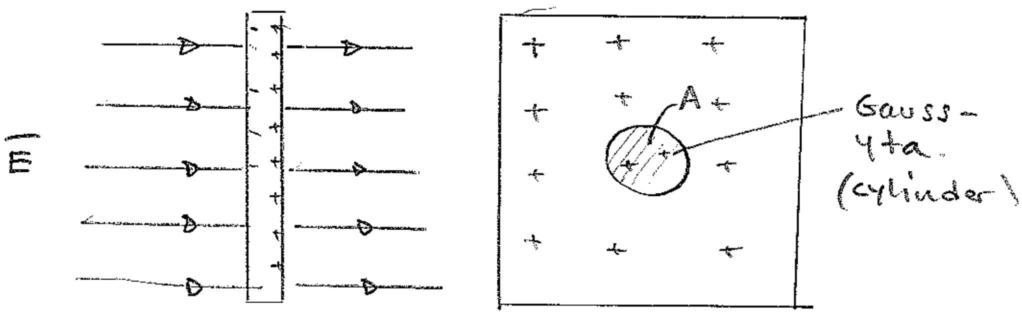
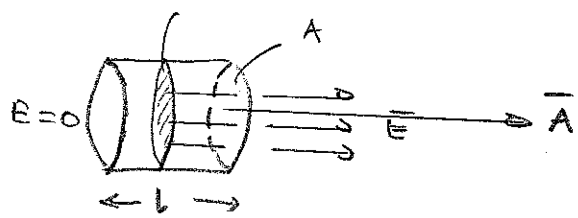
19. 44 A square plate of copper with 50.0-cm sides has no net charge and is placed in a region of uniform electric field of 80.0 kN/C directed perpendicularly to the plate. Find (a) the charge density of each face of the plate and (b) the total charge on each face.

Lösning:



här sitter de positiva laddningarna (högra ytan)



Gauss sats: $\phi = \oint \bar{E} \cdot d\bar{A} = \dfrac{q_{in}}{\epsilon_0}$

här: det enda stället där $\bar{E} \cdot d\bar{A} \neq 0$ är vid cylinderns högra "lock".
Eftersom koppar är en metall kan det inte finnas något el-fält inne i plattan

a)

$$\left.\begin{aligned} \therefore\ \oint \bar{E} \cdot d\bar{A} &= EA = \frac{q_{in}}{\epsilon_0} \\ q_{in} &= \sigma A \end{aligned}\right\} \Rightarrow EA = \frac{\sigma A}{\epsilon_0} \Rightarrow \boxed{\sigma = E \cdot \epsilon_0}$$

med siffror:

$$\sigma = 80{,}0 \cdot 10^3\ N/C \cdot 8{,}85 \cdot 10^{-12}\ C^2/Nm^2 =$$

laddn. per sida

$$= \underline{7{,}08 \cdot 10^{-7}\ C/m^2}$$

b) $Q = A_{tot} \cdot \sigma = 0{,}500^2 \cdot 7{,}08 \cdot 10^{-7}\ C = \underline{1{,}77 \cdot 10^{-7}\ C}$

Physics Now™ A solid, insulating sphere of radius a has a uniform charge density ρ and a total charge Q. Concentric with this sphere is an uncharged, conducting hollow sphere whose inner and outer radii are b and c as shown in Figure P19.65. (a) Find the magnitude of the electric field in the regions $r < a$, $a < r < b$, $b < r < c$, and $r > c$. (b) Determine the induced charge per unit area on the inner and outer surfaces of the hollow sphere.

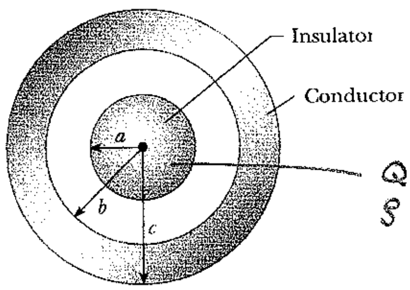


FIGURE P19.65

Lösning:

$$\phi = \oint \bar{E} \cdot d\bar{A} = \frac{q_{in}}{\epsilon_0}$$

Isolerande sfären i mitten: $\rho = \dfrac{Q}{\frac{4}{3}\pi a^3}$

Gauss sats tillämplig pga hög symmetri.

a) E:

$r < a$ $\quad E \cdot 4\pi r^2 = \dfrac{\frac{4}{3}\pi r^3 \frac{Q}{\frac{4}{3}\pi a^3}}{\epsilon_0} \Rightarrow E = \dfrac{Q}{4\pi\epsilon_0 a^3} r$

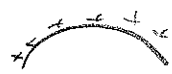
$a < r < b$ $\quad E \cdot 4\pi r^2 = \dfrac{Q}{\epsilon_0} \Rightarrow E = \dfrac{Q}{4\pi\epsilon_0} \dfrac{1}{r^2}$

$b < r < c$ (ledare) $\quad E = 0$

$r > c$ $\quad E \cdot 4\pi r^2 = \dfrac{Q}{\epsilon_0} \Rightarrow E = \dfrac{Q}{4\pi\epsilon_0} \dfrac{1}{r^2}$

---

b) Inducerad laddningstäthet σ på den ledande ihåliga sfären:

Ytterradien  $\quad E \cdot 4\pi c^2 = \dfrac{\sigma \cdot 4\pi c^2}{\epsilon_0}$

men $E = \dfrac{Q}{4\pi\epsilon_0 c^2}$ $\Bigg\} \Rightarrow$

$$\Rightarrow \frac{Q}{4\pi \cancel{\epsilon_0} c^2} \cdot \cancel{4\pi c^2} = \frac{\sigma \cdot \cancel{4\pi c^2}}{\cancel{\epsilon_0}} \Rightarrow \sigma_c = \frac{Q}{4\pi c^2}$$

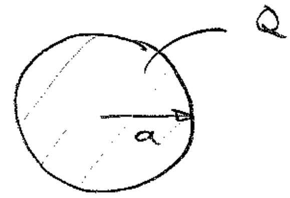
Innerradien (negativ laddning)

pss. $\sigma_b = -\dfrac{Q}{4\pi b^2}$  fältet går in i Gaussytan: −

19.

40 An insulating solid sphere of radius a has a uniform vol-ume charge density and carries a total positive charge Q. A spherical gaussian surface of radius r, which shares a common center with the insulating sphere, is inflated starting from $r = 0$. (a) Find an expression for the electric flux passing through the surface of the gaussian sphere as a function of r for $r < a$. (b) Find an expression for the electric flux for $r > a$. (c) Plot the flux versus r.

Lösning:



$$\rho = \frac{Q}{\frac{4}{3}\pi a^3}$$

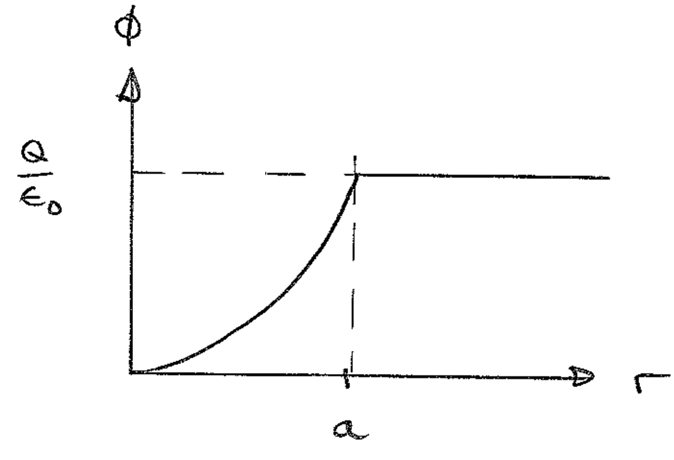
$r < a$:

$$q_{in} = \rho \cdot \frac{4}{3}\pi r^3$$

$$\phi = \frac{q_{in}}{\epsilon_0} = \frac{Q}{\epsilon_0} \frac{\frac{4}{3}\pi r^3}{\frac{4}{3}\pi a^3} = \frac{Q}{\epsilon_0 a^3} r^3$$

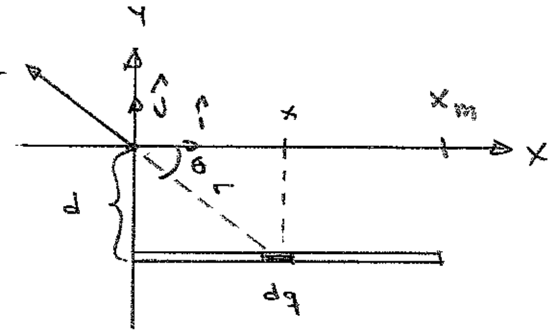
$r \geq a$

$$\phi = \frac{q_{in}}{\epsilon_0} = \frac{Q}{\epsilon_0}$$



4. A line of charge with uniform density 35.0 nC/m lies along the line $y = -15.0$ cm, between the points with coordinates $x = 0$ and $x = 40.0$ cm. Find the electric field it creates at the origin.

Lösning:



$$d\bar{E} = \frac{k_e \cdot dq}{r^2}\left(-\cos\theta\,\hat{i} + \sin\theta\,\hat{j}\right)$$

$$dq = \lambda \cdot dx \qquad \lambda = 35{,}0 \cdot 10^{-9}\ C/m$$

$$\cos\theta = \frac{x}{r}, \quad \sin\theta = \frac{d}{r}$$

$$r^2 = d^2 + x^2$$

$$\therefore\ d\bar{E} = \frac{k_e \cdot \lambda \cdot dx}{(d^2+x^2)}\left[\frac{-x\,\hat{i}}{(d^2+x^2)^{1/2}} + \frac{d\,\hat{j}}{(d^2+x^2)^{1/2}}\right]$$

$$\bar{E} = \int_0^{x_m} d\bar{E} = k_e\lambda\left[\int_0^{x_m}\frac{-x \cdot dx}{(d^2+x^2)^{3/2}}\,\hat{i} + \int_0^{x_m}\frac{d \cdot dx}{(d^2+x^2)^{3/2}}\,\hat{j}\right]$$

$\dfrac{1}{(d^2+x^2)^{3/2}}$ primitiv funktion: $\dfrac{x}{d^2(d^2+x^2)^{1/2}}$

$\dfrac{-x}{(d^2+x^2)^{3/2}}$ — " — : $\dfrac{1}{(d^2+x^2)^{1/2}}$

$$\Rightarrow \bar{E} = k_e\lambda\left\{\left[\frac{\hat{i}}{(x^2+d^2)^{1/2}}\right]_0^{x_m} + \left[\frac{d \cdot x\ \hat{j}}{(x^2+d^2)^{1/2}}\right]_0^{x_m}\right\}$$

$x_m = 0{,}40$ m $\quad d = 0{,}15$ m

$$\Rightarrow \bar{E} = 8{,}99 \cdot 10^9 \cdot 35 \cdot 10^{-9}\left[(2{,}34 - 6{,}67)\,\hat{i} + 6{,}24\,\hat{j}\right] =$$

$$= \left[-1{,}36\,\hat{i} + 1{,}96\,\hat{j}\right]\ kN/C$$

20.8

8. Given two 2.00-μC charges as shown in Figure P20.8 and a positive test charge $q = 1.28 \times 10^{-18}$ C at the origin, (a) what is the net force exerted by the two 2.00-μC charges on the test charge q? (b) What is the electric field at the origin due to the two 2.00-μC charges? (c) What is the electrical potential at the origin due to the two 2.00-μC charges?

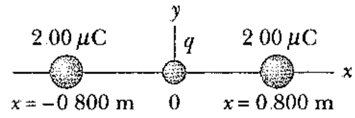


FIGURE P20.8

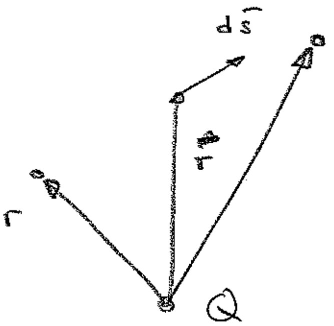
a) Nettokraft på q = 0

b) $E(0) = 0$

c) Elektriska potentialen i origo:

$$\Delta U = U_B - U_A = -q\int_A^B \bar{E}\cdot d\bar{s}$$

$$\bar{E}\cdot d\bar{s} = k_e \frac{q}{r^2}\,\hat{r}\cdot d\bar{s}$$



$$\hat{r}\cdot d\bar{s} = dr$$

$$\therefore\ \bar{E}\cdot d\bar{s} = k_e \frac{Q}{r^2}\cdot dr$$

$$\Rightarrow -\int_A^B \bar{E}\cdot d\bar{s} \equiv \Delta V = -\int_{r_A}^{r_B} k_e \frac{Q}{r^2}\cdot dr =$$

$$= k_e Q\left[\frac{1}{r_B} - \frac{1}{r_A}\right]$$

Sätt $V_\infty = 0 = V_A$

$$\Rightarrow V_B = k_e Q\,\frac{1}{r}$$

Här $V(0) =$

$$= 2 k_e Q\,\frac{1}{x} =$$

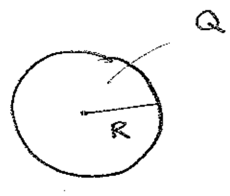
$$= \frac{2\cdot 8{,}99\cdot 10^{9}\cdot 2{,}00\cdot 10^{-6}}{0{,}800}$$

$= 45{,}0$ kV

9D

22. The electric potential inside a charged spherical conductor of radius R is given by $V = k_e Q/R$ and outside the potential is given by $V = k_e Q/r$. Using $E_r = -dV/dr$, derive the electric field (a) inside and (b) outside this charge distribution.

Lösning:



Inne i sfären: $V_{in} = k_e \frac{Q}{R}$ = konstant

Utanför sfären: $V_{out} = k_e \frac{Q}{r}$

samband mellan V och E:

$$dV = -\bar{E} \cdot \bar{dr}$$

endimensionellt

$$E = -\frac{dV}{dr}$$

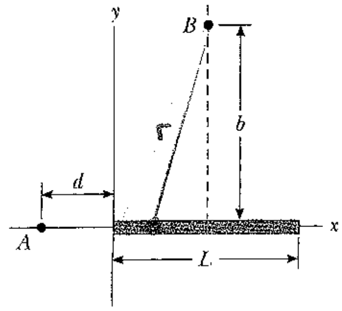
$$\therefore \quad E_{in} = -\frac{dV_{in}}{dr} = 0$$

$$E_{out} = -\frac{d}{dr}\left[k_e \frac{Q}{r}\right] = -\left[k_e \frac{-Q}{r^2}\right] =$$

$$= \underline{\underline{k_e \frac{Q}{r^2}}}$$

20.

26. For the arrangement described in Problem 20.25, calculate the electric potential at point B that lies on the perpendicular bisector of the rod a distance b above the x axis.



Lösning:

$\lambda = \alpha x \qquad \alpha > 0$

$$V = k_e \int \frac{dq}{r} \qquad r^2 = b^2 + \left(\frac{L}{2} - x\right)^2$$

$$dq = dx \cdot \lambda = \alpha x \cdot dx$$

$$\Rightarrow V = k_e \int_0^L \frac{\alpha \cdot x \cdot dx}{\left[b^2 + \left(\frac{L}{2} - x\right)^2\right]^{1/2}}$$

$x = 0 \Rightarrow z = \frac{L}{2}$

$x = L \Rightarrow z = -\frac{L}{2}$

Inför $z = \frac{L}{2} - x \Rightarrow dx = -dz$ och $x = \frac{L}{2} - z$

$$\Rightarrow V = k_e \alpha \int \frac{\left(\frac{L}{2} - z\right)(-dz)}{(b^2 + z^2)^{1/2}} = -\frac{k_e \alpha L}{2} \int \frac{dz}{(b^2 + z^2)^{1/2}} +$$

$$+ k_e \alpha \int \frac{z \cdot dz}{(b^2 + z^2)^{1/2}}$$

$\frac{1}{(b^2 + z^2)^{1/2}}$ primitiv funktion $\ln\left[z + \sqrt{z^2 + b^2}\right]$

$\frac{z}{(b^2 + z^2)^{1/2}}$ — " — $(b^2 + z^2)^{1/2}$

$$\Rightarrow V = -\frac{k_e \alpha L}{2}\left[\ln\left(z + \sqrt{z^2 + b^2}\right)\right]_{\frac{L}{2}}^{-\frac{L}{2}} + k_e \alpha \sqrt{b^2 + z^2}$$

20.26 (Forts)

$$V = -\frac{L k_e \alpha}{2} \cdot \ln\left[\frac{-\frac{L}{2}+\sqrt{\frac{L^2}{4}+b^2}}{\frac{L}{2}+\sqrt{\frac{L^2}{4}+b^2}}\right] +$$

$$+ k_e \alpha \left[\left[b^2+\left(\frac{L}{2}\right)^2\right] - \left[b^2+\left(\frac{L}{2}\right)^2\right]\right] =$$

$$= \underbrace{-\frac{k_e \alpha L}{2} \underbrace{\ln\left[\frac{\sqrt{b^2+\frac{L^2}{4}}-\frac{L}{2}}{\sqrt{b^2+\frac{L^2}{4}}+\frac{L}{2}}\right]}_{<0}}_{>0}$$