(g) The original kinetic energy all becomes either final kinetic energy or internal energy:

$$3.15\times10^5\ \mathrm{J} = 1.27\times10^5\ \mathrm{J} + \Delta E_{\mathrm{int}}$$
$$\Delta E_{\mathrm{int}} = \boxed{1.87\times10^5\ \mathrm{J}}$$

*CC1.2 (a) For the conventional car in the first part of the trip, efficiency $= \dfrac{\text{useful output energy}}{\text{input energy}}$.

$0.07 = \dfrac{33\ \mathrm{MJ}}{\text{input}}$, $\text{input} = \dfrac{33\ \mathrm{MJ}}{0.07} = 471\ \mathrm{MJ}$. Second part: $0.3 = \dfrac{33\ \mathrm{MJ}}{\text{input}}$, $\text{input} = 110\ \mathrm{MJ}$. Total

$\text{input} = 471\ \mathrm{MJ} + 110\ \mathrm{MJ} = \boxed{581\ \mathrm{MJ}}$. For the hybrid car, $0.3 = \dfrac{66\ \mathrm{MJ}}{\text{input}}$, $\text{input} = \boxed{220\ \mathrm{MJ}}$.

(b) conventional car: efficiency $= \dfrac{66\ \mathrm{MJ}}{\text{input}} = 0.114 = \boxed{11.4\%}$. This is much closer to 7% than to 30%. Hybrid car: efficiency $= \boxed{30.0\%}$

ANSWERS TO EVEN CONTEXT 1 CONCLUSION PROBLEMS

CC1.2 (a) 581 MJ, 220 MJ; (b) 11.4%, 30.0%

Chapter 8
Momentum and Collisions

CHAPTER OUTLINE

ANSWERS TO QUESTIONS

Q8.1 No. Impulse, $F\Delta t$, depends on the force and the time for which it is applied.

Q8.2 The momentum doubles since it is proportional to the speed. The kinetic energy quadruples, since it is proportional to the speed-squared.

Q8.3 The momenta of two particles will only be the same if the masses of the particles of the same.

Q8.4 (a) It does not carry force, for if it did, it could accelerate itself.

(b) It cannot deliver more kinetic energy than it possesses. This would violate the law of energy conservation.

(c) It can deliver more momentum in a collision than it possesses in its flight, by bouncing from the object it strikes.

Q8.5 The superhero has finite mass. When he throws the piano, he must recoil in the opposite direction. To hover in midair, he must continuously fly by giving downward momentum to air, rocket exhaust or something that will disturb things below.

Q8.6 No. Only in a precise head-on collision with momenta with equal magnitudes and opposite directions can both objects wind up at rest. Yes. Assume that ball 2, originally at rest, is struck squarely by an equal-mass ball 1. Then ball 2 will take off with the velocity of ball 1, leaving ball 1 at rest.

Q8.7 Interestingly, mutual gravitation brings the ball and the Earth together. As the ball moves downward, the Earth moves upward, although with an acceleration 10^{25} times smaller than that of the ball. The two objects meet, rebound, and separate. Momentum of the ball-Earth system is conserved.

Q8.8 (a) Linear momentum is conserved since there are no external forces acting on the system.

(b) Kinetic energy is not conserved because the chemical potential energy initially in the explosive is converted into kinetic energy of the pieces of the bomb.

Q8.9 Momentum conservation is not violated if we choose as our system the planet along with you. When you receive an impulse forward, the Earth receives the same size impulse backwards. The resulting acceleration of the Earth due to this impulse is significantly smaller than your acceleration forward, but the planet's backward momentum is equal in magnitude to your forward momentum.

Q8.10 Suppose car and truck move along the same line. If one vehicle overtakes the other, the faster-moving one loses more energy than the slower one gains. In a head-on collision, if the speed of the truck is less than $\frac{m_T + 3m_c}{3m_T + m_c}$ times the speed of the car, the car will lose more energy.

Q8.11 The rifle has a much lower speed than the bullet and much less kinetic energy. The butt distributes the recoil force over an area much larger than that of the bullet.

Q8.12 His impact speed is determined by the acceleration of gravity and the distance of fall, in $v_f^2 = v_i^2 - 2g(0 - y_i)$. The force exerted by the pad depends also on the unknown stiffness of the pad.

Q8.13 The product of the mass flow rate and velocity of the water determines the force the firefighters must exert.

Q8.14 The sheet stretches and pulls the two students toward each other. These effects are larger for a faster-moving egg. The time over which the egg stops is extended so that the force stopping it is never too large.

Q8.15 The planet is in motion around the sun, and thus has momentum and kinetic energy of its own. The spacecraft is directed to cross the planet's orbit behind it, so that the planet's gravity has a component pulling forward on the spacecraft. Since this is an elastic collision, and the velocity of the planet remains nearly unchanged, the probe must both increase speed and change direction for both momentum and kinetic energy to be conserved.

Q8.16 Yes. A boomerang, a kitchen stool.

Q8.17 The center of mass of the balls is in free fall, moving up and then down with the acceleration due to gravity, during the 40% of the time when the juggler's hands are empty. During the 60% of the time when the juggler is engaged in catching and tossing, the center of mass must accelerate up with a somewhat smaller average acceleration. The center of mass moves around in a little circle, making three revolutions for every one revolution that one ball makes. Letting T represent the time for one cycle and F_g the weight of one ball, we have $F_J 0.60T = 3F_g T$ and $F_J = 5F_g$. The average force exerted by the juggler is five times the weight of one ball.

Q8.18 Inflate a balloon and release it. The air escaping from the balloon gives the balloon an impulse.

Q8.19 In empty space, the center of mass of a rocket-plus-fuel system does not accelerate during a burn, because no outside force acts on this system. According to the text's 'basic expression for rocket propulsion,' the change in speed of the rocket body will be larger than the speed of the exhaust relative to the rocket, if the final mass is less than 37% of the original mass.

Q8.20 There was a time when the English favored position (a), the Germans position (b), and the French position (c). A Frenchman, Jean D'Alembert, is most responsible for showing that each theory is consistent with the others. All are equally correct. Each is useful for giving a mathematically simple solution for some problems.

SOLUTIONS TO PROBLEMS

Section 8.1 Linear Momentum and Its Conservation

P8.1 $m = 3.00 \text{ kg}$, $\vec{v} = (3.00\hat{i} - 4.00\hat{j}) \text{ m/s}$

(a) $\vec{p} = m\vec{v} = (9.00\hat{i} - 12.0\hat{j}) \text{ kg}\cdot\text{m/s}$

Thus, $\boxed{p_x = 9.00 \text{ kg}\cdot\text{m/s}}$

and $\boxed{p_y = -12.0 \text{ kg}\cdot\text{m/s}}$.

(b) $p = \sqrt{p_x^2 + p_y^2} = \sqrt{(9.00)^2 + (12.0)^2} = \boxed{15.0 \text{ kg}\cdot\text{m/s}}$

$$\theta = \tan^{-1}\left(\frac{p_y}{p_x}\right) = \tan^{-1}(-1.33) = \boxed{307°}$$

P8.2 I have mass 85.0 kg and can jump to raise my center of gravity 25.0 cm. I leave the ground with speed given by

$v_f^2 - v_i^2 = 2a(x_f - x_i)$: $\quad 0 - v_i^2 = 2(-9.80 \text{ m/s}^2)(0.250 \text{ m})$

$$v_i = 2.20 \text{ m/s}$$

Total momentum of the system of the Earth and me is conserved as I push the earth down and myself up:

$$0 = (5.98\times10^{24} \text{ kg})v_e + (85.0 \text{ kg})(2.20 \text{ m/s})$$

$$v_e \sim \boxed{10^{-23} \text{ m/s}}$$

P8.3 Assume the velocity of the blood is constant over the 0.160 s. Then the patient's body and pallet will have a constant velocity of $\frac{6\times10^{-5} \text{ m}}{0.160 \text{ s}} = 3.75\times10^{-4} \text{ m/s}$ in the opposite direction. Momentum conservation gives

$\vec{p}_{1i} + \vec{p}_{2i} = \vec{p}_{1f} + \vec{p}_{2f}$: $\quad 0 = m_{\text{blood}}\, 0.5 \text{ m/s} + 54 \text{ kg}\,(-3.75\times10^{-4} \text{ m/s})$

$$m_{\text{blood}} = 0.0405 \text{ kg} = \boxed{40.5 \text{ g}}$$

P8.4 (a) The momentum is $p = mv$, so $v = \frac{p}{m}$ and the kinetic energy is $K = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{p}{m}\right)^2 = \boxed{\frac{p^2}{2m}}$.

(b) $K = \frac{1}{2}mv^2$ implies $v = \sqrt{\frac{2K}{m}}$, so $p = mv = m\sqrt{\frac{2K}{m}} = \boxed{\sqrt{2mK}}$.

P8.5 (a) For the system of two blocks $\Delta p = 0$,

or $p_i = p_f$.

Therefore, $0 = Mv_m + (3M)(2.00\ \text{m/s})$.

Solving gives $v_m = \boxed{-6.00\ \text{m/s}}$ (motion toward the left).

(b) $\frac{1}{2}kx^2 = \frac{1}{2}Mv_M^2 + \frac{1}{2}(3M)v_{3M}^2 = \boxed{8.40\ \text{J}}$

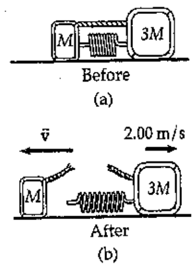

FIG. P8.5

Section 8.2 Impulse and Momentum

***P8.6** From the impulse-momentum theorem, $\bar{F}(\Delta t) = \Delta p = mv_f - mv_i$, the average force required to hold onto the child is

$$\bar{F} = \frac{m(v_f - v_i)}{(\Delta t)} = \frac{(12\ \text{kg})(0-60\ \text{mi/h})}{0.050\ \text{s} - 0}\left(\frac{1\ \text{m/s}}{2.237\ \text{mi/h}}\right) = -6.44\times10^3\ \text{N}.$$

Therefore, the magnitude of the needed retarding force is $\boxed{6.44\times10^3\ \text{N}}$, or 1 400 lb. A person cannot exert a force of this magnitude and a safety device should be used.

P8.7 (a) $I = \int F dt =$ area under curve

$$I = \frac{1}{2}(1.50\times10^{-3}\ \text{s})(18\,000\ \text{N}) = \boxed{13.5\ \text{N}\cdot\text{s}}$$

(b) $$F = \frac{13.5\ \text{N}\cdot\text{s}}{1.50\times10^{-3}\ \text{s}} = \boxed{9.00\ \text{kN}}$$

(c) From the graph, we see that $F_{max} = \boxed{18.0\ \text{kN}}$

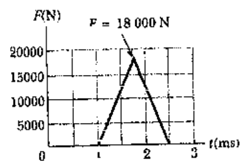

FIG. P8.7

P8.8 Assume the initial direction of the ball in the $-x$ direction.

(a) Impulse, $\vec{I} = \Delta\vec{p} = \vec{p}_f - \vec{p}_i = (0.060\,0)(40.0)\hat{i} - (0.060\,0)(50.0)(-\hat{i}) = \boxed{5.40\hat{i}\ \text{N}\cdot\text{s}}$

(b) Work $= K_f - K_i = \frac{1}{2}(0.060\,0)\left[(40.0)^2 - (50.0)^2\right] = \boxed{-27.0\ \text{J}}$

P8.9 $\Delta\vec{p} = \vec{F}\Delta t$

$$\Delta p_y = m(v_{fy} - v_{iy}) = m(v\cos60.0°) - mv\cos60.0° = 0$$
$$\Delta p_x = m(-v\sin60.0° - v\sin60.0°) = -2mv\sin60.0°$$
$$= -2(3.00\ \text{kg})(10.0\ \text{m/s})(0.866)$$
$$= -52.0\ \text{kg}\cdot\text{m/s}$$
$$F_{ave} = \frac{\Delta p_x}{\Delta t} = \frac{-52.0\ \text{kg}\cdot\text{m/s}}{0.200\ \text{s}} = \boxed{-260\ \text{N}}$$

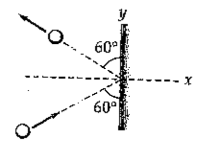

FIG. P8.9

P8.10 Take x-axis toward the pitcher

(a) $p_{ix} + I_x = p_{fx}$: $(0.200\ \text{kg})(15.0\ \text{m/s})(-\cos45.0°) + I_x = (0.200\ \text{kg})(40.0\ \text{m/s})\cos30.0°$

$I_x = 9.05\ \text{N}\cdot\text{s}$

$p_{iy} + I_y = p_{fy}$: $(0.200\ \text{kg})(15.0\ \text{m/s})(-\sin45.0°) + I_y = (0.200\ \text{kg})(40.0\ \text{m/s})\sin30.0°$

$$\vec{I} = \boxed{(9.05\hat{i} + 6.12\hat{j})\ \text{N}\cdot\text{s}}$$

(b) $\vec{I} = \frac{1}{2}(0 + \vec{F}_m)(4.00\ \text{ms}) + \vec{F}_m(20.0\ \text{ms}) + \frac{1}{2}\vec{F}_m(4.00\ \text{ms})$

$\vec{F}_m \times 24.0\times10^{-3}\ \text{s} = (9.05\hat{i} + 6.12\hat{j})\ \text{N}\cdot\text{s}$

$\vec{F}_m = \boxed{(377\hat{i} + 255\hat{j})\ \text{N}}$

P8.11 The force exerted on the water by the hose is

$$F = \frac{\Delta p_{water}}{\Delta t} = \frac{mv_f - mv_i}{\Delta t} = \frac{(0.600\ \text{kg})(25.0\ \text{m/s}) - 0}{1.00\ \text{s}} = \boxed{15.0\ \text{N}}.$$

According to Newton's third law, the water exerts a force of equal magnitude back on the hose. Thus, the gardener must apply a 15.0 N force (in the direction of the velocity of the exiting water stream) to hold the hose stationary.

P8.12 (a) Energy is conserved for the spring-mass system:

$K_i + U_{si} = K_f + U_{sf}$: $0 + \frac{1}{2}kx^2 = \frac{1}{2}mv^2 + 0$

$$v = x\sqrt{\frac{k}{m}}$$

(b) From the equation, a $\boxed{\text{smaller}}$ value of m makes $v = x\sqrt{\frac{k}{m}}$ larger.

continued on next page

(c) $I = |\vec{p}_f - \vec{p}_i| = mv_f = 0 = mx\sqrt{\frac{k}{m}} = x\sqrt{km}$

(d) From the equation, a $\boxed{\text{larger}}$ value of m makes $I = x\sqrt{km}$ larger.

(e) For the glider, $W = K_f - K_i = \frac{1}{2}mv^2 - 0 = \frac{1}{2}kx^2$

The mass makes $\boxed{\text{no difference}}$ to the work.

Section 8.3 Collisions

P8.13 (a) $mv_{1i} + 3mv_{2i} = 4mv_f$ where $m = 2.50 \times 10^4$ kg

$$v_f = \frac{4.00 + 3(2.00)}{4} = \boxed{2.50 \text{ m/s}}$$

(b) $$K_f - K_i = \frac{1}{2}(4m)v_f^2 - \left[\frac{1}{2}mv_{1i}^2 + \frac{1}{2}(3m)v_{2i}^2\right] = (2.50 \times 10^4)(12.5 - 8.00 - 6.00) = \boxed{-3.75 \times 10^4 \text{ J}}$$

P8.14 (a) The internal forces exerted by the actor do not change the total momentum of the system of the four cars and the movie actor

$$(4m)v_i = (3m)(2.00 \text{ m/s}) + m(4.00 \text{ m/s})$$

$$v_i = \frac{6.00 \text{ m/s} + 4.00 \text{ m/s}}{4} = \boxed{2.50 \text{ m/s}}$$

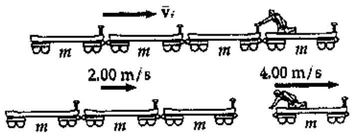

FIG. P8.14

(b) $$W_{\text{actor}} = K_f - K_i = \frac{1}{2}\left[(3m)(2.00 \text{ m/s})^2 + m(4.00 \text{ m/s})^2\right] - \frac{1}{2}(4\,m)(2.50 \text{ m/s})^2$$

$$W_{\text{actor}} = \frac{(2.50 \times 10^4 \text{ kg})}{2}(12.0 + 16.0 - 25.0)(\text{m/s})^2 = \boxed{37.5 \text{ kJ}}$$

(c) $\boxed{\text{The event considered here is the time reversal of the perfectly inelastic collision in the previous problem. The same momentum conservation equation describes both processes.}}$

P8.15 (a), (b) Let v_g and v_p be the velocity of the girl and the plank relative to the ice surface. Then we may say that $v_g - v_p$ is the velocity of the girl relative to the plank, so that

$$v_g - v_p = 1.50 \qquad (1)$$

But also we must have $m_g v_g + m_p v_p = 0$, since total momentum of the girl-plank system is zero relative to the ice surface. Therefore

$$45.0v_g + 150v_p = 0\text{, or } v_g = -3.33v_p$$

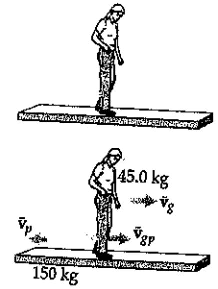

FIG. P8.15

Putting this into the equation (1) above gives $-3.33v_p - v_p = 1.50$ or $v_p = \boxed{-0.346 \text{ m/s}}$. Then $v_g = -3.33(-0.346) = \boxed{1.15 \text{ m/s}}$

P8.16 v_1, speed of m_1 at B before collision.

$$\frac{1}{2}m_1 v_1^2 = m_1 gh$$

$$v_1 = \sqrt{2(9.80)(5.00)} = 9.90 \text{ m/s}$$

v_{1f}, speed of m_1 at B just after collision.

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2}v_1 = -\frac{1}{3}(9.90) \text{ m/s} = -3.30 \text{ m/s}$$

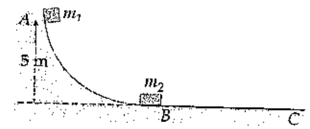

FIG. P8.16

At the highest point (after collision)

$$m_1 g h_{\max} = \frac{1}{2}m_1(-3.30)^2 \qquad h_{\max} = \frac{(-3.30 \text{ m/s})^2}{2(9.80 \text{ m/s}^2)} = \boxed{0.556 \text{ m}}$$

P8.17 For the car-truck-driver-driver system, momentum is conserved:

$$\vec{p}_{1i} + \vec{p}_{2i} = \vec{p}_{1f} + \vec{p}_{2f}: \qquad (4\,000 \text{ kg})(8 \text{ m/s})\hat{i} + (800 \text{ kg})(8 \text{ m/s})(-\hat{i}) = (4\,800 \text{ kg})v_f\hat{i}$$

$$v_f = \frac{25\,600 \text{ kg}\cdot\text{m/s}}{4\,800 \text{ kg}} = 5.33 \text{ m/s}$$

For the driver of the truck, the impulse-momentum theorem is

$$\vec{F}\Delta t = \vec{p}_f - \vec{p}_i: \qquad \vec{F}(0.120 \text{ s}) = (80 \text{ kg})(5.33 \text{ m/s})\hat{i} - (80 \text{ kg})(8 \text{ m/s})\hat{i}$$

$$\vec{F} = \boxed{1.78 \times 10^3 \text{ N}(-\hat{i}) \text{ on the truck driver}}$$

For the driver of the car, $\vec{F}(0.120 \text{ s}) = (80 \text{ kg})(5.33 \text{ m/s})\hat{i} - (80 \text{ kg})(8 \text{ m/s})(-\hat{i})$

$$\vec{F} = \boxed{8.89 \times 10^3 \text{ N}\hat{i} \text{ on the car driver}}\text{, 5 times larger.}$$

P8.18 Energy is conserved for the bob-Earth system between bottom and top of swing. At the top the stiff rod is in compression and the bob nearly at rest.

$K_i + U_i = K_f + U_f$: $\quad \frac{1}{2}Mv_b^2 + 0 = 0 + Mg2\ell$

$$v_b^2 = g4\ell \text{ so } v_b = 2\sqrt{g\ell}$$

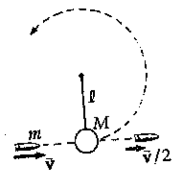

FIG. P8.18

Momentum of the bob-bullet system is conserved in the collision:

$$mv = m\frac{v}{2} + M\left(2\sqrt{g\ell}\right) \quad \boxed{v = \frac{4M}{m}\sqrt{g\ell}}$$

P8.19 (a) According to the Example in the chapter text, the fraction of total kinetic energy transferred to the moderator is

$$f_2 = \frac{4m_1m_2}{(m_1+m_2)^2}$$

where m_2 is the moderator nucleus and in this case, $m_2 = 12m_1$

$$f_2 = \frac{4m_1(12m_1)}{(13m_1)^2} = \frac{48}{169} = \boxed{0.284 \text{ or } 28.4\%}$$

of the neutron energy is transferred to the carbon nucleus.

(b) $K_C = (0.284)\left(1.6\times10^{-13}\text{ J}\right) = \boxed{4.54\times10^{-14}\text{ J}}$

$K_n = (0.716)\left(1.6\times10^{-13}\text{ J}\right) = \boxed{1.15\times10^{-13}\text{ J}}$

P8.20 We assume equal firing speeds v and equal forces F required for the two bullets to push wood fibers apart. These equal forces act backward on the two bullets.

For the first, $\quad K_i + \Delta E_{mech} = K_f \quad \frac{1}{2}\left(7.00\times10^{-3}\text{ kg}\right)v^2 - F\left(8.00\times10^{-2}\text{ m}\right) = 0$

For the second, $\quad p_i = p_f \quad \left(7.00\times10^{-3}\text{ kg}\right)v = (1.014\text{ kg})v_f$

$$v_f = \frac{\left(7.00\times10^{-3}\right)v}{1.014}$$

Again, $\quad K_i + \Delta E_{mech} = K_f$: $\quad \frac{1}{2}\left(7.00\times10^{-3}\text{ kg}\right)v^2 - Fd = \frac{1}{2}(1.014\text{ kg})v_f^2$

Substituting for v_f, $\quad \frac{1}{2}\left(7.00\times10^{-3}\text{ kg}\right)v^2 - Fd = \frac{1}{2}(1.014\text{ kg})\left(\frac{7.00\times10^{-3}v}{1.014}\right)^2$

$$Fd = \frac{1}{2}\left(7.00\times10^{-3}\right)v^2 - \frac{1}{2}\frac{\left(7.00\times10^{-3}\right)^2}{1.014}v^2$$

Substituting for v, $\quad Fd = F\left(8.00\times10^{-2}\text{ m}\right)\left(1 - \frac{7.00\times10^{-3}}{1.014}\right) \quad d = \boxed{7.94\text{ cm}}$

P8.21 At impact, momentum of the clay-block system is conserved, so:

$$mv_1 = (m_1+m_2)v_2$$

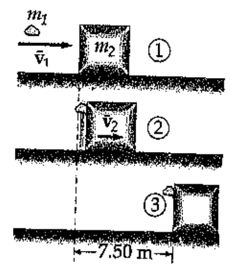

FIG. P8.21

After impact, the change in kinetic energy of the clay-block-surface system is equal to the increase in internal energy:

$$\frac{1}{2}(m_1+m_2)v_2^2 = f_f d = \mu(m_1+m_2)gd$$

$$\frac{1}{2}(0.112\text{ kg})v_2^2 = 0.650(0.112\text{ kg})\left(9.80\text{ m/s}^2\right)(7.50\text{ m})$$

$v_2^2 = 95.6\text{ m}^2/\text{s}^2 \qquad v_2 = 9.77\text{ m/s}$

$\left(12.0\times10^{-3}\text{ kg}\right)v_1 = (0.112\text{ kg})(9.77\text{ m/s}) \qquad v_1 = \boxed{91.2\text{ m/s}}$

*P8.22 (a) Using conservation of momentum, $\left(\sum\vec{p}\right)_{after} = \left(\sum\vec{p}\right)_{before}$, gives

$$\left[(4.0+10+3.0)\text{ kg}\right]v = (4.0\text{ kg})(5.0\text{ m/s}) + (10\text{ kg})(3.0\text{ m/s}) + (3.0\text{ kg})(-4.0\text{ m/s}).$$

Therefore, $v = +2.24$ m/s, or $\boxed{2.24\text{ m/s toward the right}}$.

(b) $\boxed{\text{No}}$. For example, if the 10-kg and 3.0-kg mass were to stick together first, they would move with a speed given by solving

$$(13\text{ kg})v_1 = (10\text{ kg})(3.0\text{ m/s}) + (3.0\text{ kg})(-4.0\text{ m/s}), \text{ or } v_1 = +1.38\text{ m/s}.$$

Then when this 13 kg combined mass collides with the 4.0 kg mass, we have

$$(17\text{ kg})v = (13\text{ kg})(1.38\text{ m/s}) + (4.0\text{ kg})(5.0\text{ m/s}), \text{ and } v = +2.24\text{ m/s}$$

just as in part (a). Coupling order makes no difference.

*P8.23 (a) We assume that energy is conserved in the fall of the basketball and the tennis ball. Each reaches its lowest point with a speed given by

$$\left(K+U_g\right)_{release} = \left(K+U_g\right)_{bottom}$$

$$0 + mgy_i = \frac{1}{2}mv_b^2 + 0$$

$$v_b = \sqrt{2gy_i} = \sqrt{2\left(9.8\text{ m/s}^2\right)(1.20\text{ m})} = \boxed{4.85\text{ m/s}}$$

continued on next page

(b) The two balls exert no forces on each other as they move down. They collide with each other after the basketball has its velocity reversed by the floor. We choose upward as positive. Momentum conservation:

$$(57\text{ g})(-4.85\text{ m/s})+(590\text{ g})(4.85\text{ m/s})=(57\text{ g})v_{2f}+(590\text{ g})v_{1f}$$

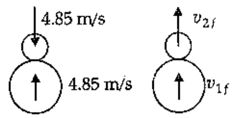

FIG. P8.23(b)

To describe the elastic character of the collision, we use the relative velocity equation

$$4.85\text{ m/s}-(-4.85\text{ m/s})=v_{2f}-v_{1f}$$

we solve by substitution

$$v_{1f}=v_{2f}-9.70\text{ m/s}$$
$$2\,580\text{ gm/s}=(57\text{ g})v_{2f}+(590\text{ g})(v_{2f}-9.70\text{ m/s})$$
$$=(57\text{ g})v_{2f}+(590\text{ g})v_{2f}-5\,720\text{ gm/s}$$
$$v_{2f}=\frac{8\,310\text{ m/s}}{647}=12.8\text{ m/s}$$

Now the tennis ball-Earth system keeps constant energy as the ball rises:

$$\frac{1}{2}(57\text{ g})(12.8\text{ m/s})^2=(57\text{ g})(9.8\text{ m/s}^2)y_f$$
$$y_f=\frac{165\text{ m}^2/\text{s}^2}{2(9.8\text{ m/s}^2)}=\boxed{8.41\text{ m}}$$

Section 8.4 Two-Dimensional Collisions

P8.24 (a) First, we conserve momentum for the system of two football players in the x direction (the direction of travel of the fullback).

$$(90.0\text{ kg})(5.00\text{ m/s})+0=(185\text{ kg})V\cos\theta$$

where θ is the angle between the direction of the final velocity V and the x axis. We find

$$V\cos\theta=2.43\text{ m/s} \qquad (1)$$

Now consider conservation of momentum of the system in the y direction (the direction of travel of the opponent).

$$(95.0\text{ kg})(3.00\text{ m/s})+0=(185\text{ kg})(V\sin\theta)$$

which gives, $V\sin\theta=1.54\text{ m/s}$ (2)

continued on next page

Divide equation (2) by (1) $\tan\theta=\dfrac{1.54}{2.43}=0.633$

From which $\boxed{\theta=32.3°}$

Then, either (1) or (2) gives $V=\boxed{2.88\text{ m/s}}$

(b) $K_i=\frac{1}{2}(90.0\text{ kg})(5.00\text{ m/s})^2+\frac{1}{2}(95.0\text{ kg})(3.00\text{ m/s})^2=1.55\times10^3\text{ J}$

$K_f=\frac{1}{2}(185\text{ kg})(2.88\text{ m/s})^2=7.67\times10^2\text{ J}$

Thus, the kinetic energy lost is $\boxed{\text{783 J into internal energy.}}$

P8.25 $p_{xf}=p_{xi}$: $mv_O\cos\theta+mv_Y\cos(90.0°-\theta)=mv_i$

$$v_O\cos\theta+v_Y\sin\theta=v_i \qquad (1)$$

$p_{yf}=p_{yi}$: $mv_O\sin\theta-mv_Y\sin(90.0°-\theta)=0$

$$v_O\sin\theta=v_Y\cos\theta \qquad (2)$$

From equation (2),

$$v_O=v_Y\left(\frac{\cos\theta}{\sin\theta}\right) \qquad (3)$$

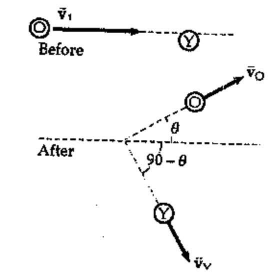

FIG. P8.25

Substituting into equation (1),

$$v_Y\left(\frac{\cos^2\theta}{\sin\theta}\right)+v_Y\sin\theta=v_i$$

so $v_Y(\cos^2\theta+\sin^2\theta)=v_i\sin\theta$, and $\boxed{v_Y=v_i\sin\theta}$.

Then, from equation (3), $\boxed{v_O=v_i\cos\theta}$.

We did not need to write down an equation expressing conservation of mechanical energy. In the problem situation, the requirement of perpendicular final velocities is equivalent to the condition of elasticity.

P8.26 We use conservation of momentum for the system of two vehicles for both northward and eastward components.

For the eastward direction:

$$M(13.0 \text{ m/s}) = 2MV_f \cos 55.0°$$

For the northward direction:

$$Mv_{2i} = 2MV_f \sin 55.0°$$

Divide the northward equation by the eastward equation to find:

$$v_{2i} = (13.0 \text{ m/s})\tan 55.0° = 18.6 \text{ m/s} = \boxed{41.5 \text{ mi/h}}$$

Thus, the driver of the north bound car was untruthful.

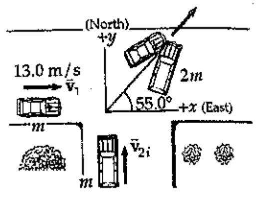

FIG. P8.26

P8.27 By conservation of momentum for the system of the two billiard balls (with all masses equal),

$$5.00 \text{ m/s} + 0 = (4.33 \text{ m/s})\cos 30.0° + v_{2fx}$$
$$v_{2fx} = 1.25 \text{ m/s}$$
$$0 = (4.33 \text{ m/s})\sin 30.0° + v_{2fy}$$
$$v_{2fy} = -2.16 \text{ m/s}$$
$$\vec{v}_{2f} = \boxed{2.50 \text{ m/s at } -60.0°}$$

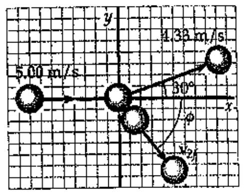

FIG. P8.27

Note that we did not need to use the fact that the collision is perfectly elastic.

P8.28 (a) $\vec{p}_i = \vec{p}_f$ so $p_{xi} = p_{xf}$

and $p_{yi} = p_{yf}$

$$mv_i = mv\cos\theta + mv\cos\phi \quad (1)$$
$$0 = mv\sin\theta + mv\sin\phi \quad (2)$$

From (2), $\sin\theta = -\sin\phi$

so $\theta = -\phi$

Furthermore, energy conservation for the system of two protons requires

$$\frac{1}{2}mv_i^2 = \frac{1}{2}mv^2 + \frac{1}{2}mv^2$$

so $\boxed{v = \frac{v_i}{\sqrt{2}}}$

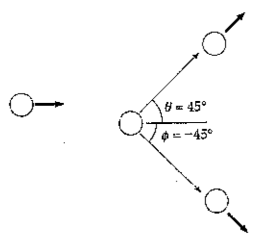

FIG. P8.28

(b) Hence, (1) gives $v_i = \frac{2v_i\cos\theta}{\sqrt{2}}$ $\quad \theta = \boxed{45.0°}$ $\quad \phi = \boxed{-45.0°}$

P8.29 $m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = (m_1 + m_2)\vec{v}_f$: $\quad 3.00(5.00)\hat{i} - 6.00\hat{j} = 5.00\vec{v}$

$$\vec{v} = \boxed{(3.00\hat{i} - 1.20\hat{j}) \text{ m/s}}$$

P8.30 (a) We write equations expressing conservation of the x and y components of momentum, with reference to the figures. Let the puck initially at rest be m_2.

$$m_1v_{1i} = m_1v_{1f}\cos\theta + m_2v_{2f}\cos\phi:$$
$$v_{2f}\cos\phi = \frac{m_1v_{1i} - m_1v_{1f}\cos\theta}{m_2}$$
$$v_{2f}\cos\phi = \frac{(0.200 \text{ kg})(2.00 \text{ m/s}) - (0.200 \text{ kg})(1.00 \text{ m/s})\cos 53.0°}{0.300 \text{ kg}}$$
$$0 = m_1v_{1f}\sin\theta - m_2v_{2f}\sin\phi: \quad v_{2f}\sin\phi = \frac{m_1v_{1f}\sin\theta}{m_2}$$
$$0 = (0.200 \text{ kg})(1.00 \text{ m/s})\sin 53.0° - (0.300 \text{ kg})(v_{2f}\sin\phi)$$

From these equations, we find $\tan\phi = \frac{\sin\phi}{\cos\phi} = \frac{0.160}{0.280} = 0.571$

$$\boxed{\phi = 29.7°}$$

Then $\quad v_{2f} = \frac{(0.160 \text{ kg}\cdot\text{m/s})}{(0.300 \text{ kg})(\sin 29.7°)} = \boxed{1.07 \text{ m/s}}$

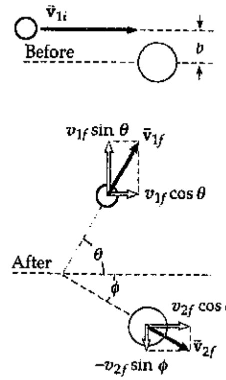

FIG. P8.30

(b) $f_{\text{lost}} = \frac{\Delta K}{K_i} = \frac{K_f - K_i}{K_i} = \boxed{-0.318}$

P8.31 $m_0 = 17.0 \times 10^{-27}$ kg $\quad \vec{v}_i = 0$ (the parent nucleus)

$m_1 = 5.00 \times 10^{-27}$ kg $\quad \vec{v}_1 = 6.00 \times 10^6\hat{j}$ m/s

$m_2 = 8.40 \times 10^{-27}$ kg $\quad \vec{v}_2 = 4.00 \times 10^6\hat{i}$ m/s

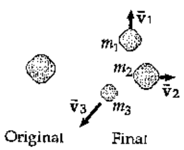

FIG. P8.31

(a) $m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3 = 0$

where $m_3 = m_0 - m_1 - m_2 = 3.60 \times 10^{-27}$ kg

$$(5.00\times10^{-27})(6.00\times10^6\hat{j}) + (8.40\times10^{-27})(4.00\times10^6\hat{i}) + (3.60\times10^{-27})\vec{v}_3 = 0$$
$$\vec{v}_3 = \boxed{(-9.33\times10^6\hat{i} - 8.33\times10^6\hat{j}) \text{ m/s}}$$

continued on next page

(b) $E=\frac{1}{2}m_1v_1^2+\frac{1}{2}m_2v_2^2+\frac{1}{2}m_3v_3^2$

$$E=\frac{1}{2}\left[\left(5.00\times10^{-27}\right)\left(6.00\times10^{6}\right)^2+\left(8.40\times10^{-27}\right)\left(4.00\times10^{6}\right)^2+\left(3.60\times10^{-27}\right)\left(12.5\times10^{6}\right)^2\right]$$

$$\boxed{E=4.39\times10^{-13}\ \text{J}}$$

Section 8.5 The Center of Mass

P8.32 The x-coordinate of the center of mass is

$$x_{CM}=\frac{\sum m_ix_i}{\sum m_i}=\frac{0+0+0+0}{(2.00\ \text{kg}+3.00\ \text{kg}+2.50\ \text{kg}+4.00\ \text{kg})}$$

$$\boxed{x_{CM}=0}$$

and the y-coordinate of the center of mass is

$$y_{CM}=\frac{\sum m_iy_i}{\sum m_i}=\frac{(2.00\ \text{kg})(3.00\ \text{m})+(3.00\ \text{kg})(2.50\ \text{m})+(2.50\ \text{kg})(0)+(4.00\ \text{kg})(-0.500\ \text{m})}{2.00\ \text{kg}+3.00\ \text{kg}+2.50\ \text{kg}+4.00\ \text{kg}}$$

$$\boxed{y_{CM}=1.00\ \text{m}}$$

P8.33 Let A_1 represent the area of the bottom row of squares, A_2 the middle square, and A_3 the top pair.

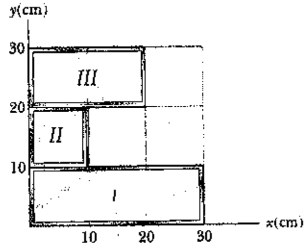

FIG. P8.33

$A=A_1+A_2+A_3$

$M=M_1+M_2+M_3$

$\frac{M_1}{A_1}=\frac{M}{A}$

$A_1=300\ \text{cm}^2,\ A_2=100\ \text{cm}^2,\ A_3=200\ \text{cm}^2,\ A=600\ \text{cm}^2$

$$M_1=M\left(\frac{A_1}{A}\right)=\frac{300\ \text{cm}^2}{600\ \text{cm}^2}M=\frac{M}{2}$$

$$M_2=M\left(\frac{A_2}{A}\right)=\frac{100\ \text{cm}^2}{600\ \text{cm}^2}M=\frac{M}{6}$$

$$M_3=M\left(\frac{A_3}{A}\right)=\frac{200\ \text{cm}^2}{600\ \text{cm}^2}M=\frac{M}{3}$$

$$x_{CM}=\frac{x_1M_1+x_2M_2+x_3M_3}{M}=\frac{15.0\ \text{cm}\left(\frac{1}{2}M\right)+5.00\ \text{cm}\left(\frac{1}{6}M\right)+10.0\ \text{cm}\left(\frac{1}{3}M\right)}{M}$$

$$x_{CM}=\boxed{11.7\ \text{cm}}$$

$$y_{CM}=\frac{\frac{1}{2}M(5.00\ \text{cm})+\frac{1}{6}M(15.0\ \text{cm})+\left(\frac{1}{3}M\right)(25.0\ \text{cm})}{M}=13.3\ \text{cm}$$

$$y_{CM}=\boxed{13.3\ \text{cm}}$$

P8.34 Take x-axis starting from the oxygen nucleus and pointing toward the middle of the V.

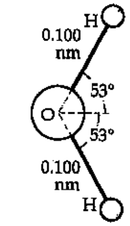

FIG. P8.34

Then $\boxed{y_{CM}=0}$

and $x_{CM}=\frac{\sum m_ix_i}{\sum m_i}=$

$$x_{CM}=\frac{0+1.008\ \text{u}(0.100\ \text{nm})\cos53.0°+1.008\ \text{u}(0.100\ \text{nm})\cos53.0°}{15.999\ \text{u}+1.008\ \text{u}+1.008\ \text{u}}$$

$$\boxed{x_{CM}=0.006\,73\ \text{nm from the oxygen nucleus}}$$

***P8.35** (a) Represent the height of a particle of mass dm within the object as y. Its contribution to the gravitational energy of the object-Earth system is $(dm)gy$. The total gravitational energy is $U_g=\int_{\text{all mass}} gy\,dm=g\int y\,dm$. For the center of mass we have $y_{CM}=\frac{1}{M}\int y\,dm$, so $U_g=gMy_{CM}$.

(b) The volume of the ramp is $\frac{1}{2}(3.6\ \text{m})(15.7\ \text{m})(64.8\ \text{m})=1.83\times10^3\ \text{m}^3$. Its mass is $\rho V=\left(3\,800\ \text{kg/m}^3\right)\left(1.83\times10^3\ \text{m}^3\right)=6.96\times10^6\ \text{kg}$. Its center of mass is above its base by one-third of its height, $y_{CM}=\frac{1}{3}15.7\ \text{m}=5.23\ \text{m}$. Then

$$U_g=Mgy_{CM}=6.96\times10^6\ \text{kg}\left(9.8\ \text{m/s}^2\right)5.23\ \text{m}=\boxed{3.57\times10^8\ \text{J}}$$

P8.36 (a) $$M=\int_0^{0.300\ \text{m}}\lambda dx=\int_0^{0.300\ \text{m}}\left[50.0\ \text{g/m}+20.0x\ \text{g/m}^2\right]dx$$

$$M=\left[50.0x\ \text{g/m}+10.0x^2\ \text{g/m}^2\right]_0^{0.300\ \text{m}}=\boxed{15.9\ \text{g}}$$

(b) $$x_{CM}=\frac{\int_{\text{all mass}}x\,dm}{M}=\frac{1}{M}\int_0^{0.300\ \text{m}}\lambda x\,dx=\frac{1}{M}\int_0^{0.300\ \text{m}}\left[50.0x\ \text{g/m}+20.0x^2\ \text{g/m}^2\right]dx$$

$$x_{CM}=\frac{1}{15.9\ \text{g}}\left[25.0x^2\ \text{g/m}+\frac{20x^3\ \text{g/m}^2}{3}\right]_0^{0.300\ \text{m}}=\boxed{0.153\ \text{m}}$$

Section 8.7 Context Connection—Rocket Propulsion

P8.41 (a) Thrust $= \left|v_e \frac{dM}{dt}\right|$ Thrust $= (2.60\times10^3\ \text{m/s})(1.50\times10^4\ \text{kg/s}) = \boxed{3.90\times10^7\ \text{N}}$

(b) $\sum F_y = \text{Thrust} - Mg = Ma$: $3.90\times10^7 - (3.00\times10^6)(9.80) = (3.00\times10^6)a$

$$a = \boxed{3.20\ \text{m/s}^2}$$

P8.42 (a) The fuel burns at a rate $\frac{dM}{dt} = \frac{12.7\ \text{g}}{1.90\ \text{s}} = 6.68\times10^{-3}\ \text{kg/s}$

Thrust $= v_e \frac{dM}{dt}$ $5.26\ \text{N} = v_e(6.68\times10^{-3}\ \text{kg/s})$

$$v_e = \boxed{787\ \text{m/s}}$$

(b) $v_f - v_i = v_e \ln\left(\frac{M_i}{M_f}\right)$: $v_f - 0 = (787\ \text{m/s})\ln\left(\frac{53.5\ \text{g} + 25.5\ \text{g}}{53.5\ \text{g} + 25.5\ \text{g} - 12.7\ \text{g}}\right)$

$$v_f = \boxed{138\ \text{m/s}}$$

P8.43 $v = v_e \ln \frac{M_i}{M_f}$

(a) $M_i = e^{v/v_e} M_f$ $M_i = e^5(3.00\times10^3\ \text{kg}) = 4.45\times10^5\ \text{kg}$

The mass of fuel and oxidizer is $\Delta M = M_i - M_f = (445 - 3.00)\times10^3\ \text{kg} = \boxed{442\ \text{metric tons}}$

(b) $\Delta M = e^2(3.00\ \text{metric tons}) - 3.00\ \text{metric tons} = \boxed{19.2\ \text{metric tons}}$

Because of the exponential, a relatively small increase in fuel and/or engine efficiency causes a large change in the amount of fuel and oxidizer required.

***P8.44** (a) From Equation 8.43, $v - 0 = v_e \ln\left(\frac{M_i}{M_f}\right) = -v_e \ln\left(\frac{M_f}{M_i}\right)$

Now, $M_f = M_i - kt$, so $v = -v_e \ln\left(\frac{M_i - kt}{M_i}\right) = -v_e \ln\left(1 - \frac{k}{M_i}t\right)$

With the definition, $T_p \equiv \frac{M_i}{k}$, this becomes

$$v(t) = \boxed{-v_e \ln\left(1 - \frac{t}{T_p}\right)}$$

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(b) With $v_e = 1\,500$ m/s, and $T_p = 144$ s, $v = -(1\,500\ \text{m/s})\ln\left(1 - \frac{t}{144\ \text{s}}\right)$

$t(s)$	$v(\text{m/s})$
0	0
20	224
40	488
60	808
80	1220
100	1780
120	2690
132	3730

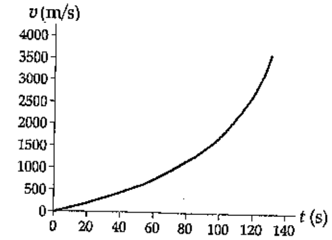

FIG. P8.44(b)

(c) $a(t) = \frac{dv}{dt} = \frac{d\left[-v_e \ln\left(1 - \frac{t}{T_p}\right)\right]}{dt} = -v_e\left(\frac{1}{1 - \frac{t}{T_p}}\right)\left(-\frac{1}{T_p}\right) = \left(\frac{v_e}{T_p}\right)\left(\frac{1}{1 - \frac{t}{T_p}}\right)$, or

$$a(t) = \boxed{\frac{v_e}{T_p - t}}$$

(d) With $v_e = 1\,500$ m/s, and $T_p = 144$ s, $a = \frac{1\,500\ \text{m/s}}{144\ \text{s} - t}$

$t(s)$	$a(\text{m/s}^2)$
0	10.4
20	12.1
40	14.4
60	17.9
80	23.4
100	34.1
120	62.5
132	125

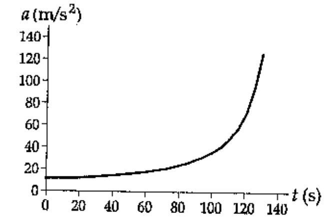

FIG. P8.44(d)

(e) $x(t) = 0 + \int_0^t v\,dt = \int_0^t \left[-v_e \ln\left(1 - \frac{t}{T_p}\right)\right]dt = v_e T_p \int_0^t \ln\left[1 - \frac{t}{T_p}\right]\left(-\frac{dt}{T_p}\right)$

$$x(t) = v_e T_p\left[\left(1 - \frac{t}{T_p}\right)\ln\left(1 - \frac{t}{T_p}\right) - \left(1 - \frac{t}{T_p}\right)\right]_0^t$$

$$x(t) = \boxed{v_e(T_p - t)\ln\left(1 - \frac{t}{T_p}\right) + v_e t}$$

continued on next page

(f) With $v_e = 1\,500$ m/s $= 1.50$ km/s, and $T_p = 144$ s,

$$x = 1.50(144 - t)\ln\left(1 - \frac{t}{144}\right) + 1.50t$$

t(s)	x(km)
0	0
20	2.19
40	9.23
60	22.1
80	42.2
100	71.7
120	115
132	153

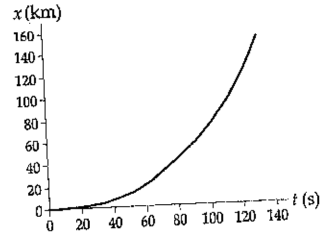

FIG. P8.44(f)

P8.45 The thrust acting on the spacecraft is

$\sum F = ma$: $\sum F = (3\,500\text{ kg})(2.50\times10^{-6})(9.80\text{ m/s}^2) = 8.58\times10^{-2}$ N

thrust $= \left(\frac{dM}{dt}\right)v_e$: $8.58\times10^{-2}\text{ N} = \left(\frac{\Delta M}{3\,600\text{ s}}\right)(70\text{ m/s})$

$$\Delta M = \boxed{4.41\text{ kg}}$$

Additional Problems

P8.46 (a) When the spring is fully compressed, each cart moves with same velocity $\vec{v}$. Apply conservation of momentum for the system of two gliders

$\vec{p}_i = \vec{p}_f$: $m_1\vec{v}_1 + m_2\vec{v}_2 = (m_1 + m_2)\vec{v}$ $\boxed{\vec{v} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2}{m_1 + m_2}}$

(b) Only conservative forces act, therefore $\Delta E = 0$. $\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 = \frac{1}{2}(m_1 + m_2)v^2 + \frac{1}{2}kx_m^2$

Substitute for v from (a) and solve for x_m.

$$x_m^2 = \frac{(m_1 + m_2)m_1v_1^2 + (m_1 + m_2)m_2v_2^2 - (m_1v_1)^2 - (m_2v_2)^2 - 2m_1m_2v_1v_2}{k(m_1 + m_2)}$$

$$x_m = \sqrt{\frac{m_1m_2\left(v_1^2 + v_2^2 - 2v_1v_2\right)}{k(m_1 + m_2)}} = \boxed{(v_1 - v_2)\sqrt{\frac{m_1m_2}{k(m_1 + m_2)}}}$$

continued on next page

(c) $m_1\vec{v}_1 + m_2\vec{v}_2 = m_1\vec{v}_{1f} + m_2\vec{v}_{2f}$

Conservation of momentum: $m_1(\vec{v}_1 - \vec{v}_{1f}) = m_2(\vec{v}_{2f} - \vec{v}_2)$ (1)

Conservation of energy: $\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 = \frac{1}{2}m_1v_{1f}^2 + \frac{1}{2}m_2v_{2f}^2$

which simplifies to: $m_1(v_1^2 - v_{1f}^2) = m_2(v_{2f}^2 - v_2^2)$

Factoring gives $m_1(\vec{v}_1 - \vec{v}_{1f})\cdot(\vec{v}_1 + \vec{v}_{1f}) = m_2(\vec{v}_{2f} - \vec{v}_2)\cdot(\vec{v}_{2f} + \vec{v}_2)$

and with the use of the momentum equation (equation (1)),

this reduces to $(\vec{v}_1 + \vec{v}_{1f}) = (\vec{v}_{2f} + \vec{v}_2)$

or $\vec{v}_{1f} = \vec{v}_{2f} + \vec{v}_2 - \vec{v}_1$ (2)

Substituting equation (2) into equation (1) and simplifying yields:

$$\vec{v}_{2f} = \boxed{\left(\frac{2m_1}{m_1 + m_2}\right)\vec{v}_1 + \left(\frac{m_2 - m_1}{m_1 + m_2}\right)\vec{v}_2}$$

Upon substitution of this expression for $\vec{v}_{2f}$ into equation 2, one finds

$$\vec{v}_{1f} = \boxed{\left(\frac{m_1 - m_2}{m_1 + m_2}\right)\vec{v}_1 + \left(\frac{2m_2}{m_1 + m_2}\right)\vec{v}_2}$$

Observe that these results are the same as Equations 8.24 and 8.25, which should have been expected since this is a perfectly elastic collision in one dimension.

P8.47 (a) $(60.0\text{ kg})4.00\text{ m/s} = (120 + 60.0)\text{ kg}v_f$

$\vec{v}_f = \boxed{1.33\text{ m/s}\,\hat{i}}$

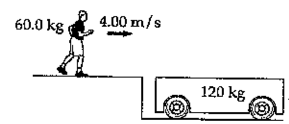

FIG. P8.47

(b) $\sum F_y = 0$: $n - (60.0\text{ kg})9.80\text{ m/s}^2 = 0$

$f_k = \mu_k n = 0.400(588\text{ N}) = 235$ N

$\vec{f}_k = \boxed{-235\text{ N}\hat{i}}$

(c) For the person, $p_i + I = p_f$

$mv_i + Ft = mv_f$

$(60.0\text{ kg})4.00\text{ m/s} - (235\text{ N})t = (60.0\text{ kg})1.33\text{ m/s}$

$t = \boxed{0.680\text{ s}}$

continued on next page

(d) person: $m\vec{v}_f - m\vec{v}_i = 60.0\text{ kg}(1.33 - 4.00)\hat{i}\text{ m/s} = \boxed{-160\text{ N}\cdot\text{s}\hat{i}}$

cart: $120\text{ kg}(1.33\text{ m/s}) - 0 = \boxed{+160\text{ N}\cdot\text{s}\hat{i}}$

(e) $x_f - x_i = \frac{1}{2}(v_i + v_f)t = \frac{1}{2}[(4.00 + 1.33)\text{ m/s}]0.680\text{ s} = \boxed{1.81\text{ m}}$

(f) $x_f - x_i = \frac{1}{2}(v_i + v_f)t = \frac{1}{2}(0 + 1.33\text{ m/s})0.680\text{ s} = \boxed{0.454\text{ m}}$

(g) $\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = \frac{1}{2}60.0\text{ kg}(1.33\text{ m/s})^2 - \frac{1}{2}60.0\text{ kg}(4.00\text{ m/s})^2 = \boxed{-427\text{ J}}$

(h) $\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = \frac{1}{2}120.0\text{ kg}(1.33\text{ m/s})^2 - 0 = \boxed{107\text{ J}}$

(i) The force exerted by the person on the cart must be equal in magnitude and opposite in direction to the force exerted by the cart on the person. The changes in momentum of the two objects must be equal in magnitude and must add to zero. Their changes in kinetic energy are different in magnitude and do not add to zero. The following represent two ways of thinking about 'why.' The distance the cart moves is different from the distance moved by the point of application of the friction force to the cart. The total change in mechanical energy for both objects together, −320 J, becomes +320 J of additional internal energy in this perfectly inelastic collision.

*P8.48 Using conservation of momentum from just before to just after the impact of the bullet with the block:

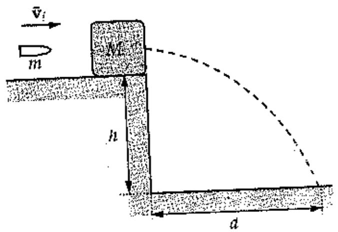

FIG. P8.48

$$mv_i = (M+m)v_f$$

or $v_i = \left(\frac{M+m}{m}\right)v_f$. (1)

The speed of the block and embedded bullet just after impact may be found using kinematic equations:

$$d = v_f t \text{ and } h = \frac{1}{2}gt^2.$$

Thus, $t = \sqrt{\frac{2h}{g}}$ and $v_f = \frac{d}{t} = d\sqrt{\frac{g}{2h}} = \sqrt{\frac{gd^2}{2h}}$.

Substituting into (1) from above gives $v_i = \boxed{\left(\frac{M+m}{m}\right)\sqrt{\frac{gd^2}{2h}}}$.

*P8.49 (a) The volume of sea water expelled in one second is $\pi r^2\ell = \pi(0.015\text{ m})^2(16\text{ m}) = 0.011\,3\text{ m}^3$. To say the same thing, the volume flow rate of the jet is $0.011\,3\text{ m}^3/\text{s}$. Its mass flow rate is $\frac{\Delta m}{\Delta t} = \rho\frac{\Delta V}{\Delta t} = 1\,030\text{ kg/m}^3(0.011\,3\text{ m}^3/\text{s}) = 11.6\text{ kg/s}$. The force the squid exerts on the water in the jet is $\frac{\Delta p}{\Delta t} = \frac{\Delta m}{\Delta t}v = 11.6\text{ kg/s}(16\text{ m/s}) = 186\text{ N}$. The water in the jet exerts a force of the same size on the squid, to give it initial acceleration $a = \frac{\sum F}{m} = \frac{186\text{ N}}{90\text{ kg}} = \boxed{2.07\text{ m/s}^2}$.

(b) For the squid in equilibrium at terminal speed.

$$186\text{ N} = R = \frac{1}{2}D\rho Av_T^2$$

$$v_T = \left(\frac{2R}{D\rho A}\right)^{1/2} = \left(\frac{2\cdot 186\text{ N}}{0.3(1\,030\text{ kg/m}^3)(800\times10^{-4}\text{ m}^2)}\right)^{1/2} = \boxed{3.88\text{ m/s}}$$

*P8.50 (a) The mass of the sleigh plus you is 270 kg. Your velocity is 7.50 m/s in the x direction. You unbolt a 15.0-kg seat and throw it back at the ravening wolves, giving it a speed of 8.00 m/s relative to you. Find the velocity of the sleigh after your action, and the velocity of the seat relative to the ground.

(b) We substitute $v_{1f} = 8\text{ m/s} - v_{2f}$

$$270\text{ kg}(7.5\text{ m/s}) = 15\text{ kg}(-8\text{ m/s} + v_{2f}) + (255\text{ kg})v_{2f}$$
$$2\,025\text{ kg}\cdot\text{m/s} = -120\text{ kg}\cdot\text{m/s} + (270\text{ kg})v_{2f}$$
$$v_{2f} = \frac{2\,145\text{ m/s}}{270} = \boxed{7.94\text{ m/s} = v_{2f}}$$
$$v_{1f} = 8\text{ m/s} - 7.94\text{ m/s} = \boxed{0.055\,6\text{ m/s} = v_{1f}}$$

The final velocity of the sleigh is $7.94\text{ m/s}\hat{i}$. That of the seat is $-0.055\,6\text{ m/s}\hat{i}$.

(c) You do work on both the sleigh and the seat, to change their kinetic energy according to

$$K_i + W = K_{1f} + K_{2f}$$
$$\frac{1}{2}(270\text{ kg})(7.5\text{ m/s})^2 + W = \frac{1}{2}(15\text{ kg})(0.055\,6\text{ m/s})^2 + \frac{1}{2}(255\text{ kg})(7.94\text{ m/s})^2$$
$$7\,594\text{ J} + W = 0.023\,1\text{ J} + 8\,047\text{ J}$$
$$W = \boxed{453\text{ J}}$$

P8.51 (a) The initial momentum of the system is zero, which remains constant throughout the motion. Therefore, when m_1 leaves the wedge, we must have

$$m_2 v_{\text{wedge}} + m_1 v_{\text{block}} = 0$$

or $$(3.00 \text{ kg})v_{\text{wedge}} + (0.500 \text{ kg})(+4.00 \text{ m/s}) = 0$$

so $$v_{\text{wedge}} = \boxed{-0.667 \text{ m/s}}$$

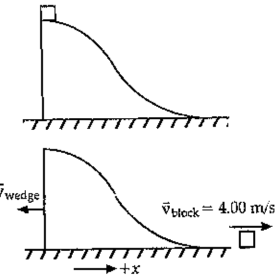

FIG. P8.51

(b) Using conservation of energy for the block-wedge-Earth system as the block slides down the smooth (frictionless) wedge, we have

$$\left[K_{\text{block}} + U_{\text{system}}\right]_i + \left[K_{\text{wedge}}\right]_i = \left[K_{\text{block}} + U_{\text{system}}\right]_f + \left[K_{\text{wedge}}\right]_f$$

or $$\left[0 + m_1 gh\right] + 0 = \left[\frac{1}{2}m_1(4.00)^2 + 0\right] + \frac{1}{2}m_2(-0.667)^2 \text{ which gives } \boxed{h = 0.952 \text{ m}}.$$

P8.52 $T = \frac{dm}{dt}(\Delta v_{\text{fuel}}) + \frac{dm}{dt}(\Delta v_{\text{air}})$: $\quad T = (3.00 \text{ kg/s})(600 \text{ m/s}) + (80.0 \text{ kg/s})(600 \text{ m/s} - 223 \text{ m/s})$

$$T = 1\,800 \text{ N} + 3.02 \times 10^4 \text{ N} = \boxed{3.20 \times 10^4 \text{ N}}$$

The delivered power is the force (or thrust) multiplied by the velocity,

$$\mathcal{P} = Tv = (3.20 \times 10^4 \text{ N})(223 \text{ m/s}) = \boxed{7.13 \times 10^6 \text{ W}}$$

***P8.53** It is interesting to compare this problem with Problem P6.50. The force exerted by the spring on each block is in magnitude $|F_s| = kx = (3.85 \text{ N/m})(0.08 \text{ m}) = 0.308 \text{ N}$.

(a) With no friction, the elastic energy in the spring becomes kinetic energy of the blocks, which have momenta of equal magnitude in opposite directions. The blocks move with constant speed after they leave the spring.

$$(K+U)_i = (K+U)_f$$

$$\frac{1}{2}kx^2 = \frac{1}{2}m_1 v_{1f}^2 + \frac{1}{2}m_2 v_{2f}^2$$

$$\frac{1}{2}(3.85 \text{ N/m})(0.08 \text{ m})^2 = \frac{1}{2}(0.25 \text{ kg})v_{1f}^2 + \frac{1}{2}(0.50 \text{ kg})v_{2f}^2$$

$$m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = m_1\vec{v}_{1f} + m_2\vec{v}_{2f}$$

$$0 = (0.25 \text{ kg})v_{1f}(-\hat{i}) + (0.50 \text{ kg})v_{2f}\hat{i}$$

$$v_{1f} = 2v_{2f}$$

$$0.012\,3 \text{ J} = \frac{1}{2}(0.25 \text{ kg})(2v_{2f})^2 + \frac{1}{2}(0.50 \text{ kg})v_{2f}^2 = \frac{1}{2}(1.5 \text{ kg})v_{2f}^2$$

$$v_{2f} = \left(\frac{0.123 \text{ J}}{0.75 \text{ kg}}\right)^{1/2} = 0.128 \text{ m/s} \quad \boxed{\vec{v}_{2f} = 0.128 \text{ m/s}\,\hat{i}}$$

$$v_{1f} = 2(0.128 \text{ m/s}) = 0.256 \text{ m/s} \quad \boxed{\vec{v}_{1f} = 0.256 \text{ m/s}(-\hat{i})}$$

continued on next page

(b) For the lighter block, $\sum F_y = ma_y$, $\quad n - 0.25 \text{ kg}(9.8 \text{ m/s}^2) = 0$, $\quad n = 2.45 \text{ N}$,
$f_k = \mu_k n = 0.1(2.45 \text{ N}) = 0.245 \text{ N}$. We assume that the maximum force of static friction is a similar size. Since 0.308 N is larger than 0.245 N, this block moves. For the heavier block, the normal force and the frictional force are twice as large: $f_k = 0.490 \text{ N}$. Since 0.308 N is less than this, the heavier block stands still. In this case, the frictional forces exerted by the floor change the momentum of the two-block system. The lighter block will gain speed as long as the spring force is larger than the friction force: that is until the spring compresses becomes x_f given by $|F_s| = kx$, $0.245 \text{ N} = (3.85 \text{ N/m})x_f$, $0.063\,6 \text{ m} = x_f$. Now for the energy of the lighter block as it moves to this maximum-speed point we have

$$K_i + U_i - f_k d = K_f + U_f$$

$$0 + 0.012\,3 \text{ J} - 0.245 \text{ N}(0.08 - 0.063\,6 \text{ m}) = \frac{1}{2}(0.25 \text{ kg})v_f^2 + \frac{1}{2}(3.85 \text{ N/m})(0.063\,6 \text{ m})^2$$

$$0.012\,3 \text{ J} - 0.004\,01 \text{ J} = \frac{1}{2}(0.25 \text{ kg})v_f^2 + 0.007\,80 \text{ J}$$

$$\left(\frac{2(0.000\,515 \text{ J})}{0.25 \text{ kg}}\right)^{1/2} = v_f = 0.064\,2 \text{ m/s}$$

Thus for the heavier block the maximum velocity is $\boxed{0}$ and for the lighter $\boxed{0.064\,2 \text{ m/s}(-\hat{i})}$.

(c) For the lighter block, $f_k = 0.462(2.45 \text{ N}) = 1.13 \text{ N}$. The force of static friction must be at least as large. The 0.308-N spring force is too small to produce motion of either block. Each has $\boxed{0}$ maximum speed.

P8.54 (a) $\vec{p}_i + \vec{F}t = \vec{p}_f$: $\quad (3.00 \text{ kg})(7.00 \text{ m/s})\hat{j} + (12.0 \text{ N}\hat{i})(5.00 \text{ s}) = (3.00 \text{ kg})\vec{v}_f$

$$\vec{v}_f = \boxed{(20.0\hat{i} + 7.00\hat{j}) \text{ m/s}}$$

(b) $\vec{a} = \frac{\vec{v}_f - \vec{v}_i}{t}$ $\quad \vec{a} = \frac{(20.0\hat{i} + 7.00\hat{j} - 7.00\hat{j}) \text{ m/s}}{5.00 \text{ s}} = \boxed{4.00\hat{i} \text{ m/s}^2}$

(c) $\vec{a} = \frac{\sum \vec{F}}{m}$: $\quad \vec{a} = \frac{12.0 \text{ N}\hat{i}}{3.00 \text{ kg}} = \boxed{4.00\hat{i} \text{ m/s}^2}$

(d) $\Delta\vec{r} = \vec{v}_i t + \frac{1}{2}\vec{a}t^2$: $\quad \Delta\vec{r} = (7.00 \text{ m/s}\,\hat{j})(5.00 \text{ s}) + \frac{1}{2}(4.00 \text{ m/s}^2\,\hat{i})(5.00 \text{ s})^2$

$$\Delta\vec{r} = \boxed{(50.0\hat{i} + 35.0\hat{j}) \text{ m}}$$

(e) $W = \vec{F}\cdot\Delta\vec{r}$: $\quad W = (12.0 \text{ N}\hat{i})\cdot(50.0 \text{ m}\hat{i} + 35.0 \text{ m}\hat{j}) = \boxed{600 \text{ J}}$

continued on next page

(f) $\frac{1}{2}mv_f^2 = \frac{1}{2}(3.00\text{ kg})(20.0\hat{i}+7.00\hat{j})\cdot(20.0\hat{i}+7.00\hat{j})\text{ m}^2/\text{s}^2$

$$\frac{1}{2}mv_f^2 = (1.50\text{ kg})(449\text{ m}^2/\text{s}^2) = \boxed{674\text{ J}}$$

(g) $\frac{1}{2}mv_i^2 + W = \frac{1}{2}(3.00\text{ kg})(7.00\text{ m/s})^2 + 600\text{ J} = \boxed{674\text{ J}}$

P8.55 Momentum conservation for the system of the two objects: $3mv_i - mv_i = mv_{1f} + 3mv_{2f}$. The relative velocity equation:

$v_{1i} - v_{2i} = -v_{1f} + v_{2f}$ $\quad \begin{cases} -v_i - v_i = -v_{1f} + v_{2f} \\ 2v_i = v_{1f} + 3v_{2f} \end{cases}$

$0 = 4v_{2f}$

$v_{1f} = \boxed{2v_i}$ $\qquad v_{2f} = \boxed{0}$

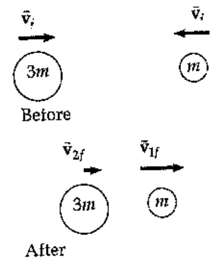

FIG. P8.55

P8.56 x-component of momentum for the system of the two objects:

$p_{1ix} + p_{2ix} = p_{1fx} + p_{2fx}$: $\quad -mv_i + 3mv_i = 0 + 3mv_{2x}$

y-component of momentum of the system: $\quad 0 + 0 = -mv_{1y} + 3mv_{2y}$

by conservation of energy of the system: $\quad +\frac{1}{2}mv_i^2 + \frac{1}{2}3mv_i^2 = \frac{1}{2}mv_{1y}^2 + \frac{1}{2}3m(v_{2x}^2 + v_{2y}^2)$

we have $\quad v_{2x} = \frac{2v_i}{3}$

also $\quad v_{1y} = 3v_{2y}$

So the energy equation becomes $\quad 4v_i^2 = 9v_{2y}^2 + \frac{4v_i^2}{3} + 3v_{2y}^2$

$$\frac{8v_i^2}{3} = 12v_{2y}^2$$

or $\quad v_{2y} = \frac{\sqrt{2}v_i}{3}$

(a) The object of mass m has final speed $\quad v_{1y} = 3v_{2y} = \boxed{\sqrt{2}v_i}$

and the object of mass 3 m moves at $\quad \sqrt{v_{2x}^2 + v_{2y}^2} = \sqrt{\frac{4v_i^2}{9} + \frac{2v_i^2}{9}}$

$$\sqrt{v_{2x}^2 + v_{2y}^2} = \boxed{\sqrt{\frac{2}{3}}v_i}$$

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(b) $\theta = \tan^{-1}\left(\frac{v_{2y}}{v_{2x}}\right)$ $\qquad \theta = \tan^{-1}\left(\frac{\sqrt{2}v_i}{3}\frac{3}{2v_i}\right) = \boxed{35.3°}$

***P8.57** (a) Each object swings down according to

$mgR = \frac{1}{2}mv_1^2$ $\qquad MgR = \frac{1}{2}Mv_1^2$ $\qquad v_1 = \sqrt{2gR}$

The collision: $-mv_1 + Mv_1 = +(m+M)v_2$

$$v_2 = \frac{M-m}{M+m}v_1$$

Swinging up: $\frac{1}{2}(M+m)v_2^2 = (M+m)gR(1-\cos 35°)$

$$v_2 = \sqrt{2gR(1-\cos 35°)}$$

$$\sqrt{2gR(1-\cos 35°)}(M+m) = (M-m)\sqrt{2gR}$$

$$0.425M + 0.425m = M - m$$

$$1.425m = 0.575M$$

$$\boxed{\frac{m}{M} = 0.403}$$

(b) No change is required if the force is different. The nature of the forces within the system of colliding objects does not affect the total momentum of the system. With strong magnetic attraction, the heavier object will be moving somewhat faster and the lighter object faster still. Their extra kinetic energy will all be immediately converted into extra internal energy when the objects latch together. Momentum conservation guarantees that none of the extra kinetic energy remains after the objects join to make them swing higher.

P8.58 (a) Use conservation of the horizontal component of momentum for the system of the shell, the cannon, and the carriage, from just before to just after the cannon firing.

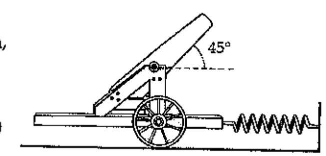

FIG. P8.58

$p_{xf} = p_{xi}$: $\quad m_{\text{shell}}v_{\text{shell}}\cos 45.0° + m_{\text{cannon}}v_{\text{recoil}} = 0$

$$(200)(125)\cos 45.0° + (5\,000)v_{\text{recoil}} = 0$$

or $\quad v_{\text{recoil}} = \boxed{-3.54\text{ m/s}}$

(b) Use conservation of energy for the system of the cannon, the carriage, and the spring from right after the cannon is fired to the instant when the cannon comes to rest.

$K_f + U_{gf} + U_{sf} = K_i + U_{gi} + U_{si}$: $\quad 0 + 0 + \frac{1}{2}kx_{\max}^2 = \frac{1}{2}mv_{\text{recoil}}^2 + 0 + 0$

$$x_{\max} = \sqrt{\frac{mv_{\text{recoil}}^2}{k}} = \sqrt{\frac{(5\,000)(-3.54)^2}{2.00\times 10^4}}\text{ m} = \boxed{1.77\text{ m}}$$

continued on next page

(c) $\left|F_{s,\,\max}\right| = kx_{\max}$ $\qquad \left|F_{s,\,\max}\right| = (2.00\times10^4\ \text{N/m})(1.77\ \text{m}) = \boxed{3.54\times10^4\ \text{N}}$

(d) No. The rail exerts a vertical external force (the normal force) on the cannon and prevents it from recoiling vertically. Momentum is not conserved in the vertical direction. The spring does not have time to stretch during the cannon firing. Thus, no external horizontal force is exerted on the system (cannon, carriage, and shell) from just before to just after firing. Momentum of this system is conserved in the horizontal direction during this interval.

P8.59 A picture one second later differs by showing five extra kilograms of sand moving on the belt.

(a) $\dfrac{\Delta p_x}{\Delta t} = \dfrac{(5.00\ \text{kg})(0.750\ \text{m/s})}{1.00\ \text{s}} = \boxed{3.75\ \text{N}}$

(b) The only horizontal force on the sand is belt friction,

so from $\quad p_{xi} + f\Delta t = p_{xf} \quad$ this is $\quad f = \dfrac{\Delta p_x}{\Delta t} = \boxed{3.75\ \text{N}}$

(c) The belt is in equilibrium:

$\sum F_x = ma_x:\quad +F_{\text{ext}} - f = 0 \qquad$ and $\qquad F_{\text{ext}} = \boxed{3.75\ \text{N}}$

(d) $W = F\Delta r\cos\theta = 3.75\ \text{N}(0.750\ \text{m})\cos 0° = \boxed{2.81\ \text{J}}$

(e) $\dfrac{1}{2}(\Delta m)v^2 = \dfrac{1}{2}5.00\ \text{kg}(0.750\ \text{m/s})^2 = \boxed{1.41\ \text{J}}$

(f) $\boxed{\text{Friction between sand and belt converts half of the input work into extra internal energy.}}$

P8.60 The force exerted by the table is equal to the change in momentum of each of the links in the chain.

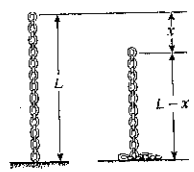

FIG. P8.60

By the calculus chain rule of derivatives,

$$F_1 = \frac{dp}{dt} = \frac{d(mv)}{dt} = v\frac{dm}{dt} + m\frac{dv}{dt}$$

We choose to account for the change in momentum of each link by having it pass from our area of interest just before it hits the table, so that

$$v\frac{dm}{dt} \neq 0 \text{ and } m\frac{dv}{dt} = 0.$$

Since the mass per unit length is uniform, we can express each link of length dx as having a mass dm:

$$dm = \frac{M}{L}dx.$$

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The magnitude of the force on the falling chain is the force that will be necessary to stop each of the elements dm.

$$F_1 = v\frac{dm}{dt} = v\left(\frac{M}{L}\right)\frac{dx}{dt} = \left(\frac{M}{L}\right)v^2$$

After falling a distance x, the square of the velocity of each link $v^2 = 2gx$ (from kinematics), hence

$$F_1 = \frac{2Mgx}{L}$$

The links already on the table have a total length x, and their weight is supported by a force F_2:

$$F_2 = \frac{Mgx}{L}$$

Hence, the *total* force on the chain is

$$F_{\text{total}} = F_1 + F_2 = \boxed{\frac{3Mgx}{L}}.$$

That is, *the total force is three times the weight of the chain on the table at that instant.*

ANSWERS TO EVEN PROBLEMS

P8.2 $\sim 10^{23}$ m/s

P8.4 (a) $\dfrac{p^2}{2m}$; (b) $\sqrt{2mK}$

P8.6 The force he would have to exert is 6.44×10^3 N

P8.8 (a) 5.40 N·s in the direction of $\vec{v}_f$; (b) −27.0 J

P8.10 (a) $(9.05\hat{i} + 6.12\hat{j})$ N·s; (b) $(377\hat{i} + 255\hat{j})$ N

P8.12 (a) see the solution; (b) small; (c) see the solution; (d) large; (e) no difference

P8.14 (a) 2.50 m/s; (b) 37.5 kJ; (c) see the solution

P8.16 0.556 m

P8.18 $\dfrac{4M}{m}\sqrt{g\ell}$

P8.20 7.94 cm

P8.22 (a) 2.24 m/s toward the right; (b) No, coupling order makes no difference

P8.24 (a) 2.88 m/s at 32.3° north of east; (b) 783 J into internal energy

P8.26 No, his speed was 41.5 mi/h

P8.28 (a) $\dfrac{v_i}{\sqrt{2}}$; (b) 45.0° and −45.0°

P8.30 (a) 1.07 m/s at −29.7°; (b) $\dfrac{\Delta K}{K_i} = -0.318$

P8.32 $x_{CM} = 0$, $y_{CM} = 1.00$ m

P8.34 $y_{CM} = 0$, $x_{CM} = 0.006\,73$ nm from the oxygen nucleus

P8.36 (a) 15.9 g; (b) 0.153 m