P5.36 (a) 3.95×10^3 N forward; (b) $3.02\ \text{m/s}^2$ forward, (c) 86.3 m/s (d) 113 m/s

P5.38 they do not, 29.4 N

P5.40 (a) 19.3°; (b) 4.21 N

P5.42 (a) see the solution; (b) see the solution; (c) 357 N; (d) see the solution; (e) 1.20

P5.44 (a) see the solution; (b) 9.80 N, $0.580\ \text{m/s}^2$

P5.46 (a) $a = 2.84\ \text{m/s}^2$, 26.5 N; (b) see the solution

P5.48 (a) $2.63\ \text{m/s}^2$; (b) 201 m; (c) 17.7 m/s

P5.50 (a) $m_2 g$; (b) $m_2 g$; (c) $\sqrt{\left(\frac{m_2}{m_1}\right) gR}$

P5.52 (a) 5.19 m/s; (b) see the solution, 555 N

P5.54 (a) 1.05×10^{-3} m/s; (b) 7.59×10^{3} s; (c) $8.88 \times 10^3\ \text{m/s}^2$; (d) 8.38 s

P5.56 (a) either 70.4° or 0°; (b) 0°

P5.58 (a) see the solution; (b) see the solution; (c) 53.0 m/s

P5.60 $\sum \vec{F} = -km\vec{v}$

Chapter 6

Energy and Energy Transfer

CHAPTER OUTLINE

ANSWERS TO QUESTIONS

Q6.1 The force is perpendicular to every increment of displacement. Therefore, $\vec{F} \cdot \Delta\vec{r} = 0$.

Q6.2 Yes. Force times distance over which the toe is in contact with the ball. No, he is no longer applying a force. Yes, both air friction and gravity do work.

Q6.3 Force of tension on a ball rotating on the end of a string. Normal force and gravitational force on an object at rest or moving across a level floor.

Q6.4 Work is only done in accelerating the ball from rest. The work is done over the effective length of the pitcher's arm—the distance his hand moves through windup and until release.

Q6.5 (a) Tension (b) Air resistance

(c) Positive in increasing velocity on the downswing. Negative in decreasing velocity on the upswing.

Q6.6 No. The vectors might be in the third and fourth quadrants, but if the angle between them is less than 90° their dot product is positive.

Q6.7 The scalar product of two vectors is positive if the angle between them is between 0 and 90°. The scalar product is negative when $90° < \theta < 180°$

Q6.8 $k' = 2k$. To stretch the smaller piece one meter, each coil would have to stretch twice as much as one coil in the original long spring, since there would be half as many coils. Assuming that the spring is ideal, twice the stretch requires twice the force.

Q6.9 Kinetic energy is always positive. Mass and squared speed are both positive. A moving object can always do positive work in striking another object and causing it to move along the same direction of motion.

Q6.10 The longer barrel will have the higher muzzle speed. Since the accelerating force acts over a longer distance, the change in kinetic energy will be larger.

Q6.11 Kinetic energy is proportional to mass. The first bullet has twice as much kinetic energy.

Q6.12 No violation. Choose the book as the system. You did work and the earth did work on the book. The average force you exerted just counterbalanced the weight of the book. The total work on the book is zero, and is equal to its overall change in kinetic energy.

Q6.13 (a) Kinetic energy is proportional to squared speed. Doubling the speed makes an object's kinetic energy four times larger.

(b) If the total work on an object is zero in some process, its speed must be the same at the final point as it was at the initial point.

Q6.14 The larger engine is unnecessary. Consider a 30-minute commute. If you travel the same speed in each car, it will take the same amount of time, expending the same amount of energy. The extra power available from the larger engine isn't used.

Q6.15 If the instantaneous power output by some agent changes continuously, its average power in a process must be equal to its instantaneous power at least one instant. If its power output is constant, its instantaneous power is always equal to its average power.

Q6.16 The rock increases in speed. The farther it has fallen, the more force it might exert on the sand at the bottom; but it might instead make a deeper crater with an equal-size average force. The farther it falls, the more work it will do in stopping. Its kinetic energy is increasing due to the work that the gravitational force does on it.

Q6.17 The normal force does no work because the angle between the normal force and the direction of motion is usually 90°. Static friction usually does no work because there is no distance through which the force is applied.

Q6.18 An argument for: As a glider moves along an airtrack, the only force that the track applies on the glider is the normal force. Since the angle between the direction of motion and the normal force is 90°, the work done must be zero, even if the track is not level.

Against: An airtrack has bumpers. When a glider bounces from the bumper at the end of the airtrack, it loses a bit of energy, as evidenced by a decreased speed. The airtrack does negative work.

SOLUTIONS TO PROBLEMS

Section 6.1 Systems and Environments

No problems in this section

Section 6.2 Work Done by a Constant Force

P6.1 (a) $W = F\Delta r\cos\theta = (16.0\text{ N})(2.20\text{ m})\cos 25.0° = \boxed{31.9\text{ J}}$

(b), (c) The normal force and the weight are both at 90° to the displacement in any time interval. Both do $\boxed{0}$ work.

(d) $\sum W = 31.9\text{ J} + 0 + 0 = \boxed{31.9\text{ J}}$

P6.2 The component of force along the direction of motion is

$$F\cos\theta = (35.0\text{ N})\cos 25.0° = 31.7\text{ N}.$$

The work done by this force is

$$W = (F\cos\theta)\Delta r = (31.7\text{ N})(50.0\text{ m}) = \boxed{1.59\times10^{3}\text{ J}}.$$

P6.3 **Method One.**

Let ϕ represent the instantaneous angle the rope makes with the vertical as it is swinging up from $\phi_i = 0$ to $\phi_f = 60°$. In an incremental bit of motion from angle ϕ to $\phi + d\phi$, the definition of radian measure implies that $\Delta r = (12\text{ m})d\phi$. The angle θ between the incremental displacement and the force of gravity is $\theta = 90° + \phi$. Then $\cos\theta = \cos(90° + \phi) = -\sin\phi$. The work done by the gravitational force on Batman is

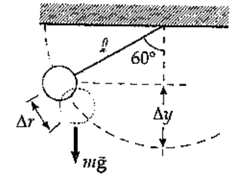

FIG. P6.3

$$W = \int_i^f F\cos\theta\,dr = \int_{\phi=0}^{\phi=60°} mg(-\sin\phi)(12\text{ m})\,d\phi$$

$$= -mg(12\text{ m})\int_0^{60°}\sin\phi\,d\phi = (-80\text{ kg})(9.8\text{ m/s}^2)(12\text{ m})(-\cos\phi)\Big|_0^{60°}$$

$$= (-784\text{ N})(12\text{ m})(-\cos 60° + 1) = \boxed{-4.70\times10^{3}\text{ J}}$$

Method Two.

The force of gravity on Batman is $mg = (80\text{ kg})(9.8\text{ m/s}^2) = 784\text{ N}$ down. Only his vertical displacement contributes to the work gravity does. His original y-coordinate below the tree limb is -12 m. His final y-coordinate is $(-12\text{ m})\cos 60° = -6\text{ m}$. His change in elevation is $-6\text{ m} - (-12\text{ m}) = 6\text{ m}$. The work done by gravity is

$$W = F\Delta r\cos\theta = (784\text{ N})(6\text{ m})\cos 180° = \boxed{-4.70\text{ kJ}}.$$

P6.4 (a) $W = mgh = (3.35\times10^{-5})(9.80)(100)\text{ J} = \boxed{3.28\times10^{-2}\text{ J}}$

(b) Since $R = mg$, $W_{\text{air resistance}} = \boxed{-3.28\times10^{-2}\text{ J}}$

Section 6.3 The Scalar Product of Two Vectors

P6.5 We must first find the angle between the two vectors. It is:

$$\theta = 360° - 118° - 90.0° - 132° = 20.0°$$

Then

$$\vec{F}\cdot\vec{v} = Fv\cos\theta = (32.8\text{ N})(0.173\text{ m/s})\cos 20.0°$$

or $\vec{F}\cdot\vec{v} = 5.33\dfrac{\text{N}\cdot\text{m}}{\text{s}} = 5.33\dfrac{\text{J}}{\text{s}} = \boxed{5.33\text{ W}}$

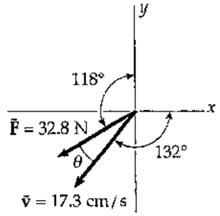

FIG. P6.5

P6.6 $\vec{A}\cdot\vec{B} = (A_x\hat{i} + A_y\hat{j} + A_z\hat{k})\cdot(B_x\hat{i} + B_y\hat{j} + B_z\hat{k})$

$$\begin{aligned}\vec{A}\cdot\vec{B} = {} & A_xB_x(\hat{i}\cdot\hat{i}) + A_xB_y(\hat{i}\cdot\hat{j}) + A_xB_z(\hat{i}\cdot\hat{k}) \\ & + A_yB_x(\hat{j}\cdot\hat{i}) + A_yB_y(\hat{j}\cdot\hat{j}) + A_yB_z(\hat{j}\cdot\hat{k}) \\ & + A_zB_x(\hat{k}\cdot\hat{i}) + A_zB_y(\hat{k}\cdot\hat{j}) + A_zB_z(\hat{k}\cdot\hat{k})\end{aligned}$$

$\vec{A}\cdot\vec{B} = \boxed{A_xB_x + A_yB_y + A_zB_z}$

P6.7 (a) $W = \vec{F}\cdot\Delta\vec{r} = F_xx + F_yy = (6.00)(3.00)\text{ N}\cdot\text{m} + (-2.00)(1.00)\text{ N}\cdot\text{m} = \boxed{16.0\text{ J}}$

(b) $\theta = \cos^{-1}\left(\dfrac{\vec{F}\cdot\Delta\vec{r}}{F\Delta r}\right) = \cos^{-1}\dfrac{16}{\sqrt{\left((6.00)^2 + (-2.00)^2\right)\left((3.00)^2 + (1.00)^2\right)}} = \boxed{36.9°}$

P6.8 $\vec{A} - \vec{B} = (3.00\hat{i} + \hat{j} - \hat{k}) - (-\hat{i} + 2.00\hat{j} + 5.00\hat{k})$

$\vec{A} - \vec{B} = 4.00\hat{i} - \hat{j} - 6.00\hat{k}$

$\vec{C}\cdot(\vec{A} - \vec{B}) = (2.00\hat{j} - 3.00\hat{k})\cdot(4.00\hat{i} - \hat{j} - 6.00\hat{k}) = 0 + (-2.00) + (+18.0) = \boxed{16.0}$

P6.9 (a) $\vec{A} = 3.00\hat{i} - 2.00\hat{j}$

$\vec{B} = 4.00\hat{i} - 4.00\hat{j}$ $\qquad \theta = \cos^{-1}\dfrac{\vec{A}\cdot\vec{B}}{AB} = \cos^{-1}\dfrac{12.0 + 8.00}{\sqrt{(13.0)(32.0)}} = \boxed{11.3°}$

(b) $\vec{B} = 3.00\hat{i} - 4.00\hat{j} + 2.00\hat{k}$

$\vec{A} = -2.00\hat{i} + 4.00\hat{j}$ $\qquad \cos\theta = \dfrac{\vec{A}\cdot\vec{B}}{AB} = \dfrac{-6.00 - 16.0}{\sqrt{(20.0)(29.0)}}$ $\qquad \theta = \boxed{156°}$

(c) $\vec{A} = \hat{i} - 2.00\hat{j} + 2.00\hat{k}$

$\vec{B} = 3.00\hat{j} + 4.00\hat{k}$ $\qquad \theta = \cos^{-1}\left(\dfrac{\vec{A}\cdot\vec{B}}{AB}\right) = \cos^{-1}\left(\dfrac{-6.00 + 8.00}{\sqrt{9.00}\cdot\sqrt{25.0}}\right) = \boxed{82.3°}$

Section 6.4 Work Done by a Varying Force

P6.10 $F_x = (8x - 16)\text{ N}$

(a) See figure to the right.

(b) $W_{net} = \dfrac{-(2.00\text{ m})(16.0\text{ N})}{2} + \dfrac{(1.00\text{ m})(8.00\text{ N})}{2} = \boxed{-12.0\text{ J}}$

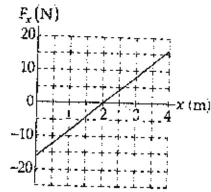

FIG. P6.10

P6.11 $W = \int F_x dx$

and W equals the area under the Force-Displacement curve

(a) For the region $0 \le x \le 5.00$ m,

$$W = \frac{(3.00\text{ N})(5.00\text{ m})}{2} = \boxed{7.50\text{ J}}$$

(b) For the region $5.00 \le x \le 10.0$,

$$W = (3.00\text{ N})(5.00\text{ m}) = \boxed{15.0\text{ J}}$$

(c) For the region $10.0 \le x \le 15.0$,

$$W = \frac{(3.00\text{ N})(5.00\text{ m})}{2} = \boxed{7.50\text{ J}}$$

(d) For the region $0 \le x \le 15.0$

$$W = (7.50 + 7.50 + 15.0)\text{ J} = \boxed{30.0\text{ J}}$$

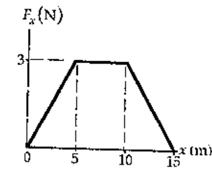

FIG. P6.11

P6.12 Compare an initial picture of the rolling car with a final picture with both springs compressed $K_i + \sum W = K_f$. Work by both springs changes the car's kinetic energy

$$K_i + \frac{1}{2}k_1\left(x_{1i}^2 - x_{1f}^2\right) + \frac{1}{2}k_2\left(x_{2i}^2 - x_{2f}^2\right) = K_f$$

$$\frac{1}{2}mv_i^2 + 0 - \frac{1}{2}(1600\text{ N/m})(0.500\text{ m})^2$$

$$+0 - \frac{1}{2}(3400\text{ N/m})(0.200\text{ m})^2 = 0$$

$$\frac{1}{2}(6\,000\text{ kg})v_i^2 - 200\text{ J} - 68.0\text{ J} = 0$$

$$v_i = \sqrt{\frac{2(268\text{ J})}{6\,000\text{ kg}}} = \boxed{0.299\text{ m/s}}$$

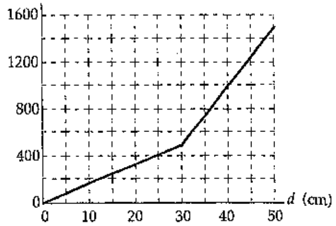

FIG. P6.12

P6.13 $k = \frac{F}{y} = \frac{Mg}{y} = \frac{(4.00)(9.80)\text{ N}}{2.50\times10^{-2}\text{ m}} = 1.57\times10^3\text{ N/m}$

(a) For 1.50 kg mass $y = \frac{mg}{k} = \frac{(1.50)(9.80)}{1.57\times10^3} = \boxed{0.938\text{ cm}}$

(b) $\text{Work} = \frac{1}{2}ky^2$

$$\text{Work} = \frac{1}{2}\left(1.57\times10^3\text{ N}\cdot\text{m}\right)\left(4.00\times10^{-2}\text{ m}\right)^2 = \boxed{1.25\text{ J}}$$

P6.14 $W = \int_i^f \vec{F}\cdot d\vec{r} = \int_0^{5\text{ m}}\left(4x\hat{i}+3y\hat{j}\right)\text{N}\cdot dx\hat{i}$

$$\int_0^{5\text{ m}}(4\text{ N/m})x\,dx + 0 = (4\text{ N/m})\left.\frac{x^2}{2}\right|_0^{5\text{ m}} = \boxed{50.0\text{ J}}$$

P6.15 (a) Spring constant is given by $F = kx$

$$k = \frac{F}{x} = \frac{(230\text{ N})}{(0.400\text{ m})} = \boxed{575\text{ N/m}}$$

(b) $\text{Work} = F_{avg}x = \frac{1}{2}(230\text{ N})(0.400\text{ m}) = \boxed{46.0\text{ J}}$

P6.16 (a) $W = \int_i^f \vec{F}\cdot d\vec{r}$

$$W = \int_0^{0.600\text{ m}}\left(15\,000\text{ N} + 10\,000x\text{ N/m} - 25\,000x^2\text{ N/m}^2\right)dx\cos 0°$$

$$W = 15\,000x + \frac{10\,000x^2}{2} - \left.\frac{25\,000x^3}{3}\right|_0^{0.600\text{ m}}$$

$$W = 9.00\text{ kJ} + 1.80\text{ kJ} - 1.80\text{ kJ} = \boxed{9.00\text{ kJ}}$$

(b) Similarly,

$$W = (15.0\text{ kN})(1.00\text{ m}) + \frac{(10.0\text{ kN/m})(1.00\text{ m})^2}{2} - \frac{(25.0\text{ kN/m}^2)(1.00\text{ m})^3}{3}$$

$W = \boxed{11.7\text{ kJ}}$, *larger by 29.6%*

***P6.17** $4.00\text{ J} = \frac{1}{2}k(0.100\text{ m})^2$

$\therefore k = 800\text{ N/m}$ and to stretch the spring to 0.200 m requires

$$\Delta W = \frac{1}{2}(800)(0.200)^2 - 4.00\text{ J} = \boxed{12.0\text{ J}}$$

***P6.18** If the weight of the first tray stretches all four springs by a distance equal to the thickness of the tray, then the proportionality expressed by Hooke's law guarantees that each additional tray will have the same effect, so that the top surface of the top tray will always have the same elevation above the floor. The weight of a tray is $0.580\text{ kg}\left(9.8\text{ m/s}^2\right) = 5.68\text{ N}$. The force $\frac{1}{4}(5.68\text{ N}) = 1.42\text{ N}$ should stretch one spring by 0.450 cm, so its spring constant is $k = \frac{|F_s|}{x} = \frac{1.42\text{ N}}{0.0045\text{ m}} = \boxed{316\text{ N/m}}$. We did not need to know the length or width of the tray.

P6.19 (a) The radius to the object makes angle θ with the horizontal, so its weight makes angle θ with the negative side of the x-axis, when we take the x-axis in the direction of motion tangent to the cylinder.

$$\sum F_x = ma_x$$
$$F - mg\cos\theta = 0$$
$$F = \boxed{mg\cos\theta}$$

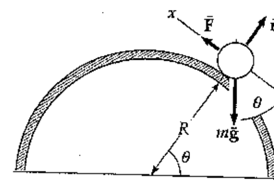

FIG. P6.19

(b) $W = \int_i^f \vec{F}\cdot d\vec{r}$

continued on next page

We use radian measure to express the next bit of displacement as $dr = Rd\theta$ in terms of the next bit of angle moved through:

$$W = \int_0^{\pi/2} mg\cos\theta R d\theta = mgR\sin\theta\Big|_0^{\pi/2}$$

$$W = mgR(1-0) = \boxed{mgR}$$

*P6.20 The same force makes both light springs stretch.

(a) The hanging mass moves down by

$$x = x_1 + x_2 = \frac{mg}{k_1} + \frac{mg}{k_2} = \boxed{mg\left(\frac{1}{k_1} + \frac{1}{k_2}\right)}$$

(b) We define the effective spring constant as

$$k = \frac{F}{x} = \frac{mg}{mg(1/k_1 + 1/k_2)} = \boxed{\left(\frac{1}{k_1} + \frac{1}{k_2}\right)^{-1}}$$

Section 6.5 Kinetic Energy and the Work-Kinetic Energy Theorem

Section 6.6 The Nonisolated System

P6.21 (a) $K_A = \frac{1}{2}(0.600 \text{ kg})(2.00 \text{ m/s})^2 = \boxed{1.20 \text{ J}}$

(b) $\frac{1}{2}mv_B^2 = K_B$. $\quad v_B = \sqrt{\frac{2K_B}{m}} = \sqrt{\frac{(2)(7.50)}{0.600}} = \boxed{5.00 \text{ m/s}}$

(c) $\sum W = \Delta K = K_B - K_A = \frac{1}{2}m\left(v_B^2 - v_A^2\right) = 7.50 \text{ J} - 1.20 \text{ J} = \boxed{6.30 \text{ J}}$

P6.22 (a) $K = \frac{1}{2}mv^2 = \frac{1}{2}(0.300 \text{ kg})(15.0 \text{ m/s})^2 = \boxed{33.8 \text{ J}}$

(b) $K = \frac{1}{2}(0.300)(30.0)^2 = \frac{1}{2}(0.300)(15.0)^2(4) = 4(33.8) = \boxed{135 \text{ J}}$

P6.23 $\vec{v}_i = \left(6.00\hat{i} - 2.00\hat{j}\right)$ m/s

(a) $v_i = \sqrt{v_{ix}^2 + v_{iy}^2} = \sqrt{40.0}$ m/s

$K_i = \frac{1}{2}mv_i^2 = \frac{1}{2}(3.00 \text{ kg})\left(40.0 \text{ m}^2/\text{s}^2\right) = \boxed{60.0 \text{ J}}$

continued on next page

(b) $\vec{v}_f = 8.00\hat{i} + 4.00\hat{j}$

$v_f^2 = \vec{v}_f \cdot \vec{v}_f = 64.0 + 16.0 = 80.0 \text{ m}^2/\text{s}^2$

$\Delta K = K_f - K_i = \frac{1}{2}m\left(v_f^2 - v_i^2\right) = \frac{3.00}{2}(80.0) - 60.0 = \boxed{60.0 \text{ J}}$

P6.24 (a) $\Delta K = K_f - K_i = \frac{1}{2}mv_f^2 - 0 = \sum W =$ (area under curve from $x = 0$ to $x = 5.00$ m)

$v_f = \sqrt{\frac{2(\text{area})}{m}} = \sqrt{\frac{2(7.50 \text{ J})}{4.00 \text{ kg}}} = \boxed{1.94 \text{ m/s}}$

(b) $\Delta K = K_f - K_i = \frac{1}{2}mv_f^2 - 0 = \sum W =$ (area under curve from $x = 0$ to $x = 10.0$ m)

$v_f = \sqrt{\frac{2(\text{area})}{m}} = \sqrt{\frac{2(22.5 \text{ J})}{4.00 \text{ kg}}} = \boxed{3.35 \text{ m/s}}$

(c) $\Delta K = K_f - K_i = \frac{1}{2}mv_f^2 - 0 = \sum W =$ (area under curve from $x = 0$ to $x = 15.0$ m)

$v_f = \sqrt{\frac{2(\text{area})}{m}} = \sqrt{\frac{2(30.0 \text{ J})}{4.00 \text{ kg}}} = \boxed{3.87 \text{ m/s}}$

P6.25 Consider the work done on the pile driver from the time it starts from rest until it comes to rest at the end of the fall. Let $d = 5.00$ m represent the distance over which the driver falls freely, and $h = 0.12$ m the distance it moves the piling.

$\sum W = \Delta K: \quad W_{\text{gravity}} + W_{\text{beam}} = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$

so $\quad (mg)(h+d)\cos 0° + (\bar{F})(d)\cos 180° = 0 - 0$.

Thus, $\quad \bar{F} = \frac{(mg)(h+d)}{d} = \frac{(2\,100 \text{ kg})(9.80 \text{ m/s}^2)(5.12 \text{ m})}{0.120 \text{ m}} = \boxed{8.78 \times 10^5 \text{ N}}$. The force on the pile driver is $\boxed{\text{upward}}$.

P6.26 (a) $K_i + \sum W = K_f = \frac{1}{2}mv_f^2$

$0 + \sum W = \frac{1}{2}\left(15.0 \times 10^{-3} \text{ kg}\right)(780 \text{ m/s})^2 = \boxed{4.56 \text{ kJ}}$

(b) $F = \frac{W}{\Delta r\cos\theta} = \frac{4.56 \times 10^3 \text{ J}}{(0.720 \text{ m})\cos 0°} = \boxed{6.34 \text{ kN}}$

(c) $a = \frac{v_f^2 - v_i^2}{2x_f} = \frac{(780 \text{ m/s})^2 - 0}{2(0.720 \text{ m})} = \boxed{422 \text{ km/s}^2}$

(d) $\sum F = ma = \left(15 \times 10^{-3} \text{ kg}\right)\left(422 \times 10^3 \text{ m/s}^2\right) = \boxed{6.34 \text{ kN}}$

P6.27 $\sum W = \Delta K = 0$: $\int_0^L mg\sin 35.0°\, dl - \int_0^d kx\, dx = 0$

$$mg\sin 35.0°(L) = \frac{1}{2}kd^2$$

$$d = \sqrt{\frac{2mg\sin 35.0°(L)}{k}}$$

$$d = \sqrt{\frac{2(12.0\text{ kg})(9.80\text{ m/s}^2)(\sin 35.0°)(3.00\text{ m})}{3.00\times 10^4\text{ N/m}}} = \boxed{0.116\text{ m}}$$

P6.28 (a) $v_f = 0.096(3\times 10^8\text{ m/s}) = 2.88\times 10^7\text{ m/s}$

$$K_f = \frac{1}{2}mv_f^2 = \frac{1}{2}(9.11\times 10^{-31}\text{ kg})(2.88\times 10^7\text{ m/s})^2 = \boxed{3.78\times 10^{-16}\text{ J}}$$

(b) $K_i + W = K_f$: $0 + F\Delta r\cos\theta = K_f$

$$F(0.028\text{ m})\cos 0° = 3.78\times 10^{-16}\text{ J}$$

$$F = \boxed{1.35\times 10^{-14}\text{ N}}$$

(c) $\sum F = ma$; $a = \frac{\sum F}{m} = \frac{1.35\times 10^{-14}\text{ N}}{9.11\times 10^{-31}\text{ kg}} = \boxed{1.48\times 10^{+16}\text{ m/s}^2}$

(d) $v_{xf} = v_{xi} + a_x t$ $2.88\times 10^7\text{ m/s} = 0 + (1.48\times 10^{16}\text{ m/s}^2)t$

$$t = \boxed{1.94\times 10^{-9}\text{ s}}$$

Check: $x_f = x_i + \frac{1}{2}(v_{xi} + v_{xf})t$

$$0.028\text{ m} = 0 + \frac{1}{2}(0 + 2.88\times 10^7\text{ m/s})t$$

$$t = 1.94\times 10^{-9}\text{ s}$$

Section 6.7 Situations Involving Kinetic Friction

P6.29 $\sum F_y = ma_y$: $n - 392\text{ N} = 0$

$n = 392\text{ N}$

$f_k = \mu_k n = (0.300)(392\text{ N}) = 118\text{ N}$

(a) $W_F = F\Delta r\cos\theta = (130)(5.00)\cos 0° = \boxed{650\text{ J}}$

(b) $\Delta E_{int} = f_k d = (118)(5.00) = \boxed{588\text{ J}}$

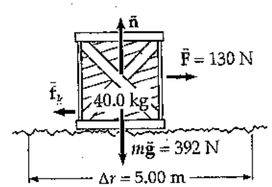

FIG. P6.29

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(c) $W_n = n\Delta r\cos\theta = (392)(5.00)\cos 90° = \boxed{0}$

(d) $W_g = mg\Delta r\cos\theta = (392)(5.00)\cos(-90°) = \boxed{0}$

(e) $\Delta K = K_f - K_i = \sum W_{other} - \Delta E_{int}$

$$\frac{1}{2}mv_f^2 - 0 = 650\text{ J} - 588\text{ J} + 0 + 0 = \boxed{62.0\text{ J}}$$

(f) $v_f = \sqrt{\frac{2K_f}{m}} = \sqrt{\frac{2(62.0\text{ J})}{40.0\text{ kg}}} = \boxed{1.76\text{ m/s}}$

P6.30 (a) $W_s = \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2 = \frac{1}{2}(500)(5.00\times 10^{-2})^2 - 0 = 0.625\text{ J}$

$$W_s = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = \frac{1}{2}mv_f^2 - 0$$

so $v_f = \sqrt{\frac{2(\sum W)}{m}} = \sqrt{\frac{2(0.625)}{2.00}}\text{ m/s} = \boxed{0.791\text{ m/s}}$

(b) $\frac{1}{2}mv_i^2 - f_k d + W_s = \frac{1}{2}mv_f^2$

$$0 - (0.350)(2.00)(9.80)(0.050\,0)\text{ J} + 0.625\text{ J} = \frac{1}{2}mv_f^2$$

$$0.282\text{ J} = \frac{1}{2}(2.00\text{ kg})v_f^2$$

$$v_f = \sqrt{\frac{2(0.282)}{2.00}}\text{ m/s} = \boxed{0.531\text{ m/s}}$$

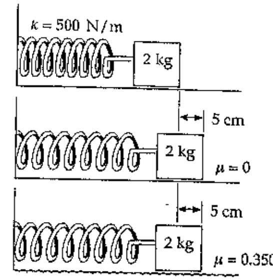

FIG. P6.30

P6.31 (a) $W_g = mg\ell\cos(90.0° + \theta)$

$$W_g = (10.0\text{ kg})(9.80\text{ m/s}^2)(5.00\text{ m})\cos 110° = \boxed{-168\text{ J}}$$

(b) $f_k = \mu_k n = \mu_k mg\cos\theta$

$\Delta E_{int} = \ell f_k = \ell\mu_k mg\cos\theta$

$\Delta E_{int} = (5.00\text{ m})(0.400)(10.0)(9.80)\cos 20.0° = \boxed{184\text{ J}}$

(c) $W_F = F\ell = (100)(5.00) = \boxed{500\text{ J}}$

(d) $\Delta K = \sum W_{other} - \Delta E_{int} = W_F + W_g - \Delta E_{int} = \boxed{148\text{ J}}$

(e) $\Delta K = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$

$$v_f = \sqrt{\frac{2(\Delta K)}{m} + v_i^2} = \sqrt{\frac{2(148)}{10.0} + (1.50)^2} = \boxed{5.65\text{ m/s}}$$

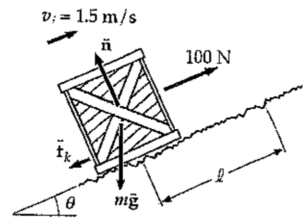

FIG. P6.31

P6.32 $\sum F_y = ma_y$: $n + (70.0\text{ N})\sin 20.0° - 147\text{ N} = 0$

$n = 123\text{ N}$

$f_k = \mu_k n = 0.300(123\text{ N}) = 36.9\text{ N}$

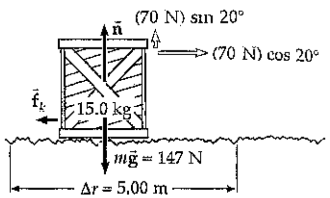

FIG. P6.32

(a) $W = F\Delta r\cos\theta = (70.0\text{ N})(5.00\text{ m})\cos 20.0° = \boxed{329\text{ J}}$

(b) $W = F\Delta r\cos\theta = (123\text{ N})(5.00\text{ m})\cos 90.0° = \boxed{0\text{ J}}$

(c) $W = F\Delta r\cos\theta = (147\text{ N})(5.00\text{ m})\cos 90.0° = \boxed{0}$

(d) $\Delta E_{int} = f_k d = (36.9\text{ N})(5.00\text{ m}) = \boxed{185\text{ J}}$

(e) $\Delta K = K_f - K_i = \sum W - \Delta E_{int} = 329\text{ J} - 185\text{ J} = \boxed{+144\text{ J}}$

P6.33 $v_i = 2.00\text{ m/s}$ $\quad\mu_k = 0.100$

$K_i - f_k d + W_{other} = K_f$: $\quad \frac{1}{2}mv_i^2 - f_k d = 0$

$\frac{1}{2}mv_i^2 = \mu_k mgd \quad d = \frac{v_i^2}{2\mu_k g} = \frac{(2.00\text{ m/s})^2}{2(0.100)(9.80)} = \boxed{2.04\text{ m}}$

Section 6.8 Power

P6.34 (a) The distance moved upward in the first 3.00 s is

$$\Delta y = \bar{v}t = \left[\frac{0 + 1.75\text{ m/s}}{2}\right](3.00\text{ s}) = 2.63\text{ m}.$$

The motor and the earth's gravity do work on the elevator car:

$$\frac{1}{2}mv_i^2 + W_{motor} + mg\Delta y\cos 180° = \frac{1}{2}mv_f^2$$
$$W_{motor} = \frac{1}{2}(650\text{ kg})(1.75\text{ m/s})^2 - 0 + (650\text{ kg})g(2.63\text{ m}) = 1.77\times 10^4\text{ J}$$

Also, $W = \bar{\mathcal{P}}\Delta t$ so $\bar{\mathcal{P}} = \frac{W}{\Delta t} = \frac{1.77\times 10^4\text{ J}}{3.00\text{ s}} = \boxed{5.91\times 10^3\text{ W}} = 7.92\text{ hp}.$

(b) When moving upward at constant speed ($v = 1.75$ m/s) the applied force equals the weight $= (650\text{ kg})(9.80\text{ m/s}^2) = 6.37\times 10^3$ N. Therefore,

$$\mathcal{P} = Fv = (6.37\times 10^3\text{ N})(1.75\text{ m/s}) = \boxed{1.11\times 10^4\text{ W}} = 14.9\text{ hp}.$$

P6.35 $\text{Power} = \frac{W}{\Delta t} \qquad \mathcal{P} = \frac{mgh}{\Delta t} = \frac{(700\text{ N})(10.0\text{ m})}{8.00\text{ s}} = \boxed{875\text{ W}}$

P6.36 (a) $\sum W = \Delta K$, but $\Delta K = 0$ because he moves at constant speed. The skier rises a vertical distance of $(60.0\text{ m})\sin 30.0° = 30.0$ m. Thus,

$$W_{in} = -W_g = (70.0\text{ kg})(9.8\text{ m/s}^2)(30.0\text{ m}) = \boxed{2.06\times 10^4\text{ J}} = \boxed{20.6\text{ kJ}}.$$

(b) The time to travel 60.0 m at a constant speed of 2.00 m/s is 30.0 s. Thus,

$$\mathcal{P}_{input} = \frac{W}{\Delta t} = \frac{2.06\times 10^4\text{ J}}{30.0\text{ s}} = \boxed{686\text{ W}} = 0.919\text{ hp}.$$

***P6.37** *energy = power × time*

For the 28.0 W bulb:

Energy used $= (28.0\text{ W})(1.00\times 10^4\text{ h}) = 280$ kilowatt·hrs

total cost $= \$17.00 + (280\text{ kWh})(\$0.080/\text{kWh}) = \$39.4$

For the 100 W bulb:

Energy used $= (100\text{ W})(1.00\times 10^4\text{ h}) = 1.00\times 10^3$ kilowatt·hrs

bulb used $= \frac{1.00\times 10^4\text{ h}}{750\text{ h/bulb}} = 13.3$

total cost $= 13.3(\$0.420) + (1.00\times 10^3\text{ kWh})(\$0.080/\text{kWh}) = \$85.6$

Savings with energy-efficient bulb $= \$85.60 - \$39.40 = \boxed{\$46.2}$

P6.38 (a) Burning 1 lb of fat releases energy $\quad 1\text{ lb}\left(\frac{454\text{ g}}{1\text{ lb}}\right)\left(\frac{9\text{ kcal}}{1\text{ g}}\right)\left(\frac{4\,186\text{ J}}{1\text{ kcal}}\right) = 1.71\times 10^7\text{ J}.$

The mechanical energy output is $\quad (1.71\times 10^7\text{ J})(0.20) = nF\Delta r\cos\theta.$

Then

$$3.42\times 10^6\text{ J} = nmg\Delta y\cos 0°$$
$$3.42\times 10^6\text{ J} = n(50\text{ kg})(9.8\text{ m/s}^2)(80\text{ steps})(0.150\text{ m})$$
$$3.42\times 10^6\text{ J} = n(5.88\times 10^3\text{ J})$$

where the number of times she must climb the steps is $n = \frac{3.42\times 10^6\text{ J}}{5.88\times 10^3\text{ J}} = \boxed{582}$.

This method is impractical compared to limiting food intake.

continued on next page

(b) Her mechanical power output is

$$\mathcal{P}=\frac{W}{\Delta t}=\frac{5.88\times10^3\ \text{J}}{65\ \text{s}}=\boxed{90.5\ \text{W}}=90.5\ \text{W}\left(\frac{1\ \text{hp}}{746\ \text{W}}\right)=\boxed{0.121\ \text{hp}}.$$

P6.39 (a) The fuel economy for walking is $\frac{1\ \text{h}}{220\ \text{kcal}}\left(\frac{3\ \text{mi}}{\text{h}}\right)\left(\frac{1\ \text{kcal}}{4\,186\ \text{J}}\right)\left(\frac{1.30\times10^8\ \text{J}}{1\ \text{gal}}\right)=\boxed{423\ \text{mi/gal}}$.

(b) For bicycling $\frac{1\ \text{h}}{400\ \text{kcal}}\left(\frac{10\ \text{mi}}{\text{h}}\right)\left(\frac{1\ \text{kcal}}{4\,186\ \text{J}}\right)\left(\frac{1.30\times10^8\ \text{J}}{1\ \text{gal}}\right)=\boxed{776\ \text{mi/gal}}$.

Section 6.9 Context Connection—Horsepower Ratings of Automobiles

P6.40 A 1 300-kg car speeds up from rest to 55.0 mi/h = 24.6 m/s in 15.0 s. The output work of the engine is equal to its final kinetic energy,

$$\frac{1}{2}(1\,300\ \text{kg})(24.6\ \text{m/s})^2=390\ \text{kJ}$$

with power $\mathcal{P}=\frac{390\,000\ \text{J}}{15.0\ \text{s}}\boxed{\sim10^4\ \text{W}}$ around 30 horsepower.

P6.41 $\mathcal{P}_a=f_a v$: $\quad f_a=\frac{\mathcal{P}_a}{v}=\frac{2.24\times10^4}{27.0}=\boxed{830\ \text{N}}$

Additional Problems

P6.42 At start, $\vec{v}=(40.0\ \text{m/s})\cos30.0°\,\hat{i}+(40.0\ \text{m/s})\sin30.0°\,\hat{j}$

At apex, $\vec{v}=(40.0\ \text{m/s})\cos30.0°\,\hat{i}+0\hat{j}=(34.6\ \text{m/s})\hat{i}$

And $K=\frac{1}{2}mv^2=\frac{1}{2}(0.150\ \text{kg})(34.6\ \text{m/s})^2=\boxed{90.0\ \text{J}}$

***P6.43** Concentration of Energy output $=(0.600\ \text{J/kg}\cdot\text{step})(60.0\ \text{kg})\left(\frac{1\ \text{step}}{1.50\ \text{m}}\right)=24.0\ \text{J/m}$

$$F=(24.0\ \text{J/m})(1\ \text{N}\cdot\text{m/J})=24.0\ \text{N}$$
$$\mathcal{P}=Fv$$
$$70.0\ \text{W}=(24.0\ \text{N})v$$
$$v=\boxed{2.92\ \text{m/s}}$$

***P6.44** As it moves at constant speed, the bicycle is in equilibrium. The forward frictional force is equal in magnitude to the air resistance, which we write as av^2 where a is a proportionality constant. The exercising woman exerts the friction force on the ground; by Newton's third law, it is this same size again. The woman's power output is $\mathcal{P}=Fv=av^3=ch$, where c is another constant and h is her heart rate. We are given $a(22\ \text{km/h})^3=c(90\ \text{beats/min})$. For her minimum heart rate we have

$av_{\min}^3=c(136\ \text{beats/min})$. By division $\left(\frac{v_{\min}}{22\ \text{km/h}}\right)^3=\frac{136}{90}$, $v_{\min}=\left(\frac{136}{90}\right)^{1/3}(22\ \text{km/h})=\boxed{25.2\ \text{km/h}}$.

Similarly, $v_{\max}=\left(\frac{166}{90}\right)^{1/3}(22\ \text{km/h})=\boxed{27.0\ \text{km/h}}$.

P6.45 (a) $x=t+2.00t^3$

Therefore,

$$v=\frac{dx}{dt}=1+6.00t^2$$
$$K=\frac{1}{2}mv^2=\frac{1}{2}(4.00)(1+6.00t^2)^2=\boxed{(2.00+24.0t^2+72.0t^4)\ \text{J}}$$

(b) $a=\frac{dv}{dt}=\boxed{(12.0t)\ \text{m/s}^2}$

$F=ma=4.00(12.0t)=\boxed{(48.0t)\ \text{N}}$

(c) $\mathcal{P}=Fv=(48.0t)(1+6.00t^2)=\boxed{(48.0t+288t^3)\ \text{W}}$

(d) $W=\int_0^{2.00}\mathcal{P}dt=\int_0^{2.00}(48.0t+288t^3)dt=\boxed{1\,250\ \text{J}}$

***P6.46** The work done by the applied force is

$$W=\int_i^f F_{\text{applied}}dx=\int_0^{x_{\max}}-\left[-(k_1x+k_2x^2)\right]dx$$
$$=\int_0^{x_{\max}}k_1x\,dx+\int_0^{x_{\max}}k_2x^2dx=k_1\left.\frac{x^2}{2}\right|_0^{x_{\max}}+k_2\left.\frac{x^3}{3}\right|_0^{x_{\max}}$$
$$=\boxed{k_1\frac{x_{\max}^2}{2}+k_2\frac{x_{\max}^3}{3}}$$

P6.47 (a) The work done by the traveler is mgh_sN where N is the number of steps he climbs during the ride.

where
$$N = (\text{time on escalator})(n)$$

and
$$(\text{time on escalator}) = \frac{h}{\text{vertical velocity of person}}$$
$$\text{vertical velocity of person} = v + nh_s$$

Then,
$$N = \frac{nh}{v+nh_s}$$

and the work done by the person becomes $W_{\text{person}} = \boxed{\dfrac{mgnhh_s}{v+nh_s}}$

(b) The work done by the escalator is

$$W_e = (\text{power})(\text{time}) = |(\text{force exerted})(\text{speed})(\text{time})| = mgvt$$

where $t = \dfrac{h}{v+nh_s}$ as above.

Thus, $W_e = \boxed{\dfrac{mgvh}{v+nh_s}}$.

As a check, the total work done on the person's body must add up to mgh, the work an elevator would do in lifting him.

It does add up as follows: $\sum W = W_{\text{person}} + W_e = \dfrac{mgnhh_s}{v+nh_s} + \dfrac{mgvh}{v+nh_s} = \dfrac{mgh(nh_s+v)}{v+nh_s} = mgh$

***P6.48** During its whole motion from $y = 10.0$ m to $y = -3.20$ mm, the force of gravity and the force of the plate do work on the ball. It starts and ends at rest

$$K_i + \sum W = K_f$$
$$0 + F_g\Delta y\cos 0° + F_p\Delta x\cos 180° = 0$$
$$mg(10.003\,2\text{ m}) - F_p(0.003\,20\text{ m}) = 0$$
$$F_p = \frac{5\text{ kg}(9.8\text{ m/s}^2)(10\text{ m})}{3.2\times10^{-3}\text{ m}} = \boxed{1.53\times10^5\text{ N upward}}$$

P6.49 $\Sigma F_x = ma_x$, $\quad kx = ma$

$$k = \frac{ma}{x} = \frac{(4.70\times10^{-3}\text{ kg})0.800(9.80\text{ m/s}^2)}{0.500\times10^{-2}\text{ m}} = \boxed{7.37\text{ N/m}}$$

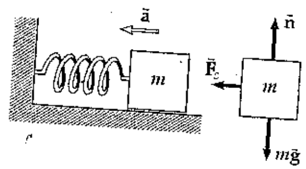

FIG. P6.49

***P6.50** The spring exerts on each block an outward force of magnitude

$$|F_s| = kx = (3.85\text{ N/m})(0.08\text{ m}) = 0.308\text{ N}.$$

For the light block on the left, the vertical forces are given by $F_g = mg = (0.25\text{ kg})(9.8\text{ m/s}^2) = 2.45\text{ N}$, $\sum F_y = 0$, $n - 2.45\text{ N} = 0$, $n = 2.45\text{ N}$. Similarly for the heavier block $n = F_g = (0.5\text{ kg})(9.8\text{ m/s}^2) = 4.9\text{ N}$.

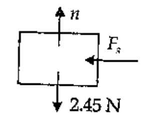

FIG. P6.50

(a) For the block on the left, $\sum F_x = ma_x$, $-0.308\text{ N} = (0.25\text{ kg})a$, $a = \boxed{-1.23\text{ m/s}^2}$. For the heavier block, $+0.308\text{ N} = (0.5\text{ kg})a$, $a = \boxed{0.616\text{ m/s}^2}$.

(b) For the block on the left, $f_k = \mu_k n = 0.1(2.45\text{ N}) = 0.245\text{ N}$

$$\sum F_x = ma_x$$
$$-0.308\text{ m/s}^2 + 0.245\text{ N} = (0.25\text{ kg})a$$
$$a = \boxed{-0.252\text{ m/s}^2}$$

if the force of static friction is not too large. For the block on the right, $f_k = \mu_k n = 0.490$ N. The maximum force of static friction would be larger, so no motion would begin and the acceleration is $\boxed{\text{zero}}$.

(c) Left block: $f_k = 0.462(2.45\text{ N}) = 1.13$ N. The maximum static friction force would be larger, so the spring force would produce no motion of this block or of the right-hand block, which could feel even more friction force. For both $a = \boxed{0}$.

P6.51 (a) $\mathscr{P} = Fv = F(v_i + at) = F\left(0 + \dfrac{F}{m}t\right) = \boxed{\left(\dfrac{F^2}{m}\right)t}$

(b) $\mathscr{P} = \left[\dfrac{(20.0\text{ N})^2}{5.00\text{ kg}}\right](3.00\text{ s}) = \boxed{240\text{ W}}$

P6.52 (a) The new length of each spring is $\sqrt{x^2+L^2}$, so its extension is $\sqrt{x^2+L^2} - L$ and the force it exerts is $k\left(\sqrt{x^2+L^2} - L\right)$ toward its fixed end. The y components of the two spring forces add to zero. Their x components add to

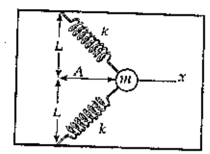

FIG. P6.52

$$\vec{F} = -2\hat{i}k\left(\sqrt{x^2+L^2} - L\right)\frac{x}{\sqrt{x^2+L^2}} = \boxed{-2kx\hat{i}\left(1 - \frac{L}{\sqrt{x^2+L^2}}\right)}.$$

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(b) $W = \int_i^f F_x dx$ $\quad$ $W = \int_A^0 -2kx\left(1 - \frac{L}{\sqrt{x^2+L^2}}\right)dx$

$W = -2k\int_A^0 x\,dx + kL\int_A^0 \left(x^2+L^2\right)^{-1/2} 2x\,dx$ $\quad$ $W = -2k\left.\frac{x^2}{2}\right|_A^0 + kL\left.\frac{\left(x^2+L^2\right)^{1/2}}{(1/2)}\right|_A^0$

$W = -0 + kA^2 + 2kL^2 - 2kL\sqrt{A^2+L^2}$ $\quad$ $W = \boxed{2kL^2 + kA^2 - 2kL\sqrt{A^2+L^2}}$

P6.53 (a) $\sum W = \Delta K$: $\quad W_s + W_g = 0$

$\frac{1}{2}kx_i^2 - 0 + mg\Delta x \cos(90°+60°) = 0$

$\frac{1}{2}\left(1.40\times 10^3 \text{ N/m}\right)\times(0.100)^2 - (0.200)(9.80)(\sin 60.0°)\Delta x = 0$

$\Delta x = \boxed{4.12 \text{ m}}$

(b) $\sum W = \Delta K + \Delta E_{int}$. $\quad W_s + W_g - \Delta E_{int} = 0$

$\frac{1}{2}kx_i^2 + mg\Delta x\cos 150° - \mu_k mg\cos 60°\,\Delta x = 0$

$\frac{1}{2}\left(1.40\times 10^3 \text{ N/m}\right)\times(0.100)^2 - (0.200)(9.80)(\sin 60.0°)\Delta x - (0.200)(9.80)(0.400)(\cos 60.0°)\Delta x = 0$

$\Delta x = \boxed{3.35 \text{ m}}$

P6.54 (a) $\vec{F}_1 = (25.0 \text{ N})\left(\cos 35.0°\,\hat{i} + \sin 35.0°\,\hat{j}\right) = \boxed{\left(20.5\hat{i} + 14.3\hat{j}\right)\text{ N}}$

$\vec{F}_2 = (42.0 \text{ N})\left(\cos 150°\,\hat{i} + \sin 150°\,\hat{j}\right) = \boxed{\left(-36.4\hat{i} + 21.0\hat{j}\right)\text{ N}}$

(b) $\sum\vec{F} = \vec{F}_1 + \vec{F}_2 = \boxed{\left(-15.9\hat{i} + 35.3\hat{j}\right)\text{ N}}$

(c) $\vec{a} = \frac{\sum\vec{F}}{m} = \boxed{\left(-3.18\hat{i} + 7.07\hat{j}\right)\text{ m/s}^2}$

(d) $\vec{v}_f = \vec{v}_i + \vec{a}t = \left(4.00\hat{i} + 2.50\hat{j}\right)\text{ m/s} + \left(-3.18\hat{i} + 7.07\hat{j}\right)\left(\text{m/s}^2\right)(3.00 \text{ s})$

$\vec{v}_f = \boxed{\left(-5.54\hat{i} + 23.7\hat{j}\right)\text{ m/s}}$

(e) $\vec{r}_f = \vec{r}_i + \vec{v}_i t + \frac{1}{2}\vec{a}t^2$

$\vec{r}_f = 0 + \left(4.00\hat{i} + 2.50\hat{j}\right)(\text{m/s})(3.00 \text{ s}) + \frac{1}{2}\left(-3.18\hat{i} + 7.07\hat{j}\right)\left(\text{m/s}^2\right)(3.00 \text{ s})^2$

$\Delta\vec{r} = \vec{r}_f = \boxed{\left(-2.30\hat{i} + 39.3\hat{j}\right)\text{ m}}$

continued on next page

(f) $K_f = \frac{1}{2}mv_f^2 = \frac{1}{2}(5.00 \text{ kg})\left[(5.54)^2 + (23.7)^2\right]\left(\text{m/s}^2\right) = \boxed{1.48 \text{ kJ}}$

(g) $K_f = \frac{1}{2}mv_i^2 + \sum\vec{F}\cdot\Delta\vec{r}$

$K_f = \frac{1}{2}(5.00 \text{ kg})\left[(4.00)^2 + (2.50)^2\right](\text{m/s})^2 + \left[(-15.9 \text{ N})(-2.30 \text{ m}) + (35.3 \text{ N})(39.3 \text{ m})\right]$

$K_f = 55.6 \text{ J} + 1\,426 \text{ J} = \boxed{1.48 \text{ kJ}}$

P6.55 $K_i + W_s + W_g = K_f$

$\frac{1}{2}mv_i^2 + \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2 + mg\Delta x\cos\theta = \frac{1}{2}mv_f^2$

$0 + \frac{1}{2}kx_i^2 - 0 + mgx_i\cos 100° = \frac{1}{2}mv_f^2$

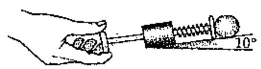

FIG. P6.55

$\frac{1}{2}(1.20 \text{ N/cm})(5.00 \text{ cm})(0.050\,0 \text{ m}) - (0.100 \text{ kg})\left(9.80 \text{ m/s}^2\right)(0.050\,0 \text{ m})\sin 10.0° = \frac{1}{2}(0.100 \text{ kg})v^2$

$0.150 \text{ J} - 8.51\times 10^{-3} \text{ J} = (0.050\,0 \text{ kg})v^2$

$v = \sqrt{\frac{0.141}{0.050\,0}} = \boxed{1.68 \text{ m/s}}$

P6.56 (a)

F(N)	L(mm)	F(N)	L(mm)
2.00	15.0	14.0	112
4.00	32.0	16.0	126
6.00	49.0	18.0	149
8.00	64.0	20.0	175
10.0	79.0	22.0	190
12.0	98.0		

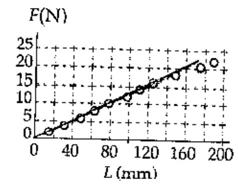

FIG. P6.56

(b) A straight line fits the first eight points, together with the origin. By least-square fitting, its slope is $0.125 \text{ N/mm} \pm 2\% = \boxed{125 \text{ N/m}} \pm 2\%$. In $F = kx$, the spring constant is $k = \frac{F}{x}$, the same as the slope of the F-versus-x graph.

(c) $F = kx = (125 \text{ N/m})(0.105 \text{ m}) = \boxed{13.1 \text{ N}}$

P6.57 If positive F represents an outward force, (same as direction as r), then

$$W=\int_i^f \vec{\mathbf{F}}\cdot d\vec{\mathbf{r}}=\int_{r_i}^{r_f}\left(2F_0\sigma^{13}r^{-13}-F_0\sigma^7r^{-7}\right)dr$$

$$W=\frac{2F_0\sigma^{13}r^{-12}}{-12}-\frac{F_0\sigma^7r^{-6}}{-6}\Bigg|_{r_i}^{r_f}$$

$$W=\frac{-F_0\sigma^{13}\left(r_f^{-12}-r_i^{-12}\right)}{6}+\frac{F_0\sigma^7\left(r_f^{-6}-r_i^{-6}\right)}{6}=\frac{F_0\sigma^7}{6}\left[r_f^{-6}-r_i^{-6}\right]-\frac{F_0\sigma^{13}}{6}\left[r_f^{-12}-r_i^{-12}\right]$$

$$W=1.03\times10^{-77}\left[r_f^{-6}-r_i^{-6}\right]-1.89\times10^{-134}\left[r_f^{-12}-r_i^{-12}\right]$$

$$W=1.03\times10^{-77}\left[1.88\times10^{-6}-2.44\times10^{-4}\right]10^{60}-1.89\times10^{-134}\left[3.54\times10^{-12}-5.96\times10^{-8}\right]10^{120}$$

$$W=-2.49\times10^{-21}\text{ J}+1.12\times10^{-21}\text{ J}=\boxed{-1.37\times10^{-21}\text{ J}}$$

P6.58 (a) $\Delta E_{\text{int}}=-\Delta K=-\frac{1}{2}m\left(v_f^2-v_i^2\right)$: $\Delta E_{\text{int}}=-\frac{1}{2}(0.400\text{ kg})\left[(6.00)^2-(8.00)^2\right](\text{m/s})^2=\boxed{5.60\text{ J}}$

(b) $\Delta E_{\text{int}}=fd=\mu_k mg(2\pi r)$: $5.60\text{ J}=\mu_k(0.400\text{ kg})(9.80\text{ m/s}^2)2\pi(1.50\text{ m})$

Thus, $\mu_k=\boxed{0.152}$.

(c) After N revolutions, the object comes to rest and $K_f=0$.

Thus, $\Delta E_{\text{int}}=-\Delta K=-0+K_i=\frac{1}{2}mv_i^2$

or $\mu_k mg\left[N(2\pi r)\right]=\frac{1}{2}mv_i^2$.

This gives $N=\dfrac{\frac{1}{2}mv_i^2}{\mu_k mg(2\pi r)}=\dfrac{\frac{1}{2}(8.00\text{ m/s})^2}{(0.152)(9.80\text{ m/s}^2)2\pi(1.50\text{ m})}=\boxed{2.28\text{ rev}}$.

P6.59 We evaluate $\int_{12.8}^{23.7}\frac{375dx}{x^3+3.75x}$ by calculating

$$\frac{375(0.100)}{(12.8)^3+3.75(12.8)}+\frac{375(0.100)}{(12.9)^3+3.75(12.9)}+\dots\frac{375(0.100)}{(23.6)^3+3.75(23.6)}=0.806$$

and

$$\frac{375(0.100)}{(12.9)^3+3.75(12.9)}+\frac{375(0.100)}{(13.0)^3+3.75(13.0)}+\dots\frac{375(0.100)}{(23.7)^3+3.75(23.7)}=0.791.$$

The answer must be between these two values. We may find it more precisely by using a value for Δx smaller than 0.100. Thus, we find the integral to be $\boxed{0.799\text{ N}\cdot\text{m}}$.

P6.60 $\mathscr{P}\Delta t=W=\Delta K=\dfrac{(\Delta m)v^2}{2}$

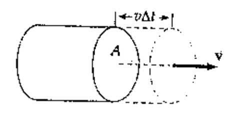

FIG. P6.60

The density is $\rho=\dfrac{\Delta m}{\text{vol}}=\dfrac{\Delta m}{A\Delta x}$

Substituting this into the first equation and solving for $\mathscr{P}$, since $\dfrac{\Delta x}{\Delta t}=v$,

for a constant speed, we get $\boxed{\mathscr{P}=\dfrac{\rho Av^3}{2}}$.

Also, since $\mathscr{P}=Fv$, $\boxed{F=\dfrac{\rho Av^2}{2}}$.

Our model predicts the same proportionalities as the empirical equation, and gives $D=1$ for the drag coefficient. Air actually slips around the moving object, instead of accumulating in front of it. For this reason, the drag coefficient is not necessarily unity. It is typically less than one for a streamlined object and can be greater than one if the airflow around the object is complicated.

***P6.61** $\mathscr{P}=\frac{1}{2}D\rho\pi r^2v^3$

(a) $\mathscr{P}_a=\frac{1}{2}1\left(1.20\text{ kg/m}^3\right)\pi(1.5\text{ m})^2(8\text{ m/s})^3=\boxed{2.17\times10^3\text{ W}}$

(b) $\dfrac{\mathscr{P}_b}{\mathscr{P}_a}=\dfrac{v_b^3}{v_a^3}=\left(\dfrac{24\text{ m/s}}{8\text{ m/s}}\right)^3=3^3=27$

$\mathscr{P}_b=27\left(2.17\times10^3\text{ W}\right)=\boxed{5.86\times10^4\text{ W}}$

***P6.62** (a) So long as the spring force is greater than the friction force, the block will be gaining speed. The block slows down when the friction force becomes the greater. It has maximum speed when $-kx_a-f_k=ma=0$.

$-\left(1.0\times10^3\text{ N/m}\right)x_a-4.0\text{ N}=0$ $\boxed{x=-4.0\times10^{-3}\text{ m}}$

(b) By the same logic,

$-\left(1.0\times10^3\text{ N/m}\right)x_b-10.0\text{ N}=0$ $\boxed{x=-1.0\times10^{-2}\text{ m}}$

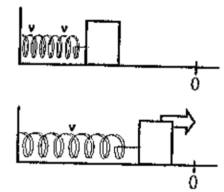

FIG. P6.62

ANSWERS TO EVEN PROBLEMS

P6.2 1.59×10^3 J

P6.4 (a) 32.8 mJ; (b) −32.8 mJ

P6.6 see the solution

P6.8 16.0

P6.10 (a) see the solution; (b) −12.0 J

P6.12 0.299 m/s

P6.14 50.0 J

P6.16 (a) 9.00 kJ; (b) 11.7 kJ, larger by 29.6%

P6.18 see the solution, 316 N/m

P6.20 (a) $mg\left(\frac{1}{k_1}+\frac{1}{k_2}\right)$; (b) $\left(\frac{1}{k_1}+\frac{1}{k_2}\right)^{-1}$

P6.22 (a) 33.8 J; (b) 135 J

P6.24 (a) 1.94 m/s; (b) 3.35 m/s; (c) 3.87 m/s

P6.26 (a) 4.56 kJ; (b) 6.34 kN; (c) 422 km/s^2; (d) 6.34 kN

P6.28 (a) 3.78×10^{-16} J; (b) 1.35×10^{-14} N; (c) $1.48\times10^{+16}$ m/s^2; (d) 1.94×10^{-9} s

P6.30 (a) 0.791 m/s; (b) 0.531 m/s

P6.32 (a) 329 J; (b) 0; (c) 0; (d) 185 J; (e) 144 J

P6.34 (a) 5.91×10^3 W; (b) it is 53.0% of 1.11×10^4 W

P6.36 (a) 20.6 kJ; (b) 686 W

P6.38 (a) 582, impractical; (b) 90.5 W = 0.121 hp

P6.40 $\sim10^4$ W

P6.42 90.0 J

P6.44 25.2 km/h, 27.0 km/h

P6.46 $k_1\frac{x_{max}^2}{2}+k_2\frac{x_{max}^3}{3}$

P6.48 1.53×10^5 N upward

P6.50 (a) -1.23 m/s^2 and 0.616 m/s^2; (b) -0.252 m/s^2 and 0; (c) 0 and 0

P6.52 (a) see the solution; (b) $2kL^2+kA^2-2kL\sqrt{A^2+L^2}$

P6.54 (a) $\vec{F}_1=(20.5\hat{i}+14.3\hat{j})$ N, $\vec{F}_2=(-36.4\hat{i}+21.0\hat{j})$ N; (b) $(-15.9\hat{i}+35.3\hat{j})$ N; (c) $(-3.18\hat{i}+7.07\hat{j})$ m/s^2; (d) $(-5.54\hat{i}+23.7\hat{j})$ m/s; (e) $(-2.30\hat{i}+39.3\hat{j})$ m; (f) 1.48 kJ; (g) 1.48 kJ

P6.56 (a) see the solution; (b) 125 N/m ± 2%, (c) 13.1 N

P6.58 (a) 5.60 J; (b) 0.152; (c) 2.28 rev

P6.60 see the solution

P6.62 (a) $x=-4.0$ mm; (b) -1.0 cm

Chapter 7 Potential Energy

CHAPTER OUTLINE

7.1	Potential Energy of a System
7.2	The Isolated System
7.3	Conservative and Nonconservative Forces
7.4	Conservative Forces and Potential Energy
7.5	The Nonisolated System in Steady State
7.6	Potential Energy for Gravitational and Electric Forces
7.7	Energy Diagrams and Stability of Equilibrium
7.8	Context Connection—Potential Energy in Fuels

ANSWERS TO QUESTIONS

Q7.1 The final speed of the children will not depend on the slide length or the presence of bumps if there is no friction. If there is friction, a longer slide will result in a lower final speed. Bumps will have the same effect as they effectively lengthen the distance over which friction can do work, to decrease the total mechanical energy of the children.

Q7.2 Total energy is the sum of kinetic and potential energies. Potential energy can be negative, so the sum of kinetic plus potential can also be negative.

Q7.3 Both agree on the *change* in potential energy, and the kinetic energy. They may disagree on the value of gravitational potential energy, depending on their choice of a zero point.

Q7.4 (a) mgh is provided by the muscles.

(b) No further energy is supplied to the object-Earth system, but some chemical energy must be supplied to the muscles as they keep the weight aloft.

(c) The object loses energy mgh, giving it back to the muscles, where most of it becomes internal energy.

Q7.5 Lift a book from a low shelf to place it on a high shelf. The net change in its kinetic energy is zero, but the book-Earth system increases in gravitational potential energy. Stretch a rubber band to encompass the ends of a ruler. It increases in elastic energy. Rub your hands together or let a pearl drift down at constant speed in a bottle of shampoo. Each system (two hands; pearl and shampoo) increases in internal energy.

Q7.6 Three potential energy terms will appear in the expression of total mechanical energy, one for each conservative force. If you write an equation with initial energy on one side and final energy on the other, the equation contains six potential-energy terms.

Q7.7 (a) It does if it makes the object's speed change, but not if it only makes the direction of the velocity change.

(b) Yes, according to Newton's second law.

Q7.8 The original kinetic energy of the skidding car can be degraded into kinetic energy of random molecular motion in the tires and the road: it is internal energy. If the brakes are used properly, the same energy appears as internal energy in the brake shoes and drums.

Q7.9 All the energy is supplied by foodstuffs that gained their energy from the sun.

Q7.10 Elastic potential energy of plates under stress plus gravitational energy is released when the plates "slip". It is carried away by mechanical waves.

Q7.11 The total energy of the ball-Earth system is conserved. Since the system initially has gravitational energy mgh and no kinetic energy, the ball will again have zero kinetic energy when it returns to its original position. Air resistance will cause the ball to come back to a point slightly below its initial position. On the other hand, if anyone gives a forward push to the ball anywhere along its path, the demonstrator will have to duck.

Q7.12 Kinetic energy is greatest at the starting point. Gravitational energy is a maximum at the top of the flight of the ball.

Q7.13 Gravitational energy is proportional to mass, so it doubles.

Q7.14 In stirring cake batter and in weightlifting, your body returns to the same conformation after each stroke. During each stroke chemical energy is irreversibly converted into output work (and internal energy). This observation proves that muscular forces are nonconservative.

Q7.15 Let the gravitational energy be zero at the lowest point in the motion. If you start the vibration by pushing down on the block (2), its kinetic energy becomes extra elastic potential energy in the spring (U_s). After the block starts moving up at its lower turning point (3), this energy becomes both kinetic energy (K) and gravitational potential energy (U_g), and then just gravitational energy when the block is at its greatest height (1). The energy then turns back into kinetic and elastic potential energy, and the cycle repeats.

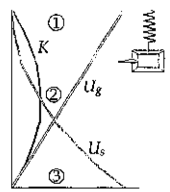

FIG. Q7.15

Q7.16 Chemical energy in the fuel turns into internal energy as the fuel burns. Most of this leaves the car by heat through the walls of the engine and by matter transfer in the exhaust gases. Some leaves the system of fuel by work done to push down the piston. Of this work, a little results in internal energy in the bearings and gears, but most becomes work done on the air to push it aside. The work on the air immediately turns into internal energy in the air. If you use the windshield wipers, you take energy from the crankshaft and turn it into extra internal energy in the glass and wiper blades and wiper-motor coils. If you turn on the air conditioner, your end effect is to put extra energy out into the surroundings. You must apply the brakes at the end of your trip. As soon as the sound of the engine has died away, all you have to show for it is thermal pollution.

Q7.17 A graph of potential energy versus position is a straight horizontal line for a particle in neutral equilibrium. The graph represents a constant function.

Q7.18 The ball is in neutral equilibrium.

SOLUTIONS TO PROBLEMS

Section 7.1 Potential Energy of a System

P7.1 (a) With our choice for the zero level for potential energy when the car is at point B,

$$\boxed{U_B = 0}.$$

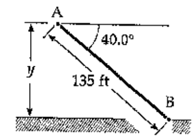

FIG. P7.1

When the car is at point A, the potential energy of the car-Earth system is given by

$$U_A = mgy$$

where y is the vertical height above zero level. With 135 ft = 41.1 m, this height is found as:

$$y = (41.1\text{ m})\sin 40.0° = 26.4\text{ m}.$$

Thus,

$$U_A = (1\,000\text{ kg})(9.80\text{ m/s}^2)(26.4\text{ m}) = \boxed{2.59\times10^5\text{ J}}.$$

The change in potential energy as the car moves from A to B is

$$U_B - U_A = 0 - 2.59\times10^5\text{ J} = \boxed{-2.59\times10^5\text{ J}}.$$

(b) With our choice of the zero level when the car is at point A, we have $\boxed{U_A = 0}$. The potential energy when the car is at point B is given by $U_B = mgy$ where y is the vertical distance of point B below point A. In part (a), we found the magnitude of this distance to be 26.5 m. Because this distance is now below the zero reference level, it is a negative number.

Thus,

$$U_B = (1\,000\text{ kg})(9.80\text{ m/s}^2)(-26.5\text{ m}) = \boxed{-2.59\times10^5\text{ J}}.$$

The change in potential energy when the car moves from A to B is

$$U_B - U_A = -2.59\times10^5\text{ J} - 0 = \boxed{-2.59\times10^5\text{ J}}.$$

P7.2 (a) We take the zero configuration of system potential energy with the child at the lowest point of the arc. When the string is held horizontal initially, the initial position is 2.00 m above the zero level. Thus,

$$U_g = mgy = (400\text{ N})(2.00\text{ m}) = \boxed{800\text{ J}}.$$

(b) From the sketch, we see that at an angle of 30.0° the child is at a vertical height of $(2.00\text{ m})(1-\cos 30.0°)$ above the lowest point of the arc. Thus,

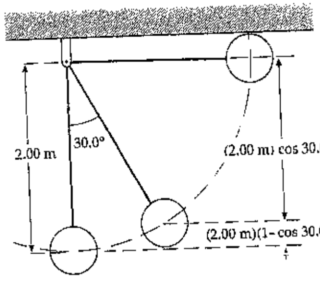

FIG. P7.2

$$U_g = mgy = (400\text{ N})(2.00\text{ m})(1-\cos 30.0°) = \boxed{107\text{ J}}.$$

(c) The zero level has been selected at the lowest point of the arc. Therefore, $\boxed{U_g = 0}$ at this location.

P7.3 The volume flow rate is the volume of water going over the falls each second:

$$3\text{ m}(0.5\text{ m})(1.2\text{ m/s}) = 1.8\text{ m}^3/\text{s}$$

The mass flow rate is $\frac{m}{t} = \rho\frac{V}{t} = (1\,000\text{ kg/m}^3)(1.8\text{ m}^3/\text{s}) = 1\,800\text{ kg/s}$

If the stream has uniform width and depth, the speed of the water below the falls is the same as the speed above the falls. Then no kinetic energy, but only gravitational energy is available for conversion into internal and electric energy.

The input power is $\mathcal{P}_{in} = \frac{\text{energy}}{\Delta t} = \frac{mgy}{\Delta t} = \frac{m}{\Delta t}gy = (1\,800\text{ kg/s})(9.8\text{ m/s}^2)(5\text{ m}) = 8.82\times10^4\text{ J/s}$

The output power is $\mathcal{P}_{useful} = (\text{efficiency})\mathcal{P}_{in} = 0.25(8.82\times10^4\text{ W}) = \boxed{2.20\times10^4\text{ W}}$

The efficiency of electric generation at Hoover Dam is about 85% with a head of water (vertical drop) of 174 m. Intensive research is underway to improve the efficiency of low head generators.

Section 7.2 The Isolated System

***P7.4** (a) One child in one jump converts chemical energy into mechanical energy in the amount that her body has as gravitational energy at the top of her jump:
$mgy = 36\text{ kg}(9.81\text{ m/s}^2)(0.25\text{ m}) = 88.3\text{ J}$. For all of the jumps of the children the energy is
$12(1.05\times10^6)88.3\text{ J} = \boxed{1.11\times10^9\text{ J}}$.

(b) The seismic energy is modeled as $E = \frac{0.01}{100}1.11\times10^9\text{ J} = 1.11\times10^5\text{ J}$, making the Richter magnitude $\frac{\log E - 4.8}{1.5} = \frac{\log 1.11\times10^5 - 4.8}{1.5} = \frac{5.05-4.8}{1.5} = \boxed{0.2}$.

P7.5 (a) $U_i + K_i = U_f + K_f$: $\quad mgh + 0 = mg(2R) + \frac{1}{2}mv^2$

$$g(3.50R) = 2g(R) + \frac{1}{2}v^2$$

$$\boxed{v = \sqrt{3.00gR}}$$

(b) $\sum F = m\frac{v^2}{R}$ $\quad n + mg = m\frac{v^2}{R}$

$$n = m\left[\frac{v^2}{R} - g\right] = m\left[\frac{3.00gR}{R} - g\right] = 2.00mg$$

$$n = 2.00(5.00\times10^{-3}\text{ kg})(9.80\text{ m/s}^2)$$

$$= \boxed{0.098\,0\text{ N downward}}$$

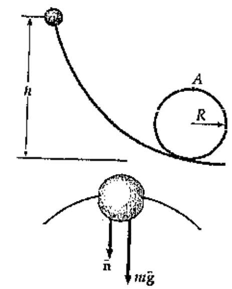

FIG. P7.5

P7.6 (a) Energy of the particle-Earth system is conserved as the particle moves between point P and the apex of the trajectory. Since the horizontal component of velocity is constant,

$$\frac{1}{2}mv_i^2 = \frac{1}{2}mv_{ix}^2 + \frac{1}{2}mv_{iy}^2$$

$$= \frac{1}{2}mv_{ix}^2 + mgh$$

$$v_{iy} = \sqrt{2(9.80)(20.0)} = \boxed{19.8\text{ m/s}}$$

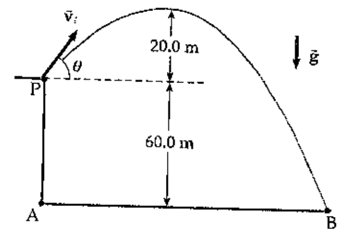

FIG. P7.6

(b) $\Delta K|_{P\to B} = W_g = mg(60.0\text{ m}) = (0.500\text{ kg})(9.80\text{ m/s}^2)(60.0\text{ m}) = \boxed{294\text{ J}}$

(c) Now let the final point be point B. $\quad v_{xi} = v_{xf} = 30.0\text{ m/s}$

$$\Delta K|_{P\to B} = \frac{1}{2}mv_{yf}^2 - \frac{1}{2}mv_{yi}^2 = 294\text{ J}$$

$$v_{yf}^2 = \frac{2}{m}(294) + v_{yi}^2 = 1\,176 + 392$$

$$v_{yf} = -39.6\text{ m/s}$$

$$\vec{v}_B = \boxed{(30.0\text{ m/s})\hat{i} - (39.6\text{ m/s})\hat{j}}$$

P7.7 From leaving ground to the highest point, $\quad K_i + U_i = K_f + U_f$

$$\frac{1}{2}m(6.00\text{ m/s})^2 + 0 = 0 + m(9.80\text{ m/s}^2)y$$

The mass makes no difference: $\quad \therefore y = \frac{(6.00\text{ m/s})^2}{(2)(9.80\text{ m/s}^2)} = \boxed{1.84\text{ m}}$

P7.8 (a) Energy of the object-Earth system is conserved as the object moves between the release point and the lowest point. We choose to measure heights from $y=0$ at the top end of the string.

$$\left(K+U_g\right)_i=\left(K+U_g\right)_f \qquad 0+mgy_i=\frac{1}{2}mv_f^2+mgy_f$$

$$\left(9.8\ \text{m/s}^2\right)(-2\ \text{m}\cos 30°)=\frac{1}{2}v_f^2+\left(9.8\ \text{m/s}^2\right)(-2\ \text{m})$$

$$v_f=\sqrt{2\left(9.8\ \text{m/s}^2\right)(2\ \text{m})(1-\cos 30°)}=\boxed{2.29\ \text{m/s}}$$

(b) Choose the initial point at $\theta=30°$ and the final point at $\theta=15°$:

$$0+mg(-L\cos 30°)=\frac{1}{2}mv_f^2+mg(-L\cos 15°)$$

$$v_f=\sqrt{2gL(\cos 15°-\cos 30°)}=\sqrt{2\left(9.8\ \text{m/s}^2\right)(2\ \text{m})(\cos 15°-\cos 30°)}=\boxed{1.98\ \text{m/s}}$$

P7.9 Using conservation of energy for the system of the Earth and the two objects

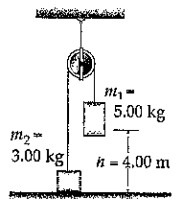

FIG. P7.9

(a) $$(5.00\ \text{kg})g(4.00\ \text{m})=(3.00\ \text{kg})g(4.00\ \text{m})+\frac{1}{2}(5.00+3.00)v^2$$

$$v=\sqrt{19.6}=\boxed{4.43\ \text{m/s}}$$

(b) Now we apply conservation of energy for the system of the 3.00 kg object and the Earth during the time interval between the instant when the string goes slack and the instant at which the 3.00 kg object reaches its highest position in its free fall.

$$\frac{1}{2}(3.00)v^2=mg\,\Delta y=3.00g\Delta y$$

$$\Delta y=1.00\ \text{m}$$

$$y_{\max}=4.00\ \text{m}+\Delta y=\boxed{5.00\ \text{m}}$$

***P7.10** (a) $$(\Delta K)_{A\to B}=\sum W=W_g=mg\Delta h=mg(5.00-3.20)$$

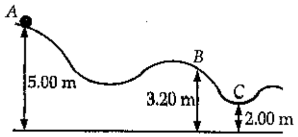

FIG. P7.10

$$\frac{1}{2}mv_B^2-\frac{1}{2}mv_A^2=m(9.80)(1.80)$$

$$v_B=\boxed{5.94\ \text{m/s}}$$

Similarly, $v_C=\sqrt{v_A^2+2g(5.00-2.00)}=\boxed{7.67\ \text{m/s}}$

(b) $$W_g\big|_{A\to C}=mg(3.00\ \text{m})=\boxed{147\ \text{J}}$$

P7.11 (a) The force needed to hang on is equal to the force F the trapeze bar exerts on the performer. From the free-body diagram for the performer's body, as shown,

$F-mg\cos\theta=m\frac{v^2}{\ell}$ or $F=mg\cos\theta+m\frac{v^2}{\ell}$. Apply conservation of mechanical energy of the performer-Earth system as the performer moves between the starting point and any later point:

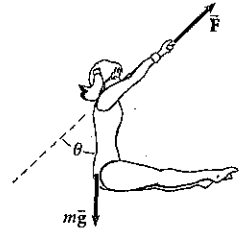

FIG. P7.11

$$mg(\ell-\ell\cos\theta_i)=mg(\ell-\ell\cos\theta)+\frac{1}{2}mv^2$$

Solve for $\frac{mv^2}{\ell}$ and substitute into the force equation to obtain $F=\boxed{mg(3\cos\theta-2\cos\theta_i)}$

(b) At the bottom of the swing, $\theta=0°$ so

$$F=mg(3-2\cos\theta_i)$$

$$F=2mg=mg(3-2\cos\theta_i)$$

which gives $\theta_i=\boxed{60.0°}$.

P7.12 The force of tension and subsequent force of compression in the rod do no work on the ball, since they are perpendicular to each step of displacement. Consider energy conservation of the ball-Earth system between the instant just after you strike the ball and the instant when it reaches the top. The speed at the top is zero if you hit it just hard enough to get it there.

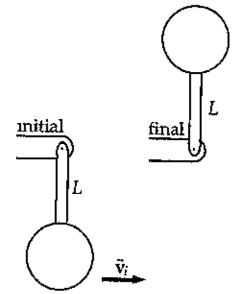

FIG. P7.12

$$K_i+U_{gi}=K_f+U_{gf}: \qquad \frac{1}{2}mv_i^2+0=0+mg(2L)$$

$$v_i=\sqrt{4gL}=\sqrt{4(9.80)(0.770)}$$

$$v_i=\boxed{5.49\ \text{m/s}}$$

*P7.13 (a) The moving sewage possesses kinetic energy in the same amount as it enters and leaves the pump. The work of the pump increases the gravitational energy of the sewage-Earth system. We take the equation $K_i + U_{gi} + W_{\text{pump}} = K_f + U_{gf}$, subtract out the K terms, and choose $U_{gi} = 0$ at the bottom of the sump, to obtain $W_{\text{pump}} = mgy_f$. Now we differentiate through with respect to time:

$$\mathscr{P}_{\text{pump}} = \frac{\Delta m}{\Delta t} g y_f = \rho \frac{\Delta V}{\Delta t} g y_f$$

$$= 1\,050\ \text{kg/m}^3 \left(1.89\times10^6\ \text{L/d}\right)\left(\frac{1\ \text{m}^3}{1\,000\ \text{L}}\right)\left(\frac{1\ \text{d}}{86\,400\ \text{s}}\right)\left(\frac{9.80\ \text{m}}{\text{s}^2}\right)(5.49\ \text{m}) = \boxed{1.24\times10^3\ \text{W}}$$

(b) $$\text{efficiency} = \frac{\text{useful output work}}{\text{total input work}} = \frac{\text{useful output work}/\Delta t}{\text{total input work}/\Delta t}$$

$$= \frac{\text{mechanical output power}}{\text{input electric power}} = \frac{1.24\ \text{kW}}{5.90\ \text{kW}} = \boxed{0.209} = 20.9\%$$

The remaining power, $5.90 - 1.24\ \text{kW} = 4.66\ \text{kW}$ is the rate at which internal energy is injected into the sewage and the surroundings of the pump.

Section 7.3 Conservative and Nonconservative Forces

P7.14 (a) $W = \int \vec{\mathbf{F}}\cdot d\vec{\mathbf{r}}$ and if the force is constant, this can be written as

$W = \vec{\mathbf{F}}\cdot\int d\vec{\mathbf{r}} = \boxed{\vec{\mathbf{F}}\cdot\left(\vec{\mathbf{r}}_f - \vec{\mathbf{r}}_i\right), \text{ which depends only on end points, not path.}}$

(b) $$W = \int \vec{\mathbf{F}}\cdot d\vec{\mathbf{r}} = \int\left(3\hat{\mathbf{i}} + 4\hat{\mathbf{j}}\right)\cdot\left(dx\hat{\mathbf{i}} + dy\hat{\mathbf{j}}\right) = (3.00\ \text{N})\int_0^{5.00\ \text{m}} dx + (4.00\ \text{N})\int_0^{5.00\ \text{m}} dy$$

$$W = (3.00\ \text{N})x\Big|_0^{5.00\ \text{m}} + (4.00\ \text{N})y\Big|_0^{5.00\ \text{m}} = 15.0\ \text{J} + 20.0\ \text{J} = \boxed{35.0\ \text{J}}$$

The same calculation applies for all paths.

P7.15 (a) $$W_{OA} = \int_0^{5.00\ \text{m}} dx\hat{\mathbf{i}}\cdot\left(2y\hat{\mathbf{i}} + x^2\hat{\mathbf{j}}\right) = \int_0^{5.00\ \text{m}} 2y\,dx$$

and since along this path, $y = 0$ $\quad W_{OA} = 0$

$$W_{AC} = \int_0^{5.00\ \text{m}} dy\hat{\mathbf{j}}\cdot\left(2y\hat{\mathbf{i}} + x^2\hat{\mathbf{j}}\right) = \int_0^{5.00\ \text{m}} x^2 dy.$$

For $x = 5.00$ m, $\quad W_{AC} = 125\ \text{J}$

and $\quad W_{OAC} = 0 + 125 = \boxed{125\ \text{J}}$

continued on next page

(b) $$W_{OB} = \int_0^{5.00\ \text{m}} dy\hat{\mathbf{j}}\cdot\left(2y\hat{\mathbf{i}} + x^2\hat{\mathbf{j}}\right) = \int_0^{5.00\ \text{m}} x^2 dy$$

since along this path, $x = 0$. $\quad W_{OB} = 0$

$$W_{BC} = \int_0^{5.00\ \text{m}} dx\hat{\mathbf{i}}\cdot\left(2y\hat{\mathbf{i}} + x^2\hat{\mathbf{j}}\right) = \int_0^{5.00\ \text{m}} 2y\,dx$$

since $y = 5.00$ m, $\quad W_{BC} = 50.0\ \text{J}$

$$W_{OBC} = 0 + 50.0 = \boxed{50.0\ \text{J}}$$

(c) $$W_{OC} = \int\left(dx\hat{\mathbf{i}} + dy\hat{\mathbf{j}}\right)\cdot\left(2y\hat{\mathbf{i}} + x^2\hat{\mathbf{j}}\right) = \int\left(2y\,dx + x^2 dy\right)$$

Since $x = y$ along OC, $$W_{OC} = \int_0^{5.00\ \text{m}}\left(2x + x^2\right)dx = \boxed{66.7\ \text{J}}$$

(d) F is $\boxed{\text{nonconservative}}$ since the work done is path dependent.

P7.16 Choose the zero point of gravitational potential energy of the object-spring-Earth system as the configuration in which the object comes to rest. Then because the incline is frictionless, we have $E_B = E_A$: $K_B + U_{gB} + U_{sB} = K_A + U_{gA} + U_{sA}$ or $0 + mg(d + x)\sin\theta + 0 = 0 + 0 + \frac{1}{2}kx^2$. Solving for d gives $d = \boxed{\dfrac{kx^2}{2mg\sin\theta} - x}$.

P7.17 From conservation of energy for the block-spring-Earth system,

$$U_{gt} = U_{si},$$

or

$$(0.250\ \text{kg})\left(9.80\ \text{m/s}^2\right)h = \left(\frac{1}{2}\right)(5\,000\ \text{N/m})(0.100\ \text{m})^2$$

This gives a maximum height $h = \boxed{10.2\ \text{m}}$.

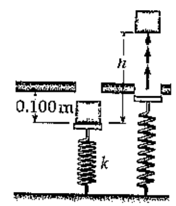

FIG. P7.17

***P7.18** (a) For a 5-m cord the spring constant is described by $F = kx$. $mg = k(1.5\text{ m})$. For a longer cord of length L the stretch distance is longer so the spring constant is smaller in inverse proportion:

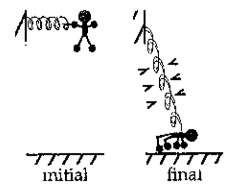

FIG. P7.18

$$k = \frac{5\text{ m}}{L}\frac{mg}{1.5\text{ m}} = 3.33\,mg/L$$
$$(K + U_g + U_s)_i = (K + U_g + U_s)_f$$
$$0 + mgy_i + 0 = 0 + mgy_f + \frac{1}{2}kx_f^2$$
$$mg(y_i - y_f) = \frac{1}{2}kx_f^2 = \frac{1}{2}3.33\frac{mg}{L}x_f^2$$

here $y_i - y_f = 55\text{ m} = L + x_f$

$$55.0\text{ m}L = \frac{1}{2}3.33(55.0\text{ m} - L)^2$$
$$55.0\text{ m}L = 5.04\times10^3\text{ m}^2 - 183\text{ m}L + 1.67L^2$$
$$0 = 1.67L^2 - 238L + 5.04\times10^3 = 0$$
$$L = \frac{238 \pm \sqrt{238^2 - 4(1.67)(5.04\times10^3)}}{2(1.67)} = \frac{238 \pm 152}{3.33} = \boxed{25.8\text{ m}}$$

only the value of L less than 55 m is physical.

(b) $k = 3.33\dfrac{mg}{25.8\text{ m}}$ $\quad x_{\max} = x_f = 55.0\text{ m} - 25.8\text{ m} = 29.2\text{ m}$

$\sum F = ma$ $\quad +kx_{\max} - mg = ma$

$$3.33\frac{mg}{25.8\text{ m}}29.2\text{ m} - mg = ma$$
$$a = 2.77g = \boxed{27.1\text{ m/s}^2}$$

P7.19 (a) $U_f = K_i - K_f + U_i$ $\quad U_f = 30.0 - 18.0 + 10.0 = \boxed{22.0\text{ J}}$

$$\boxed{E = 40.0\text{ J}}$$

(b) Yes, $\Delta E_{\text{mech}} = \Delta K + \Delta U$ is not equal to zero. For conservative forces $\Delta K + \Delta U = 0$.

***P7.20** (a) Simplified, the equation is $0 = (9\,700\text{ N/m})x^2 - (450.8\text{ N})x - 1\,395\text{ N}\cdot\text{m}$. Then

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{450.8\text{ N} \pm \sqrt{(450.8\text{ N})^2 - 4(9\,700\text{ N/m})(-1\,395\text{ N}\cdot\text{m})}}{2(9\,700\text{ N/m})}$$
$$= \frac{450.8\text{ N} \pm 7\,370\text{ N}}{19\,400\text{ N/m}} = \boxed{0.403\text{ m or } -0.357\text{ m}}$$

(b) One possible problem statement: From a perch at a height of 2.80 m above the top of the pile of mattresses, a 46.0-kg child jumps nearly straight upward with speed 2.40 m/s. The mattresses behave as a linear spring with force constant 19.4 kN/m. Find the maximum amount by which they are compressed when the child lands on them. Physical meaning: The positive value of x represents the maximum spring compression. The negative value represents the maximum extension of the equivalent spring if the child sticks to the top of the mattress pile as he rebounds upward without friction.

P7.21 The distance traveled by the ball from the top of the arc to the bottom is πR. The work done by the non-conservative force, the force exerted by the pitcher, is $\Delta E = F\Delta r\cos 0° = F(\pi R)$. We shall assign the gravitational energy of the ball-Earth system to be zero with the ball at the bottom of the arc. Then $\Delta E_{\text{mech}} = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 + mgy_f - mgy_i$ becomes $\frac{1}{2}mv_f^2 = \frac{1}{2}mv_i^2 + mgy_i + F(\pi R)$ or

$$v_f = \sqrt{v_i^2 + 2gy_i + \frac{2F(\pi R)}{m}} = \sqrt{(15.0)^2 + 2(9.80)(1.20) + \frac{2(30.0)\pi(0.600)}{0.250}}$$
$$v_f = \boxed{26.5\text{ m/s}}$$

***P7.22** (a) Maximum speed occurs after the needle leaves the spring, before it enters the body.

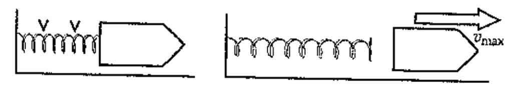

FIG. P7.22(a)

$$K_i + U_i - f_k d = K_f + U_f.$$

We assume the needle is fired horizontally

$$0 + \frac{1}{2}kx^2 - 0 = \frac{1}{2}mv_{\max}^2 + 0$$
$$\frac{1}{2}(375\text{ N/m})(0.081\text{ m})^2 = \frac{1}{2}(0.005\,6\text{ kg})v_{\max}^2$$
$$\left(\frac{2(1.23\text{ J})}{0.005\,6\text{ kg}}\right)^{1/2} = v_{\max} = \boxed{21.0\text{ m/s}}$$

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(b) The same energy of 1.23 J as in part (a) now becomes partly internal energy in the soft tissue, partly internal energy in the organ, and partly kinetic energy of the needle just before it runs into the stop. We make up a work-energy equation to describe this process:

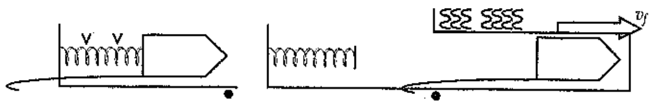

FIG. P7.22(b)

$$K_i + U_i - f_{k1}d_1 - f_{k2}d_2 = K_f + U_f$$

$$1 + \frac{1}{2}kx^2 - f_{k1}d_1 - f_{k2}d_2 = \frac{1}{2}mv_f^2 + 0$$

$$1.23\text{ J} - 7.60\text{ N}(0.024\text{ m}) - 9.20\text{ N}(0.035\text{ m}) = \frac{1}{2}(0.005\,6\text{ kg})v_f^2$$

$$\left(\frac{2(1.23\text{ J} - 0.182\text{ J} - 0.322\text{ J})}{0.005\,6\text{ kg}}\right)^{1/2} = v_f = \boxed{16.1\text{ m/s}}$$

P7.23 $U_i + K_i + \Delta E_{mech} = U_f + K_f$: $m_2gh - fh = \frac{1}{2}m_1v^2 + \frac{1}{2}m_2v^2$

$$f = \mu n = \mu m_1 g$$

$$m_2gh - \mu m_1gh = \frac{1}{2}(m_1 + m_2)v^2$$

$$v^2 = \frac{2(m_2 - \mu m_1)(hg)}{m_1 + m_2}$$

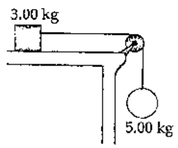

FIG. P7.23

$$v = \sqrt{\frac{2(9.80\text{ m/s}^2)(1.50\text{ m})[5.00\text{ kg} - 0.400(3.00\text{ kg})]}{8.00\text{ kg}}} = \boxed{3.74\text{ m/s}}$$

P7.24 We begin with Equation 6.20 for the isolated child-wheelchair-Earth system,

$$\Delta K + \Delta U + \Delta E_{int} = 0 \quad (1)$$

We recognize that the change in potential energy is of two types: gravitational and chemical potential energy stored in the body of the child from past meals. We also recognize that the change in internal energy will be due to the friction force (air resistance and rolling resistance) as the child rolls down the hill. Therefore, we write (1) as,

$$\Delta K + \Delta U_g + \Delta U_{body} + f_k d = 0 \quad (2)$$

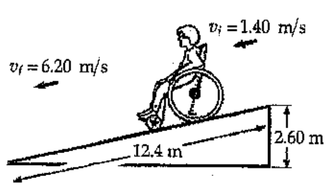

FIG. P7.24

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The change in potential energy in the body of the child will be due to work done within the system by the child on the wheels of the chair. Consequently, the requested answer, the work done by the child, is $W_{child} = -\Delta U_{body}$. Therefore, (2) can be expressed as,

$$W_{child} = \Delta K + \Delta U_g + f_k d = \left(\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2\right) + (mgh_f - mgh_i) + f_k d$$

$$= \frac{1}{2}m(v_f^2 - v_i^2) + mg(h_f - h_i) + f_k d$$

$$= \frac{1}{2}(47.0\text{ kg})\left[(6.20\text{ m/s})^2 - (1.40\text{ m/s})^2\right] + (47.0\text{ kg})(9.80\text{ m/s}^2)(0 - 2.60\text{ m}) + (41.0\text{ N})(12.4\text{ m})$$

$$= \boxed{168\text{ J}}$$

P7.25 (a) $\Delta K = \frac{1}{2}m(v_f^2 - v_i^2) = -\frac{1}{2}mv_i^2 = \boxed{-160\text{ J}}$

(b) $\Delta U = mg(3.00\text{ m})\sin 30.0° = \boxed{73.5\text{ J}}$

(c) The mechanical energy converted due to friction is 86.5 J

$$f = \frac{86.5\text{ J}}{3.00\text{ m}} = \boxed{28.8\text{ N}}$$

(d) $f = \mu_k n = \mu_k mg\cos 30.0° = 28.8\text{ N}$

$$\mu_k = \frac{28.8\text{ N}}{(5.00\text{ kg})(9.80\text{ m/s}^2)\cos 30.0°} = \boxed{0.679}$$

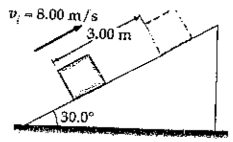

FIG. P7.25

P7.26 Consider the whole motion: $K_i + U_i + \Delta E_{mech} = K_f + U_f$

(a) $0 + mgy_i - f_1\Delta x_1 - f_2\Delta x_2 = \frac{1}{2}mv_f^2 + 0$

$$(80.0\text{ kg})(9.80\text{ m/s}^2)1\,000\text{ m} - (50.0\text{ N})(800\text{ m}) - (3\,600\text{ N})(200\text{ m}) = \frac{1}{2}(80.0\text{ kg})v_f^2$$

$$784\,000\text{ J} - 40\,000\text{ J} - 720\,000\text{ J} = \frac{1}{2}(80.0\text{ kg})v_f^2$$

$$v_f = \sqrt{\frac{2(24\,000\text{ J})}{80.0\text{ kg}}} = \boxed{24.5\text{ m/s}}$$

(b) $\boxed{\text{Yes}}$ this is too fast for safety.

(c) Now in the same energy equation as in part (a), Δx_2 is unknown, and $\Delta x_1 = 1\,000\text{ m} - \Delta x_2$:

$$784\,000\text{ J} - (50.0\text{ N})(1\,000\text{ m} - \Delta x_2) - (3\,600\text{ N})\Delta x_2 = \frac{1}{2}(80.0\text{ kg})(5.00\text{ m/s})^2$$

$$784\,000\text{ J} - 50\,000\text{ J} - (3\,550\text{ N})\Delta x_2 = 1\,000\text{ J}$$

$$\Delta x_2 = \frac{733\,000\text{ J}}{3\,550\text{ N}} = \boxed{206\text{ m}}$$

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(d) Really the air drag will depend on the skydiver's speed. It will be larger than her 784 N weight only after the chute is opened. It will be nearly equal to 784 N before she opens the chute and again before she touches down, whenever she moves near terminal speed.

P7.27 (a) $(K+U)_i + \Delta E_{\text{mech}} = (K+U)_f$:

$$0+\frac{1}{2}kx^2 - f\Delta x = \frac{1}{2}mv^2 + 0$$

$$\frac{1}{2}(8.00\text{ N/m})(5.00\times10^{-2}\text{ m})^2 - (3.20\times10^{-2}\text{ N})(0.150\text{ m}) = \frac{1}{2}(5.30\times10^{-3}\text{ kg})v^2$$

$$v=\sqrt{\frac{2(5.20\times10^{-3}\text{ J})}{5.30\times10^{-3}\text{ kg}}} = \boxed{1.40\text{ m/s}}$$

(b) When the spring force just equals the friction force, the ball will stop speeding up. Here $|\vec{F}_s| = kx$; the spring is compressed by

$$\frac{3.20\times10^{-2}\text{ N}}{8.00\text{ N/m}} = 0.400\text{ cm}$$

and the ball has moved

$$5.00\text{ cm} - 0.400\text{ cm} = \boxed{4.60\text{ cm from the start.}}$$

(c) Between start and maximum speed points,

$$\frac{1}{2}kx_i^2 - f\Delta x = \frac{1}{2}mv^2 + \frac{1}{2}kx_f^2$$

$$\frac{1}{2}8.00(5.00\times10^{-2})^2 - (3.20\times10^{-2})(4.60\times10^{-2}) = \frac{1}{2}(5.30\times10^{-3})v^2 + \frac{1}{2}8.00(4.00\times10^{-3})^2$$

$$v = \boxed{1.79\text{ m/s}}$$

P7.28 $\sum F_y = n - mg\cos 37.0° = 0$

$\therefore n = mg\cos 37.0° = 391\text{ N}$

$f = \mu n = 0.250(391\text{ N}) = 97.8\text{ N}$

$-f\Delta x = \Delta E_{\text{mech}}$

$(-97.8)(20.0) = \Delta U_A + \Delta U_B + \Delta K_A + \Delta K_B$

$\Delta U_A = m_A g(h_f - h_i) = (50.0)(9.80)(20.0\sin 37.0°) = 5.90\times10^3$

$\Delta U_B = m_B g(h_f - h_i) = (100)(9.80)(-20.0) = -1.96\times10^4$

$\Delta K_A = \frac{1}{2}m_A(v_f^2 - v_i^2)$

$\Delta K_B = \frac{1}{2}m_B(v_f^2 - v_i^2) = \frac{m_B}{m_A}\Delta K_A = 2\Delta K_A$

Adding and solving, $\Delta K_A = \boxed{3.92\text{ kJ}}$.

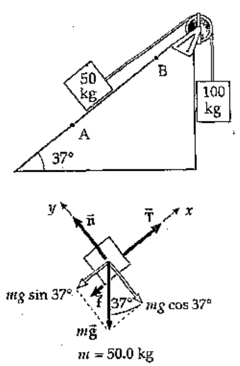

FIG. P7.28

P7.29 (a) The object moved down distance $1.20\text{ m} + x$. Choose $y=0$ at its lower point.

$$K_i + U_{gi} + U_{si} + \Delta E_{\text{mech}} = K_f + U_{gf} + U_{sf}$$

$$0 + mgy_i + 0 + 0 = 0 + 0 + \frac{1}{2}kx^2$$

$$(1.50\text{ kg})(9.80\text{ m/s}^2)(1.20\text{ m}+x) = \frac{1}{2}(320\text{ N/m})x^2$$

$$0 = (160\text{ N/m})x^2 - (14.7\text{ N})x - 17.6\text{ J}$$

$$x = \frac{14.7\text{ N} \pm \sqrt{(-14.7\text{ N})^2 - 4(160\text{ N/m})(-17.6\text{ N}\cdot\text{m})}}{2(160\text{ N/m})}$$

$$x = \frac{14.7\text{ N} \pm 107\text{ N}}{320\text{ N/m}}$$

The negative root tells how high the object will rebound if it is instantly glued to the spring. We want

$$x = \boxed{0.381\text{ m}}$$

(b) From the same equation,

$$(1.50\text{ kg})(1.63\text{ m/s}^2)(1.20\text{ m}+x) = \frac{1}{2}(320\text{ N/m})x^2$$

$$0 = 160x^2 - 2.44x - 2.93$$

The positive root is $x = \boxed{0.143\text{ m}}$.

(c) The equation expressing the energy version of the nonisolated system model has one more term:

$$mgy_i - f\Delta x = \frac{1}{2}kx^2$$

$$(1.50\text{ kg})(9.80\text{ m/s}^2)(1.20\text{ m}+x) - 0.700\text{ N}(1.20\text{ m}+x) = \frac{1}{2}(320\text{ N/m})x^2$$

$$17.6\text{ J} + 14.7\text{ N}x - 0.840\text{ J} - 0.700\text{ N}x = 160\text{ N/m}x^2$$

$$160x^2 - 14.0x - 16.8 = 0$$

$$x = \frac{14.0 \pm \sqrt{(14.0)^2 - 4(160)(-16.8)}}{320}$$

$$x = \boxed{0.371\text{ m}}$$

P7.30 The total mechanical energy of the skysurfer-Earth system is

$$E_{\text{mech}} = K + U_g = \frac{1}{2}mv^2 + mgh.$$

Since the skysurfer has constant speed,

$$\frac{dE_{\text{mech}}}{dt} = mv\frac{dv}{dt} + mg\frac{dh}{dt} = 0 + mg(-v) = -mgv.$$

The rate the system is losing mechanical energy is then

$$\left|\frac{dE_{\text{mech}}}{dt}\right| = mgv = (75.0\text{ kg})(9.80\text{ m/s}^2)(60.0\text{ m/s}) = \boxed{44.1\text{ kW}}.$$

Section 7.4 Conservative Forces and Potential Energy

P7.31 (a) $W = \int F_x dx = \int_1^{5.00\text{ m}} (2x+4)dx = \left(\frac{2x^2}{2} + 4x\right)\Bigg|_1^{5.00\text{ m}} = 25.0 + 20.0 - 1.00 - 4.00 = \boxed{40.0\text{ J}}$

(b) $\Delta K + \Delta U = 0 \qquad \Delta U = -\Delta K = -W = \boxed{-40.0\text{ J}}$

(c) $\Delta K = K_f - \frac{mv_1^2}{2} \qquad K_f = \Delta K + \frac{mv_1^2}{2} = \boxed{62.5\text{ J}}$

P7.32 (a) $U = -\int_0^x (-Ax + Bx^2)dx = \boxed{\frac{Ax^2}{2} - \frac{Bx^3}{3}}$

(b) $\Delta U = -\int_{2.00\text{ m}}^{3.00\text{ m}} F dx = \frac{A\left[(3.00)^2 - (2.00)^2\right]}{2} - \frac{B\left[(3.00)^3 - (2.00)^3\right]}{3} = \boxed{\frac{5.00}{2}A - \frac{19.0}{3}B}$

$\Delta K = \boxed{\left(-\frac{5.00}{2}A + \frac{19.0}{3}B\right)}$

P7.33 $U(r) = \frac{A}{r}$

$F_r = -\frac{\partial U}{\partial r} = -\frac{d}{dr}\left(\frac{A}{r}\right) = \boxed{\frac{A}{r^2}}$. The positive value indicates a force of repulsion if A is positive.

P7.34 $F_x = -\frac{\partial U}{\partial x} = -\frac{\partial(3x^3y - 7x)}{\partial x} = -(9x^2y - 7) = 7 - 9x^2y$

$F_y = -\frac{\partial U}{\partial y} = -\frac{\partial(3x^3y - 7x)}{\partial y} = -(3x^3 - 0) = -3x^3$

Thus, the force acting at the point (x, y) is $\vec{F} = F_x\hat{i} + F_y\hat{j} = \boxed{(7 - 9x^2y)\hat{i} - 3x^3\hat{j}}$.

Section 7.5 The Nonisolated System in Steady State

No problems in this section

Section 7.6 Potential Energy for Gravitational and Electric Forces

Note: Problems 20.10, 20.13, 20.18, 20.19, 20.20, and 20.62 in Chapter 20 can be assigned with Section 7.6.

P7.35 (a) $U = -\frac{GM_Em}{r} = -\frac{(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(5.98\times10^{24}\text{ kg})(100\text{ kg})}{(6.37 + 2.00)\times10^6\text{ m}} = \boxed{-4.77\times10^9\text{ J}}$

(b), (c) Planet and satellite exert forces of equal magnitude on each other, directed downward on the satellite and upward on the planet.

$$F = \frac{GM_Em}{r^2} = \frac{(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(5.98\times10^{24}\text{ kg})(100\text{ kg})}{(8.37\times10^6\text{ m})^2} = \boxed{569\text{ N}}$$

P7.36 $U = -G\frac{Mm}{r}$ and $g = \frac{GM_E}{R_E^2}$ so that

$$\Delta U = -GMm\left(\frac{1}{3R_E} - \frac{1}{R_E}\right) = \frac{2}{3}mgR_E$$

$$\Delta U = \frac{2}{3}(1000\text{ kg})(9.80\text{ m/s}^2)(6.37\times10^6\text{ m}) = \boxed{4.17\times10^{10}\text{ J}}$$

P7.37 The height attained is not small compared to the radius of the Earth, so $U = mgy$ does not apply; $U = -\frac{GM_1M_2}{r}$ does. From launch to apogee at height h,

$K_i + U_i + \Delta E_{\text{mech}} = K_f + U_f$: $\quad \frac{1}{2}M_pv_i^2 - \frac{GM_EM_p}{R_E} + 0 = 0 - \frac{GM_EM_p}{R_E + h}$

$$\frac{1}{2}(10.0\times10^3\text{ m/s})^2 - (6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)\left(\frac{5.98\times10^{24}\text{ kg}}{6.37\times10^6\text{ m}}\right)$$

$$= -(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)\left(\frac{5.98\times10^{24}\text{ kg}}{6.37\times10^6\text{ m} + h}\right)$$

$$(5.00\times10^7\text{ m}^2/\text{s}^2) - (6.26\times10^7\text{ m}^2/\text{s}^2) = \frac{-3.99\times10^{14}\text{ m}^3/\text{s}^2}{6.37\times10^6\text{ m} + h}$$

$$6.37\times10^6\text{ m} + h = \frac{3.99\times10^{14}\text{ m}^3/\text{s}^2}{1.26\times10^7\text{ m}^2/\text{s}^2} = 3.16\times10^7\text{ m}$$

$$\boxed{h = 2.52\times10^7\text{ m}}$$

P7.38 (a) $U_{Tot} = U_{12} + U_{13} + U_{23} = 3U_{12} = 3\left(-\frac{Gm_1m_2}{r_{12}}\right)$

$$U_{Tot} = -\frac{3(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(5.00\times10^{-3}\ \text{kg})^2}{0.300\ \text{m}} = \boxed{-1.67\times10^{-14}\ \text{J}}$$

(b) $\boxed{\text{At the center}}$ of the equilateral triangle

Section 7.7 Energy Diagrams and Stability of Equilibrium

P7.39 (a) F_x is zero at points A, C and E; F_x is positive at point B and negative at point D.

(b) A and E are unstable, and C is stable.

(c)

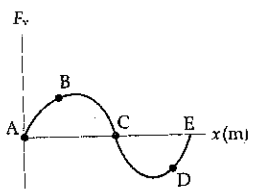

FIG. P7.39(c)

P7.40 (a) There is an equilibrium point wherever the graph of potential energy is horizontal:

At $r = 1.5$ mm and 3.2 mm, the equilibrium is stable.

At $r = 2.3$ mm, the equilibrium is unstable.

A particle moving out toward $r \to \infty$ approaches neutral equilibrium.

(b) The system energy E cannot be less than –5.6 J. The particle is bound if $\boxed{-5.6\ \text{J} \le E < 1\ \text{J}}$.

(c) If the system energy is –3 J, its potential energy must be less than or equal to –3 J. Thus, the particle's position is limited to $\boxed{0.6\ \text{mm} \le r \le 3.6\ \text{mm}}$.

(d) $K + U = E$. Thus, $K_{max} = E - U_{min} = -3.0\ \text{J} - (-5.6\ \text{J}) = \boxed{2.6\ \text{J}}$.

(e) Kinetic energy is a maximum when the potential energy is a minimum, at $\boxed{r = 1.5\ \text{mm}}$.

(f) $-3\ \text{J} + W = 1\ \text{J}$. Hence, the binding energy is $W = \boxed{4\ \text{J}}$.

P7.41 (a) When the mass moves distance x, the length of each spring changes from L to $\sqrt{x^2+L^2}$, so each exerts force $k\left(\sqrt{x^2+L^2}-L\right)$ towards its fixed end. The y-components cancel out and the x components add to:

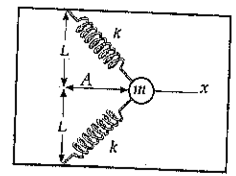

FIG. P7.41(a)

$$F_x = -2k\left(\sqrt{x^2+L^2}-L\right)\left(\frac{x}{\sqrt{x^2+L^2}}\right) = -2kx + \frac{2kLx}{\sqrt{x^2+L^2}}$$

Choose $U = 0$ at $x = 0$. Then at any point the potential energy of the system is

$$U(x) = -\int_0^x F_x dx = -\int_0^x \left(-2kx + \frac{2kLx}{\sqrt{x^2+L^2}}\right)dx = 2k\int_0^x x\,dx - 2kL\int_0^x \frac{x}{\sqrt{x^2+L^2}}dx$$

$$U(x) = \boxed{kx^2 + 2kL\left(L - \sqrt{x^2+L^2}\right)}$$

(b) $U(x) = 40.0x^2 + 96.0\left(1.20 - \sqrt{x^2+1.44}\right)$

For negative x, $U(x)$ has the same value as for positive x. The only equilibrium point (i.e., where $F_x = 0$) is $\boxed{x = 0}$.

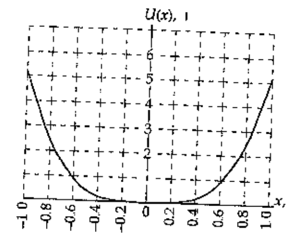

FIG. P7.41(b)

(c) $K_i + U_i + \Delta E_{mech} = K_f + U_f$

$$0 + 0.400\ \text{J} + 0 = \frac{1}{2}(1.18\ \text{kg})v_f^2 + 0$$

$v_f = \boxed{0.823\ \text{m/s}}$

Section 7.8 Context Connection—Potential Energy in Fuels

***P7.42** $v = 100\ \text{km/h} = 27.8\ \text{m/s}$

The retarding force due to air resistance is

$$R = \frac{1}{2}D\rho Av^2 = \frac{1}{2}(0.330)(1.20\ \text{kg/m}^3)(2.50\ \text{m}^2)(27.8\ \text{m/s})^2 = 382\ \text{N}$$

Comparing the energy of the car at two points along the hill,

$$K_i + U_{gi} + \Delta E = K_f + U_{gf}$$

or $$K_i + U_{gi} + \Delta W_e - R(\Delta s) = K_f + U_{gf}$$

where ΔW_e is the work input from the engine. Thus,

$$\Delta W_e = R(\Delta s) + (K_f - K_i) + (U_{gf} - U_{gi})$$

continued on next page

Recognizing that $K_f = K_i$ and dividing by the travel time Δt gives the required power input from the engine as

$$\mathcal{P} = \left(\frac{\Delta W_e}{\Delta t}\right) = R\left(\frac{\Delta s}{\Delta t}\right) + mg\left(\frac{\Delta y}{\Delta t}\right) = Rv + mgv\sin\theta$$

$$\mathcal{P} = (382\text{ N})(27.8\text{ m/s}) + (1\,500\text{ kg})(9.80\text{ m/s}^2)(27.8\text{ m/s})\sin 3.20°$$

$$\mathcal{P} = \boxed{33.4\text{ kW} = 44.8\text{ hp}}$$

*P7.43 The energy per mass values are:

gasoline: 44 MJ/kg

hydrogen: 142 MJ/kg, larger by $\frac{142}{44} = 3.2$ times

hay: 17 MJ/kg, smaller than gasoline by $\frac{44}{17} = 2.6$ times

battery: $\frac{\text{energy}}{\text{mass}} = \frac{\mathcal{P}\Delta t}{m} = \frac{1\,200\text{ W}(3\,600\text{ s})}{16\text{ kg}} = 0.27$ MJ/kg, smaller than hay by $\frac{17}{0.27} = 63$ times.

Thus in the order requested, the energy densities are

$$\boxed{\begin{array}{l}0.27\text{ MJ/kg for a battery}\\ 17\text{ MJ/kg for hay is 63 times larger}\\ 44\text{ MJ/kg for gasoline is 2.6 times larger still}\\ 142\text{ MJ/kg for hydrogen is 3.2 times larger still}\end{array}}$$

*P7.44 (a) $\frac{\mathcal{P}}{\text{area}} = \frac{\text{energy}}{\Delta t \cdot \text{area}} = \frac{600\text{ kWh}}{(30\text{ d})(13\text{ m})(9.5\text{ m})} = \frac{600\times10^3\text{ (J/s)h}}{30\text{ d}(123.5\text{ m}^2)}\left(\frac{1\text{ d}}{24\text{ h}}\right) = \boxed{6.75\text{ W/m}^2}$

(b) The car uses gasoline at the rate 55 $\text{mi/h}\left(\frac{\text{gal}}{25\text{ mi}}\right)$. Its rate of energy conversion is

$$\mathcal{P} = 44\times10^6\text{ J/kg}\left(\frac{2.54\text{ kg}}{1\text{ gal}}\right)(55\text{ mi/h})\left(\frac{\text{gal}}{25\text{ mi}}\right)\left(\frac{1\text{ h}}{3\,600\text{ s}}\right) = 6.83\times10^4\text{ W}$$. Its power-per-footprint-area is $\frac{\mathcal{P}}{\text{area}} = \frac{6.83\times10^4\text{ W}}{2.10\text{ m}(4.90\text{ m})} = \boxed{6.64\times10^3\text{ W/m}^2}$.

(c) For an automobile of typical weight and power to run on sunlight, it would have to carry a solar panel huge compared to its own size. Rather than running a conventional car, it is much more natural to use solar energy for agriculture, lighting, space heating, water heating, and small appliances.

Additional Problems

P7.45 At a pace I could keep up for a half-hour exercise period, I climb two stories up, traversing forty steps each 18 cm high, in 20 s. My output work becomes the final gravitational energy of the system of the Earth and me,

$$mgy = (85\text{ kg})(9.80\text{ m/s}^2)(40\times0.18\text{ m}) = 6\,000\text{ J}$$

making my sustainable power $\frac{6\,000\text{ J}}{20\text{ s}} = \boxed{\sim10^2\text{ W}}$.

P7.46 m = mass of pumpkin
R = radius of silo top

$$\sum F_r = ma_r \Rightarrow n - mg\cos\theta = -m\frac{v^2}{R}$$

When the pumpkin first loses contact with the surface, $n = 0$. Thus, at the point where it leaves the surface: $v^2 = Rg\cos\theta$.

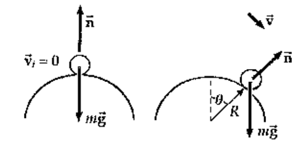

FIG. P7.46

Choose $U_g = 0$ in the $\theta = 90.0°$ plane. Then applying conservation of energy for the pumpkin-Earth system between the starting point and the point where the pumpkin leaves the surface gives

$$K_f + U_{gf} = K_i + U_{gi}$$

$$\frac{1}{2}mv^2 + mgR\cos\theta = 0 + mgR$$

Using the result from the force analysis, this becomes

$\frac{1}{2}mRg\cos\theta + mgR\cos\theta = mgR$, which reduces to

$\cos\theta = \frac{2}{3}$, and gives $\theta = \cos^{-1}(2/3) = \boxed{48.2°}$

as the angle at which the pumpkin will lose contact with the surface.

*P7.47 When block B moves up by 1 cm, block A moves down by 2 cm and the separation becomes 3 cm. We then choose the final point to be when B has moved up by $\frac{h}{3}$ and has speed $\frac{v_A}{2}$. Then A has moved down $\frac{2h}{3}$ and has speed v_A.

$$(K_A + K_B + U_g)_i = (K_A + K_B + U_g)_f$$

$$0 + 0 + 0 = \frac{1}{2}mv_A^2 + \frac{1}{2}m\left(\frac{v_A}{2}\right)^2 + \frac{mgh}{3} - \frac{mg2h}{3}$$

$$\frac{mgh}{3} = \frac{5}{8}mv_A^2$$

$$\boxed{v_A = \sqrt{\frac{8gh}{15}}}$$

P7.48 (a) $U_A = mgR = (0.200\text{ kg})(9.80\text{ m/s}^2)(0.300\text{ m}) = \boxed{0.588\text{ J}}$

(b) $K_A + U_A = K_B + U_B$

$K_B = K_A + U_A - U_B = mgR = \boxed{0.588\text{ J}}$

(c) $v_B = \sqrt{\dfrac{2K_B}{m}} = \sqrt{\dfrac{2(0.588\text{ J})}{0.200\text{ kg}}} = \boxed{2.42\text{ m/s}}$

(d) $U_C = mgh_C = (0.200\text{ kg})(9.80\text{ m/s}^2)(0.200\text{ m}) = \boxed{0.392\text{ J}}$

$K_C = K_A + U_A - U_C = mg(h_A - h_C)$

$K_C = (0.200\text{ kg})(9.80\text{ m/s}^2)(0.300 - 0.200)\text{ m} = \boxed{0.196\text{ J}}$

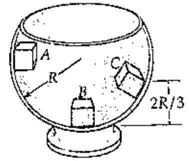

FIG. P7.48

P7.49 (a) $K_B = \dfrac{1}{2}mv_B^2 = \dfrac{1}{2}(0.200\text{ kg})(1.50\text{ m/s})^2 = \boxed{0.225\text{ J}}$

(b) $\Delta E_{mech} = \Delta K + \Delta U = K_B - K_A + U_B - U_A$

$= K_B + mg(h_B - h_A)$

$= 0.225\text{ J} + (0.200\text{ kg})(9.80\text{ m/s}^2)(0 - 0.300\text{ m})$

$= 0.225\text{ J} - 0.588\text{ J} = \boxed{-0.363\text{ J}}$

(c) It's possible to estimate an effective coefficient of friction, but not to find the actual value of μ since n and f vary with position.

P7.50 $k = 2.50 \times 10^4\text{ N/m}$, $m = 25.0\text{ kg}$

$x_A = -0.100\text{ m}$, $U_g\big|_{x=0} = U_s\big|_{x=0} = 0$

(a) $E_{mech} = K_A + U_{gA} + U_{sA}$ $E_{mech} = 0 + mgx_A + \dfrac{1}{2}kx_A^2$

$E_{mech} = (25.0\text{ kg})(9.80\text{ m/s}^2)(-0.100\text{ m})$

$+\dfrac{1}{2}(2.50 \times 10^4\text{ N/m})(-0.100\text{ m})^2$

$E_{mech} = -24.5\text{ J} + 125\text{ J} = \boxed{100\text{ J}}$

(b) Since only conservative forces are involved, the total energy of the child-pogo-stick-Earth system at point C is the same as that at point A.

$K_C + U_{gC} + U_{sC} = K_A + U_{gA} + U_{sA}$: $0 + (25.0\text{ kg})(9.80\text{ m/s}^2)x_C + 0 = 0 - 24.5\text{ J} + 125\text{ J}$

$x_C = \boxed{0.410\text{ m}}$

continued on next page

(c) $K_B + U_{gB} + U_{sB} = K_A + U_{gA} + U_{sA}$: $\dfrac{1}{2}(25.0\text{ kg})v_B^2 + 0 + 0 = 0 + (-24.5\text{ J}) + 125\text{ J}$

$v_B = \boxed{2.84\text{ m/s}}$

(d) K and v are at a maximum when $a = \sum F/m = 0$ (i.e., when the magnitude of the upward spring force equals the magnitude of the downward gravitational force).

This occurs at $x < 0$ where $k|x| = mg$

or $|x| = \dfrac{(25.0\text{ kg})(9.8\text{ m/s}^2)}{2.50 \times 10^4\text{ N/m}} = 9.80 \times 10^{-3}\text{ m}$

Thus, $K = K_{max}$ at $x = \boxed{-9.80\text{ mm}}$

(e) $K_{max} = K_A + \left(U_{gA} - U_g\big|_{x=-9.80\text{ mm}}\right) + \left(U_{sA} - U_s\big|_{x=-9.80\text{ mm}}\right)$

or $\dfrac{1}{2}(25.0\text{ kg})v_{max}^2 = (25.0\text{ kg})(9.80\text{ m/s}^2)\left[(-0.100\text{ m}) - (-0.009\,8\text{ m})\right]$

$+\dfrac{1}{2}(2.50 \times 10^4\text{ N/m})\left[(-0.100\text{ m})^2 - (-0.009\,8\text{ m})^2\right]$

yielding $v_{max} = \boxed{2.85\text{ m/s}}$

P7.51 $\Delta E_{mech} = -fd$

$E_f - E_i = -f \cdot d_{BC}$

$\dfrac{1}{2}kx^2 - mgh = -\mu mgd_{BC}$

$\mu = \dfrac{mgh - \frac{1}{2}kx^2}{mgd_{BC}} = \boxed{0.328}$

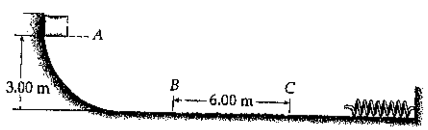

FIG. P7.51

P7.52 (a) $\vec{\mathbf{F}} = -\dfrac{d}{dx}\left(-x^3 + 2x^2 + 3x\right)\hat{\mathbf{i}} = \boxed{\left(3x^2 - 4x - 3\right)\hat{\mathbf{i}}}$

(b) $F = 0$

when $x = \boxed{1.87\text{ and } -0.535}$

(c) The stable point is at

$x = -0.535$ point of minimum $U(x)$.

The unstable point is at

$x = 1.87$ maximum in $U(x)$.

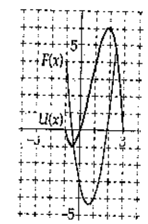

FIG. P7.52

P7.53 $(K+U)_i = (K+U)_f$

$$0 + (30.0\text{ kg})(9.80\text{ m/s}^2)(0.200\text{ m}) + \frac{1}{2}(250\text{ N/m})(0.200\text{ m})^2$$
$$= \frac{1}{2}(50.0\text{ kg})v^2 + (20.0\text{ kg})(9.80\text{ m/s}^2)(0.200\text{ m})\sin 40.0°$$

$$58.8\text{ J} + 5.00\text{ J} = (25.0\text{ kg})v^2 + 25.2\text{ J}$$

$$\boxed{v = 1.24\text{ m/s}}$$

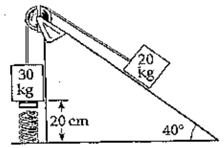

FIG. P7.53

P7.54 (a) Between the second and the third picture, $\Delta E_{mech} = \Delta K + \Delta U$

$$-\mu mgd = -\frac{1}{2}mv_i^2 + \frac{1}{2}kd^2$$

$$\frac{1}{2}(50.0\text{ N/m})d^2 + 0.250(1.00\text{ kg})(9.80\text{ m/s}^2)d - \frac{1}{2}(1.00\text{ kg})(3.00\text{ m/s})^2 = 0$$

$$d = \frac{[-2.45 \pm 21.35]\text{ N}}{50.0\text{ N/m}} = \boxed{0.378\text{ m}}$$

(b) Between picture two and picture four, $\Delta E_{mech} = \Delta K + \Delta U$

$$-f(2d) = \frac{1}{2}mv^2 - \frac{1}{2}mv_i^2$$

$$v = \sqrt{(3.00\text{ m/s})^2 - \frac{2}{(1.00\text{ kg})}(2.45\text{ N})(2)(0.378\text{ m})}$$

$$= \boxed{2.30\text{ m/s}}$$

(c) For the motion from picture two to picture five, $\Delta E_{mech} = \Delta K + \Delta U$

$$-f(D+2d) = -\frac{1}{2}(1.00\text{ kg})(3.00\text{ m/s})^2$$

$$D = \frac{9.00\text{ J}}{2(0.250)(1.00\text{ kg})(9.80\text{ m/s}^2)} - 2(0.378\text{ m}) = \boxed{1.08\text{ m}}$$

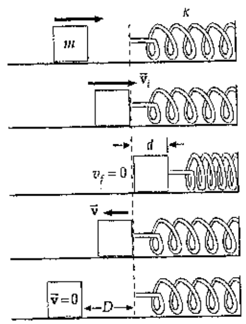

FIG. P7.54

P7.55 (a) Initial compression of spring: $\frac{1}{2}kx^2 = \frac{1}{2}mv^2$

$$\frac{1}{2}(450\text{ N/m})(\Delta x)^2 = \frac{1}{2}(0.500\text{ kg})(12.0\text{ m/s})^2$$

$$\therefore \Delta x = \boxed{0.400\text{ m}}$$

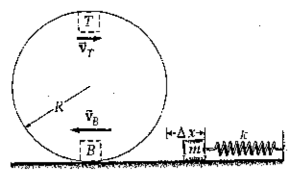

FIG. P7.55

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(b) Speed of block at top of track: $\Delta E_{mech} = -fd$

$$\left(mgh_T + \frac{1}{2}mv_T^2\right) - \left(mgh_B + \frac{1}{2}mv_B^2\right) = -f(\pi R)$$

$$(0.500\text{ kg})(9.80\text{ m/s}^2)(2.00\text{ m}) + \frac{1}{2}(0.500\text{ kg})v_T^2 - \frac{1}{2}(0.500\text{ kg})(12.0\text{ m/s})^2$$
$$= -(7.00\text{ N})(\pi)(1.00\text{ m})$$

$$0.250v_T^2 = 4.21$$

$$\therefore v_T = \boxed{4.10\text{ m/s}}$$

(c) Does block fall off at or before top of track? Block falls if $a_r < g$

$$a_r = \frac{v_T^2}{R} = \frac{(4.10)^2}{1.00} = 16.8\text{ m/s}^2$$

Therefore $a_r > g$ and the $\boxed{\text{block stays on the track}}$.

P7.56 Let λ represent the mass of each one meter of the chain and T represent the tension in the chain at the table edge. We imagine the edge to act like a frictionless and massless pulley.

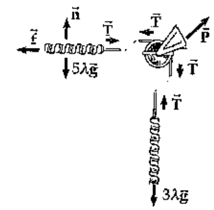

FIG. P7.56

(a) For the five meters on the table with motion impending,

$\sum F_y = 0$: $\quad +n - 5\lambda g = 0 \qquad n = 5\lambda g$

$f_s \le \mu_s n = 0.6(5\lambda g) = 3\lambda g$

$\sum F_x = 0$: $\quad +T - f_s = 0 \qquad T = f_s \qquad T \le 3\lambda g$

The maximum value is barely enough to support the hanging segment according to

$\sum F_y = 0$: $\quad +T - 3\lambda g = 0 \qquad T = 3\lambda g$

so it is at this point that the chain starts to slide.

(b) Let x represent the variable distance the chain has slipped since the start.

Then length $(5-x)$ remains on the table, with now

$\sum F_y = 0$: $\quad +n - (5-x)\lambda g = 0 \qquad n = (5-x)\lambda g$

$$f_k = \mu_k n = 0.4(5-x)\lambda g = 2\lambda g - 0.4x\lambda g$$

Consider energies of the chain-Earth system at the initial moment when the chain starts to slip, and a final moment when $x = 5$, when the last link goes over the brink. Measure heights above the final position of the leading end of the chain. At the moment the final link slips off, the center of the chain is at $y_f = 4$ meters.

Originally, 5 meters of chain is at height 8 m and the middle of the dangling segment is at height $8 - \frac{3}{2} = 6.5$ m.

continued on next page

$$K_i + U_i + \Delta E_{mech} = K_f + U_f: \quad 0 + (m_1 g y_1 + m_2 g y_2)_i - \int_i^f f_k dx = \left(\frac{1}{2}mv^2 + mgy\right)_f$$

$$(5\lambda g)8 + (3\lambda g)6.5 - \int_0^5 (2\lambda g - 0.4x\lambda g)dx = \frac{1}{2}(8\lambda)v^2 + (8\lambda g)4$$

$$40.0g + 19.5g - 2.00g\int_0^5 dx + 0.400g\int_0^5 x\,dx = 4.00v^2 + 32.0g$$

$$27.5g - 2.00gx\Big|_0^5 + 0.400g\frac{x^2}{2}\Big|_0^5 = 4.00v^2$$

$$27.5g - 2.00g(5.00) + 0.400g(12.5) = 4.00v^2$$

$$22.5g = 4.00v^2$$

$$v = \sqrt{\frac{(22.5\text{ m})(9.80\text{ m/s}^2)}{4.00}} = \boxed{7.42\text{ m/s}}$$

P7.57 The geometry reveals $D = L\sin\theta + L\sin\phi$, $50.0\text{ m} = 40.0\text{ m}(\sin 50° + \sin\phi)$, $\phi = 28.9°$

(a) From takeoff to alighting for the Jane-Earth system

$$(K + U_g)_i + W_{wind} = (K + U_g)_f$$

$$\frac{1}{2}mv_i^2 + mg(-L\cos\theta) + FD(-1) = 0 + mg(-L\cos\phi)$$

$$\frac{1}{2}50\text{ kg }v_i^2 + 50\text{ kg}(9.8\text{ m/s}^2)(-40\text{ m}\cos 50°) - 110\text{ N}(50\text{ m}) = 50\text{ kg}(9.8\text{ m/s}^2)(-40\text{ m}\cos 28.9°)$$

$$\frac{1}{2}50\text{ kg }v_i^2 - 1.26\times10^4\text{ J} - 5.5\times10^3\text{ J} = -1.72\times10^4\text{ J}$$

$$v_i = \sqrt{\frac{2(947\text{ J})}{50\text{ kg}}} = \boxed{6.15\text{ m/s}}$$

(b) For the swing back

$$\frac{1}{2}mv_i^2 + mg(-L\cos\phi) + FD(+1) = 0 + mg(-L\cos\theta)$$

$$\frac{1}{2}130\text{ kg }v_i^2 + 130\text{ kg}(9.8\text{ m/s}^2)(-40\text{ m}\cos 28.9°) + 110\text{ N}(50\text{ m})$$

$$= 130\text{ kg}(9.8\text{ m/s}^2)(-40\text{ m}\cos 50°)$$

$$\frac{1}{2}130\text{ kg }v_i^2 - 4.46\times10^4\text{ J} + 5\,500\text{ J} = -3.28\times10^4\text{ J}$$

$$v_i = \sqrt{\frac{2(6\,340\text{ J})}{130\text{ kg}}} = \boxed{9.87\text{ m/s}}$$

P7.58 Case I: Surface is frictionless $\quad \frac{1}{2}mv^2 = \frac{1}{2}kx^2$

$$k = \frac{mv^2}{x^2} = \frac{(5.00\text{ kg})(1.20\text{ m/s})^2}{10^{-2}\text{ m}^2} = 7.20\times10^2\text{ N/m}$$

Case II: Surface is rough, $\quad \mu_k = 0.300$

$$\frac{1}{2}mv^2 = \frac{1}{2}kx^2 - \mu_k mgx$$

$$\frac{5.00\text{ kg}}{2}v^2 = \frac{1}{2}(7.20\times10^2\text{ N/m})(10^{-1}\text{ m})^2 - (0.300)(5.00\text{ kg})(9.80\text{ m/s}^2)(10^{-1}\text{ m})$$

$$\boxed{v = 0.923\text{ m/s}}$$

***P7.59** (a) $(K + U_g)_A = (K + U_g)_B$

$$0 + mgy_A = \frac{1}{2}mv_B^2 + 0 \qquad v_B = \sqrt{2gy_A} = \sqrt{2(9.8\text{ m/s}^2)6.3\text{ m}} = \boxed{11.1\text{ m/s}}$$

(b) $a_c = \frac{v^2}{r} = \frac{(11.1\text{ m/s})^2}{6.3\text{ m}} = \boxed{19.6\text{ m/s}^2\text{ up}}$

(c) $\sum F_y = ma_y \qquad +n_B - mg = ma_c$

$$n_B = 76\text{ kg}(9.8\text{ m/s}^2 + 19.6\text{ m/s}^2) = \boxed{2.23\times10^3\text{ N up}}$$

(d) $W = F\Delta r\cos\theta = 2.23\times10^3\text{ N}(0.450\text{ m})\cos 0° = \boxed{1.01\times10^3\text{ J}}$

This quantity of work represents chemical energy, previously in the skateboarder's body, that is converted into extra kinetic energy:

$$(K + U_g + U_{chemical})_B = (K + U_g + U_{chemical})_C$$

$$\frac{1}{2}mv_B^2 + 0 + U_{chemical,B} - U_{chemical,C} = K_C + mg(y_C - y_B)$$

$$\frac{1}{2}(76\text{ kg})(11.1\text{ m/s})^2 + 1.01\times10^3\text{ J} - 76\text{ kg}(9.8\text{ m/s}^2)(0.450\text{ m}) = K_C$$

(e) $(K + U_g)_C = (K + U_g)_D$

$$5.36\times10^3\text{ J} = \frac{1}{2}(76\text{ kg})v_D^2 + 76\text{ kg}(9.8\text{ m/s}^2)(5.85\text{ m})$$

$$\sqrt{\frac{(5.36\times10^3\text{ J} - 4.36\times10^3\text{ J})2}{76\text{ kg}}} = v_D = \boxed{5.14\text{ m/s}}$$

continued on next page

(f) $(K+U_g)_D = (K+U_g)_E$ where E is the apex of his motion

$$\frac{1}{2}mv_D^2 + 0 = 0 + mg(y_E - y_D) \qquad y_E - y_D = \frac{v_D^2}{2g} = \frac{(5.14 \text{ m/s})^2}{2(9.8 \text{ m/s}^2)} = \boxed{1.35 \text{ m}}$$

(g) Consider the motion with constant acceleration between takeoff and touchdown. The time is the positive root of

$$y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2$$

$$-2.34 \text{ m} = 0 + 5.14 \text{ m/s}\, t + \frac{1}{2}(-9.8 \text{ m/s}^2)t^2$$

$$4.9t^2 - 5.14t - 2.34 = 0$$

$$t = \frac{5.14 \pm \sqrt{5.14^2 - 4(4.9)(-2.34)}}{9.8} = \boxed{1.39 \text{ s}}$$

P7.60 If the spring is just barely able to lift the lower block from the table, the spring lifts it through no noticeable distance, but exerts on the block a force equal to its weight Mg. The extension of the spring, from $|\vec{F}_s| = kx$, must be Mg/k. Between an initial point at release and a final point when the moving block first comes to rest, we have

$$K_i + U_{gi} + U_{si} = K_f + U_{gf} + U_{sf}: \qquad 0 + mg\left(-\frac{4mg}{k}\right) + \frac{1}{2}k\left(\frac{4mg}{k}\right)^2 = 0 + mg\left(\frac{Mg}{k}\right) + \frac{1}{2}k\left(\frac{Mg}{k}\right)^2$$

$$-\frac{4m^2g^2}{k} + \frac{8m^2g^2}{k} = \frac{mMg^2}{k} + \frac{M^2g^2}{2k}$$

$$4m^2 = mM + \frac{M^2}{2}$$

$$\frac{M^2}{2} + mM - 4m^2 = 0$$

$$M = \frac{-m \pm \sqrt{m^2 - 4\left(\frac{1}{2}\right)(-4m^2)}}{2\left(\frac{1}{2}\right)} = -m \pm \sqrt{9m^2}$$

Only a positive mass is physical, so we take $M = m(3-1) = \boxed{2m}$.

***P7.61** (a) Energy is conserved in the swing of the pendulum, and the stationary peg does no work. So the ball's speed does not change when the string hits or leaves the peg, and the ball swings equally high on both sides.

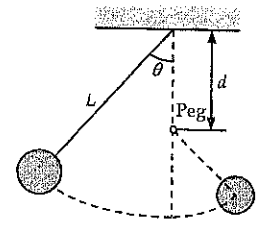

FIG. P7.61

(b) Relative to the point of suspension, $U_i = 0$, $U_f = -mg[d - (L-d)]$. From this we find that

$-mg(2d - L) + \frac{1}{2}mv^2 = 0$. Also for centripetal motion,

$mg = \frac{mv^2}{R}$ where $R = L - d$. Upon solving, we get $\boxed{d = \frac{3L}{5}}$.

P7.62 (a) At the top of the loop the car and riders are in free fall:

$$\sum F_y = ma_y: \qquad mg \text{ down} = \frac{mv^2}{R} \text{ down}$$

$$v = \sqrt{Rg}$$

Energy of the car-riders-Earth system is conserved between release and top of loop:

$$K_i + U_{gi} = K_f + U_{gf}: \qquad 0 + mgh = \frac{1}{2}mv^2 + mg(2R)$$

$$gh = \frac{1}{2}Rg + g(2R)$$

$$\boxed{h = 2.50R}$$

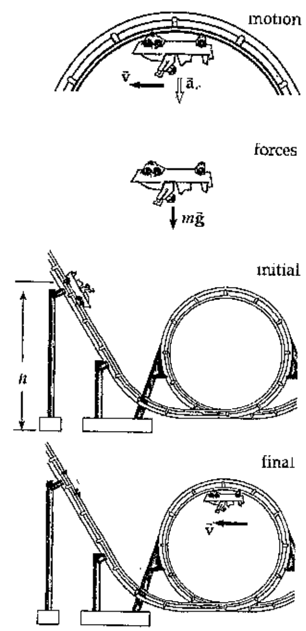

FIG. P7.62

(b) Let h now represent the height $\geq 2.5R$ of the release point. At the bottom of the loop we have

$$mgh = \frac{1}{2}mv_b^2 \quad \text{or} \quad v_b^2 = 2gh$$

$$\sum F_y = ma_y: \qquad n_b - mg = \frac{mv_b^2}{R}\text{(up)}$$

$$n_b = mg + \frac{m(2gh)}{R}$$

At the top of the loop, $mgh = \frac{1}{2}mv_t^2 + mg(2R)$

$$v_t^2 = 2gh - 4gR$$

continued on next page

$\sum F_y = ma_y$: $\quad -n_t - mg = -\dfrac{mv_t^2}{R}$

$$n_t = -mg + \frac{m}{R}(2gh - 4gR)$$

$$n_t = \frac{m(2gh)}{R} - 5mg$$

Then the normal force at the bottom is larger by

$$n_b - n_t = mg + \frac{m(2gh)}{R} - \frac{m(2gh)}{R} + 5mg = \boxed{6mg}$$

P7.63 (a) Conservation of energy for the sled-rider-Earth system, between A and C:

$$K_i + U_{gi} = K_f + U_{gf}$$

$$\frac{1}{2}m(2.5\text{ m/s})^2 + m(9.80\text{ m/s}^2)(9.76\text{ m}) = \frac{1}{2}mv_C^2 + 0$$

$$v_C = \sqrt{(2.5\text{ m/s})^2 + 2(9.80\text{ m/s}^2)(9.76\text{ m})} = \boxed{14.1\text{ m/s}}$$

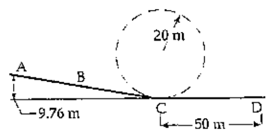

FIG. P7.63(a)

(b) Incorporating the loss of mechanical energy during the portion of the motion in the water, we have, for the entire motion between A and D (the rider's stopping point),

$$K_i + U_{gi} - f_k d = K_f + U_{gf}: \quad \frac{1}{2}(80\text{ kg})(2.5\text{ m/s})^2 + (80\text{ kg})(9.80\text{ m/s}^2)(9.76\text{ m}) - f_k\Delta x = 0 + 0$$

$$-f_k d = \boxed{-7.90 \times 10^3\text{ J}}$$

(c) The water exerts a frictional force $\quad f_k = \dfrac{7.90\times10^3\text{ J}}{d} = \dfrac{7.90\times10^3\text{ N}\cdot\text{m}}{50\text{ m}} = 158\text{ N}$

and also a normal force of $\quad n = mg = (80\text{ kg})(9.80\text{ m/s}^2) = 784\text{ N}$

The magnitude of the water force is $\quad \sqrt{(158\text{ N})^2 + (784\text{ N})^2} = \boxed{800\text{ N}}$

(d) The angle of the slide is

$$\theta = \sin^{-1}\frac{9.76\text{ m}}{54.3\text{ m}} = 10.4°$$

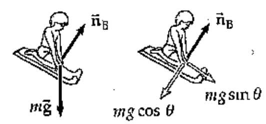

FIG. P7.63(d)

For forces perpendicular to the track at B,

$\sum F_y = ma_y$: $\quad n_B - mg\cos\theta = 0$

$$n_B = (80.0\text{ kg})(9.80\text{ m/s}^2)\cos 10.4° = \boxed{771\text{ N}}$$

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(e) $\sum F_y = ma_y$: $\quad +n_C - mg = \dfrac{mv_C^2}{r}$

$$n_C = (80.0\text{ kg})(9.80\text{ m/s}^2) + \frac{(80.0\text{ kg})(14.1\text{ m/s})^2}{20\text{ m}}$$

$$n_C = \boxed{1.57\times10^3\text{ N up}}$$

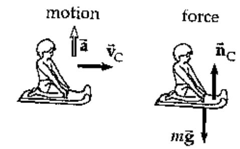

FIG. P7.63(e)

The rider pays for the thrills of a giddy height at A, and a high speed and tremendous splash at C. As a bonus, he gets the quick change in direction and magnitude among the forces we found in parts (d), (e), and (c).

***P7.64** (a) $U_g = mgy = (64\text{ kg})(9.8\text{ m/s}^2)y = \boxed{(627\text{ N})y}$

(b) At the original height and at all heights above $65\text{ m} - 25.8\text{ m} = 39.2\text{ m}$, the cord is unstretched and $\boxed{U_s = 0}$. Below 39.2 m, the cord extension x is given by $x = 39.2\text{ m} - y$, so the elastic energy is $U_s = \frac{1}{2}kx^2 = \boxed{\frac{1}{2}(81\text{ N/m})(39.2\text{ m} - y)^2}$.

(c) For $y > 39.2$ m, $U_g + U_s = \boxed{(627\text{ N})y}$

For $y \le 39.2$ m,

$$U_g + U_s = (627\text{ N})y + 40.5\text{ N/m}(1\,537\text{ m}^2 - (78.4\text{ m})y + y^2)$$

$$= \boxed{(40.5\text{ N/m})y^2 - (2\,550\text{ N})y + 62\,200\text{ J}}$$

(d)

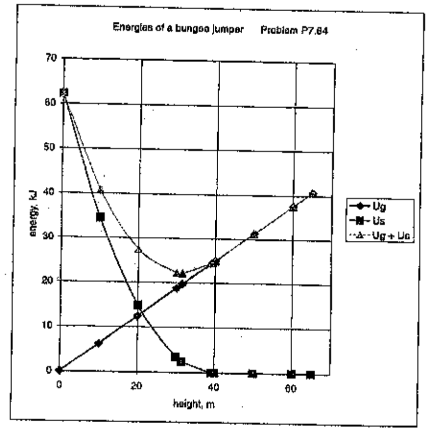

FIG. P7.64(d)

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(e) At minimum height, the jumper has zero kinetic energy and the same total energy as at his starting point. $K_i + U_i = K_f + U_f$ becomes

$$627\text{ N}(65\text{ m}) = (40.5\text{ N/m})y_f^2 - (2\,550\text{ N})y_f + 62\,200\text{ J}$$

$$0 = 40.5y_f^2 - 2\,550y_f + 21\,500$$

$$y_f = \boxed{10.0\text{ m}}\ [\text{the root } 52.9\text{ m is unphysical.}]$$

(f) The total potential energy has a minimum, representing a $\boxed{\text{stable equilibrium}}$ position. To find it, we require $\frac{dU}{dy} = 0$:

$$\frac{d}{dy}\left(40.5y^2 - 2\,550y + 62\,200\right) = 0 = 81y - 2\,550$$

$$y = \boxed{31.5\text{ m}}$$

(g) Maximum kinetic energy occurs at minimum potential energy. Between the takeoff point and this location we have $K_i + U_i = K_f + U_f$

$$0 + 40\,800\text{ J} = \frac{1}{2}(64\text{ kg})v_{max}^2 + 40.5(31.5)^2 - 2\,550(31.5) + 62\,200$$

$$v_{max} = \left(\frac{2(40\,800 - 22\,200)}{64}\right)^{1/2} = \boxed{24.1\text{ m/s}}$$

ANSWERS TO EVEN PROBLEMS

P7.2 (a) 800 J; (b) 107 J; (c) 0

P7.4 (a) 1.11×10^9 J; (b) 0.2

P7.6 (a) 19.8 m/s; (b) 294 J; (c) $(30.0\hat{i} - 39.6\hat{j})$ m/s

P7.8 (a) 2.29 m/s; (b) 1.98 m/s

P7.10 (a) $v_B = 5.94$ m/s, $v_C = 7.67$ m/s; (b) 147 J

P7.12 5.49 m/s

P7.14 (a) see the solution; (b) 35.0 J

P7.16 $d = \frac{kx^2}{2mg\sin\theta} - x$

P7.18 (a) 25.8 m; (b) 27.1 m/s^2

P7.20 (a) 0.403 m or −0.357 m; (b) see the solution

P7.22 (a) 21.0 m/s; (b) 16.1 m/s

P7.24 168 J

P7.26 (a) 24.5 m/s; (b) yes; (c) 206 m; (d) unrealistic, see the solution

P7.28 3.92 kJ

P7.30 44.1 kW

P7.32 (a) $\frac{Ax^2}{2} - \frac{Bx^3}{3}$; (b) $\Delta U = \frac{5A}{2} - \frac{19B}{3}$; $\Delta K = -\frac{5A}{2} + \frac{19B}{3}$

P7.34 $(7 - 9x^2y)\hat{i} - 3x^3\hat{j}$

P7.36 4.17×10^{10} J

P7.38 (a) -1.67×10^{-14} J; (b) at the center

P7.40 (a) $r = 1.5$ mm stable, 2.3 mm unstable, 3.2 mm stable, $r \to \infty$ neutral; (b) $-5.6\text{ J} \le E < 1\text{ J}$; (c) $0.6\text{ mm} \le r \le 3.6\text{ mm}$; (d) 2.6 J; (e) 1.5 mm; (f) 4 J

P7.42 33.4 kW = 44.8 hp

P7.44 (a) 6.75 W/m^2; (b) 6.64×10^3 W/m^2; (c) see solution

P7.46 48.2°

P7.48 (a) 0.588 J; (b) 0.588 J; (c) 2.42 m/s; (d) $U_C = 0.392$ J, $K_C = 0.196$ J

P7.50 (a) 100 J; (b) 0.410 m; (c) 2.84 m/s; (d) −9.80 mm; (e) 2.85 m/s

P7.52 (a) $(3x^2 - 4x - 3)\hat{i}$; (b) $x = 1.87$ and −0.535; (c) see the solution

P7.54 (a) 0.378 m; (b) 2.30 m/s; (c) 1.08 m

P7.56 (a) see the solution; (b) 7.42 m/s

P7.58 0.923 m/s

P7.60 $2m$

P7.62 (a) $2.5R$; (b) see the solution

P7.64 (a) $(627\text{ N})y$; (b) $U_s = 0$ for $y > 39.2$ m and $U_s = \frac{1}{2}(81\text{ N/m})(39.2\text{ m} - y)^2$ for $y \le 39.2$ m; (c) $U_g + U_s = (627\text{ N})y$, for $y > 39.2$ m $U_g + U_s = (40.5\text{ N/m})y^2 - (2\,550\text{ N})y + 62\,200\text{ J}$ for $y \le 39.2$ m; (d) see the solution; (e) 10.0 m; (f) yes: stable equilibrium at 31.5 m; (g) 24.1 m/s

CONTEXT 1 CONCLUSION SOLUTIONS TO PROBLEMS

***CC1.1** (a) $K = \frac{1}{2}mv^2 = \frac{1}{2}(1\,300\text{ kg})(22\text{ m/s})^2 = \boxed{3.15 \times 10^5\text{ J}}$

(b) Energy delivered to battery $= 0.70(3.15 \times 10^5\text{ J}) = \boxed{2.20 \times 10^5\text{ J}}$

(c) Energy returned to battery $= 0.85(2.20 \times 10^5\text{ J}) = \boxed{1.87 \times 10^5\text{ J}}$

(d) Kinetic energy produced by motor $= 0.68(1.87 \times 10^5\text{ J}) = \boxed{1.27 \times 10^5\text{ J}}$

(e) $v = \left(\frac{2K}{m}\right)^{1/2} = \left(\frac{2 \times 1.27 \times 10^5\text{ J}}{1\,300\text{ kg}}\right)^{1/2} = \boxed{14.0\text{ m/s}}$

(f) $e = \frac{\text{useful energy output}}{\text{total energy input}} = \frac{1.27 \times 10^5\text{ J}}{3.15 \times 10^5\text{ J}} = \boxed{0.405} = 40.5\%$

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