related by

continued on next page

$$\frac{I_2}{I_1} = \frac{DA_2^{\,2}}{DA_1^{\,2}} = \frac{r_1C}{r_2C}$$

$$\frac{A_2}{A_1} = \sqrt{\frac{r_1}{r_2}}$$

$$A_2 = A_1\sqrt{\frac{r_1}{r_2}} = 5.0\text{ cm}\sqrt{\frac{10\text{ km}}{20\text{ km}}} = \boxed{3.5\text{ cm}}$$

CC3.2 As in Equation 13.23, the rate of energy transfer in a seismic wave is proportional to the speed and to the amplitude squared. We write $\mathcal{P} = FvA^2$, where F is some constant. If no wave energy is reflected or turns into internal energy, $Fv_{bedrock}A^2_{bedrock} = Fv_{mudfill}A^2_{mudfill}$

$$\frac{v_{mudfill}}{v_{bedrock}} = \left(\frac{A_{bedrock}}{A_{mudfill}}\right)^2 = \left(\frac{A_{bedrock}}{5A_{bedrock}}\right)^2 = \frac{1}{25}$$

$\boxed{\text{The speed decreases by a factor of 25.}}$

CC3.3 **METHOD ONE**

From the graph, we have for the speed of S waves $v_S = \frac{395\text{ km}}{100\text{ s}} = 3.95$ km/s, and for the speed of P waves $v_P = \frac{400\text{ km}}{48\text{ s}} = 8.33$ km/s. From the data of station 1 we can find a value for the time the quake started: $15\text{ h:}46\text{ min:}06\text{ s} - \frac{200\text{ km}}{8.33\text{ km/s}} = 15\text{ h:}45\text{ min:}66\text{ s} - 24\text{ s} = 15\text{ h:}45\text{ min:}42\text{ s}$. Similarly from the data of the other stations, the quake began at $15{:}46{:}01 - \frac{160\text{ km}}{8.33\text{ km/s}} = 15{:}45{:}41.8$ or $15{:}45{:}54 - \frac{105\text{ s}}{8.33} = 15{:}45{:}41.4$. For the most probable value for the actual time we take the average, $15{:}45{:}41.7 \pm 0.3$ s. Then the S-wave arrival time should be

$$15{:}45{:}41.7 + \frac{200\text{ km}}{3.95\text{ km/s}} = \boxed{15{:}46{:}32 \text{ for station 1,}}$$

$$15{:}45{:}41.7 + \frac{160\text{ s}}{3.95} = \boxed{15{:}46{:}22 \text{ for station 2,}}$$

$$15{:}45{:}41.7 + \frac{105\text{ s}}{3.95} = \boxed{15{:}46{:}08 \text{ for station 3,}}$$

all with uncertainties of $\boxed{\pm1\text{ s}}$

METHOD TWO

With no significant loss of precision, we can use the graph of travel times to read the S wave arrival times almost directly.

For station #1, locate 200 km on the horizontal axis. Vertically above it, read the size of the space between the P and S lines as 27 s. Add this S wave delay time to the P wave arrival time, 15:46:06, to obtain 15:46:33 as the S wave arrival time at station #1.

Similarly for station #2, the S wave should arrive at 21 s + 15:46:01 = 15:46:22.

For station #3, the graph shows that at range 105 km an S wave arrives 14 s after a P wave, placing it at 15:45:54 + 14 = 15:46:08.