Chapter 16

Temperature and the Kinetic Theory of Gases

CHAPTER OUTLINE

ANSWERS TO QUESTIONS

Q16.1 The copper's temperature drops and the water temperature rises until both temperatures are the same. Then the metal and the water are in thermal equilibrium

Q16.2 The astronaut is referring to the temperature of the lunar surface, specifically a 400°F difference A thermometer would register the temperature of the thermometer liquid. Since there is no atmosphere in the moon, the thermometer will not read a realistic temperature unless it is placed into the lunar soil.

Q16.3 If the amalgam had a larger coefficient of expansion than your tooth, it would expand more than the cavity in your tooth when you take a sip of your ever-beloved coffee, resulting in a broken or cracked tooth! As you ice down your now excruciatingly painful broken tooth, the amalgam would contract more than the cavity in your tooth and fall out, leaving the nerve roots exposed Isn't it nice that your dentist knows thermodynamics?

Q16.4 The measurements made with the heated steel tape will be too short—but only by a factor of 5×10^{-5} of the measured length.

Q16.5 (a) One mole of H_2 has a mass of 2.016 0 g

(b) One mole of He has a mass of 4.002 6 g.

(c) One mole of CO has a mass of 28.010 g.

Q16.6 The ideal gas law, $PV = nRT$ predicts zero volume at absolute zero. This is incorrect because the ideal gas law cannot work all the way down to or below the temperature at which gas turns to liquid, or in the case of CO_2, a solid.

Q16.7 Call the process isobaric cooling or isobaric contraction. The rubber wall is easy to stretch The air inside is nearly at atmospheric pressure originally and stays at atmospheric pressure as the wall moves in, just maintaining equality of pressure outside and inside. The air is nearly an ideal gas to start with, but $PV = nRT$ soon fails Volume will drop by a larger factor than temperature as the water vapor liquefies and then freezes, as the carbon dioxide turns to snow, as the argon turns to slush, and as the oxygen liquefies. From the outside, you see contraction to a small fraction of the original volume.

Q16.8 Cylinder A must be at lower pressure If the gas is thin, it will be at one-third the absolute pressure of B

Q16.9 At high temperature and pressure, the steam inside exerts large forces on the pot and cover. Strong latches hold them together, but they would explode apart if you tried to open the hot cooker

Q16.10 (a) The water level in the cave rises by a smaller distance than the water outside, as the trapped air is compressed Air can escape from the cave if the rock is not completely airtight, and also by dissolving in the water.

(b) The ideal cave stays completely full of water at low tide. The water in the cave is supported by atmospheric pressure on the free water surface outside

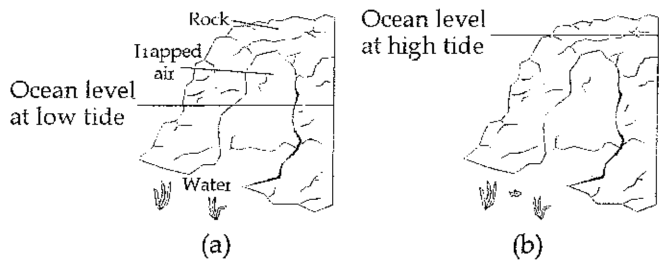

FIG. Q16.10

Q16.11 The sphere expands when heated, so that it no longer fits through the ring. With the sphere still hot, you can separate the sphere and ring by heating the ring. This more surprising result occurs because the thermal expansion of the ring is not like the inflation of a blood-pressure cuff Rather, it is like a photographic enlargement; every linear dimension, including the hole diameter, increases by the same factor. The reason for this is that the atoms everywhere, including those around the inner circumference, push away from each other The only way that the atoms can accommodate the greater distances is for the circumference—and corresponding diameter—to grow. This property was once used to fit metal rims to wooden wagon and horse-buggy wheels. If the ring is heated and the sphere left at room temperature, the sphere would pass through the ring with more space to spare

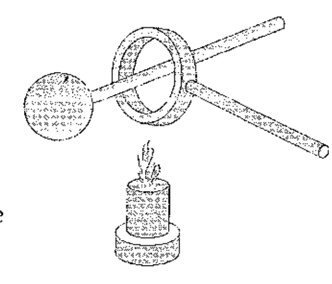

FIG. Q16.11

Q16.12 The coefficient of expansion of metal is larger than that of glass When hot water is run over the jar, both the glass and the lid expand, but at different rates. Since *all* dimensions expand, there will be a certain temperature at which the inner diameter of the lid has expanded more than the top of the jar, and the lid will be easier to remove

Q16.13 The alcohol evaporates, absorbing energy from the skin to lower the skin temperature.

Q16.14 The molecules of all different kinds collide with the walls of the container, so molecules of all different kinds exert partial pressures that contribute to the total pressure. The molecules can be so small that they collide with one another relatively rarely and each kind exerts partial pressure as if the other kinds of molecules were absent. If the molecules collide with one another often, the collisions exactly conserve momentum and so do not affect the net force on the walls.

Q16.15 The volume of the balloon will decrease The pressure inside the balloon is nearly equal to the constant exterior atmospheric pressure Then from $PV = nRT$, volume must decrease in proportion to the absolute temperature. Call the process isobaric contraction

Q16.16 The dry air is denser. Since the air and the water vapor are at the same temperature, they have the same kinetic energy per molecule For a controlled experiment, the humid and dry air are at the same pressure, so the number of molecules per unit volume must be the same for both The water molecule has a smaller molecular mass (18 0 u) than any of the gases that make up the air, so the humid air must have the smaller mass per unit volume.

Q16.17 Suppose the balloon rises into air uniform in temperature. The air cannot be uniform in pressure because the lower layers support the weight of all the air above them. The rubber in a typical balloon is easy to stretch and stretches or contracts until interior and exterior pressures are nearly equal. So as the balloon rises it expands This is an isothermal expansion, with P decreasing as V increases by the same factor in $PV = nRT$ If the rubber wall is very strong it will eventually contain the helium at higher pressure than the air outside but at the same density, so that the balloon will stop rising. More likely, the rubber will stretch and break, releasing the helium to keep rising and "boil out" of the Earth's atmosphere

Q16.18 (a) Average molecular kinetic energy increases by a factor of 3.

(b) The rms speed increases by a factor of $\sqrt{3}$.

(c) Average momentum change increases by $\sqrt{3}$

(d) Rate of collisions increases by a factor of $\sqrt{3}$ since the mean free path remains unchanged.

(e) Pressure increases by a factor of 3.

SOLUTIONS TO PROBLEMS

Section 16.1 Temperature and the Zeroth Law of Thermodynamics

No problems in this section

Section 16.2 Thermometers and Temperature Scales

P16.1 Since we have a linear graph, the pressure is related to the temperature as $P = A + BT$, where A and B are constants. To find A and B, we use the data

$$0\,900 \text{ atm} = A + (-80.0°\text{C})B \quad (1)$$

$$1\,635 \text{ atm} = A + (78.0°\text{C})B \quad (2)$$

Solving (1) and (2) simultaneously,

we find $A = 1\,272$ atm

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and $B = 4.652 \times 10^{-3}$ atm/°C

Therefore, $P = 1.272 \text{ atm} + \left(4.652 \times 10^{-3} \text{ atm/°C}\right)T$

(a) At absolute zero $P = 0 = 1.272 \text{ atm} + \left(4.652 \times 10^{-3} \text{ atm/°C}\right)T$

which gives $\boxed{T = -273°\text{C}}$.

(b) At the freezing point of water $P = 1.272 \text{ atm} + 0 = \boxed{1.27 \text{ atm}}$.

(c) And at the boiling point $P = 1.272 \text{ atm} + \left(4.652 \times 10^{-3} \text{ atm/°C}\right)(100°\text{C}) = \boxed{1.74 \text{ atm}}$

P16.2 (a) To convert from Fahrenheit to Celsius, we use $T_C = \frac{5}{9}(T_F - 32.0) = \frac{5}{9}(98.6 - 32.0) = \boxed{37.0°\text{C}}$

and the Kelvin temperature is found as $T = T_C + 273 = \boxed{310 \text{ K}}$

(b) In a fashion identical to that used in (a), we find $T_C = \boxed{-20.6°\text{C}}$

and $T = \boxed{253 \text{ K}}$

P16.3 (a) $T_F = \frac{9}{5}T_C + 32.0°\text{F} = \frac{9}{5}(-195.81) + 32.0 = \boxed{-320°\text{F}}$

(b) $T = T_C + 273.15 = -195.81 + 273.15 = \boxed{77.3 \text{ K}}$

P16.4 (a) $\Delta T = 450°\text{C} = 450°\text{C}\left(\frac{212°\text{F} - 32.0°\text{F}}{100°\text{C} - 0.00°\text{C}}\right) = \boxed{810°\text{F}}$

(b) $\Delta T = 450°\text{C} = \boxed{450 \text{ K}}$

Section 16.3 Thermal Expansion of Solids and Liquids

***P16.5** $\Delta L = \alpha L_i \Delta T = 11 \times 10^{-6} (°\text{C})^{-1}(1\,300 \text{ km})[35°\text{C} - (-73°\text{C})] = \boxed{1.54 \text{ km}}$

The expansion can be compensated for by mounting the pipeline on rollers and placing Ω-shaped loops between straight sections. They bend as the steel changes length.

P16.6 For the dimensions to increase, $\Delta L = \alpha L_i \Delta T$

$$1.00 \times 10^{-2} \text{ cm} = 1.30 \times 10^{-4} °\text{C}^{-1}(2.20 \text{ cm})(T - 20.0°\text{C})$$
$$T = \boxed{55.0°\text{C}}$$

***P16.7** $\Delta L = \alpha L_i \Delta T = \left(22 \times 10^{-6}/\text{C°}\right)(2.40 \text{ cm})(30°\text{C}) = \boxed{1.58 \times 10^{-3} \text{ cm}}$

***P16.8** We consider the expansion of the horizontal strip of vinyl lying between the snug nailhead and a nail near the far end of the wall, where the siding panel can slide by 1.40 cm under the nailhead:

$$\Delta L = \alpha L_i \Delta T$$

$$0.014\text{ m} = \alpha(15\text{ m})[38°\text{C} - (-5°\text{C})]$$

$$\alpha = \boxed{2.17\times10^{-5}(°\text{C})^{-1}}$$

P16.9 (a) $\Delta L = \alpha L_i \Delta T = 9.00\times10^{-6}\,°\text{C}^{-1}(30.0\text{ cm})(65.0°\text{C}) = \boxed{0.176\text{ mm}}$

(b) $\Delta L = \alpha L_i \Delta T = 9.00\times10^{-6}\,°\text{C}^{-1}(1.50\text{ cm})(65.0°\text{C}) = \boxed{8.78\times10^{-4}\text{ cm}}$

(c) $\Delta V = 3\alpha V_i \Delta T = 3\left(9.00\times10^{-6}\,°\text{C}^{-1}\right)\left(\dfrac{30.0(\pi)(1.50)^2}{4}\text{ cm}^3\right)(65.0°\text{C}) = \boxed{0.0930\text{ cm}^3}$

P16.10 The horizontal section expands according to $\Delta L = \alpha L_i \Delta T$.

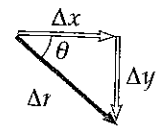

FIG. P16.10

$$\Delta x = \left(17\times10^{-6}\,°\text{C}^{-1}\right)(28.0\text{ cm})(46.5°\text{C} - 18.0°\text{C}) = 1.36\times10^{-2}\text{ cm}$$

The vertical section expands similarly by

$$\Delta y = \left(17\times10^{-6}\,°\text{C}^{-1}\right)(134\text{ cm})(28.5°\text{C}) = 6.49\times10^{-2}\text{ cm}$$

The vector displacement of the pipe elbow has magnitude

$$\Delta r = \sqrt{\Delta x^2 + \Delta y^2} = \sqrt{(0.136\text{ mm})^2 + (0.649\text{ mm})^2} = 0.663\text{ mm}$$

and is directed to the right below the horizontal at angle

$$\theta = \tan^{-1}\left(\frac{\Delta y}{\Delta x}\right) = \tan^{-1}\left(\frac{0.649\text{ mm}}{0.136\text{ mm}}\right) = 78.2°$$

$$\boxed{\Delta r = 0.663\text{ mm to the right at }78.2°\text{ below the horizontal}}$$

P16.11 (a) $\Delta A = 2\alpha A_i \Delta T$: $\Delta A = 2\left(17.0\times10^{-6}\,°\text{C}^{-1}\right)(0.0800\text{ m})^2(50.0°\text{C})$

$$\Delta A = 1.09\times10^{-5}\text{ m}^2 = \boxed{0.109\text{ cm}^2}$$

(b) The length of each side of the hole has increased. Thus, this represents an $\boxed{\text{increase}}$ in the area of the hole.

P16.12 $\Delta V = (\beta - 3\alpha)V_i\Delta T = \left(5.81\times10^{-4} - 3\left(11.0\times10^{-6}\right)\right)(50.0\text{ gal})(20.0) = \boxed{0.548\text{ gal}}$

P16.13 (a) $\Delta V = V_t\beta_t\Delta T - V_{Al}\beta_{Al}\Delta T = (\beta_t - 3\alpha_{Al})V_i\Delta T$

$$= \left(9.00\times10^{-4} - 0.720\times10^{-4}\right)^\circ\text{C}^{-1}\left(2\,000\text{ cm}^3\right)(60.0^\circ\text{C})$$

$$\Delta V = \boxed{99.4\text{ cm}^3} \text{ overflows.}$$

(b) The whole new volume of turpentine is

$$2\,000\text{ cm}^3 + 9.00\times10^{-4}\,^\circ\text{C}^{-1}\left(2\,000\text{ cm}^3\right)(60.0^\circ\text{C}) = 2\,108\text{ cm}^3$$

so the fraction lost is $\dfrac{99.4\text{ cm}^3}{2\,108\text{ cm}^3} = 4.71\times10^{-2}$

and this fraction of the cylinder's depth will be empty upon cooling:

$$4.71\times10^{-2}(20.0\text{ cm}) = \boxed{0.943\text{ cm}}$$

P16.14 (a) $L = L_i(1+\alpha\Delta T)$: $5.050\text{ cm} = 5.000\text{ cm}\left[1 + 24.0\times10^{-6}\,^\circ\text{C}^{-1}(T - 20.0^\circ\text{C})\right]$

$$T = \boxed{437^\circ\text{C}}$$

(b) We must get $L_{Al} = L_{Brass}$ for some ΔT, or

$$L_{i,\,Al}(1+\alpha_{Al}\Delta T) = L_{i\,Brass}(1+\alpha_{Brass}\Delta T)$$

$$5.000\text{ cm}\left[1+\left(24.0\times10^{-6}\,^\circ\text{C}^{-1}\right)\Delta T\right] = 5.050\text{ cm}\left[1+\left(19.0\times10^{-6}\,^\circ\text{C}^{-1}\right)\Delta T\right]$$

Solving for ΔT, $\Delta T = 2\,080^\circ\text{C}$,

so $\boxed{T = 2\,100^\circ\text{C}}$

This will not work because $\boxed{\text{aluminum melts at }660^\circ\text{C}}$.

Section 16.4 Macroscopic Description of an Ideal Gas

P16.15 Mass of gold abraded: $|\Delta m| = 3.80\text{ g} - 3.35\text{ g} = 0.45\text{ g} = (0.45\text{ g})\left(\dfrac{1\text{ kg}}{10^3\text{ g}}\right) = 4.5\times10^{-4}\text{ kg}$.

Each atom has mass $m_0 = 197\text{ u} = 197\text{ u}\left(\dfrac{1.66\times10^{-27}\text{ kg}}{1\text{ u}}\right) = 3.27\times10^{-25}\text{ kg}$

Now, $|\Delta m| = |\Delta N|m_0$, and the number of atoms missing is

$$|\Delta N| = \frac{|\Delta m|}{m_0} = \frac{4.5\times10^{-4}\text{ kg}}{3.27\times10^{-25}\text{ kg}} = 1.38\times10^{21}\text{ atoms.}$$

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The rate of loss is

$$\frac{|\Delta N|}{\Delta t}=\frac{1.38\times10^{21}\ \text{atoms}}{50\ \text{yr}}\left(\frac{1\ \text{yr}}{365.25\ \text{d}}\right)\left(\frac{1\ \text{d}}{24\ \text{h}}\right)\left(\frac{1\ \text{h}}{60\ \text{min}}\right)\left(\frac{1\ \text{min}}{60\ \text{s}}\right)$$

$$\frac{|\Delta N|}{\Delta t}=\boxed{8.72\times10^{11}\ \text{atoms/s}}$$

P16.16 One mole of helium contains Avogadro's number of molecules and has a mass of 4.00 g. Let us call m_0 the mass of one atom, and we have

$$N_A m_0 = 4.00\ \text{g/mol}$$

or

$$m_0=\frac{4.00\ \text{g/mol}}{6.02\times10^{23}\ \text{molecules/mol}}=6.64\times10^{-24}\ \text{g/molecule}$$

$$m_0=\boxed{6.64\times10^{-27}\ \text{kg}}$$

P16.17 (a) Initially, $P_iV_i=n_iRT_i$ $\quad(1.00\ \text{atm})V_i=n_iR(10.0+273.15)\ \text{K}$

Finally, $P_fV_f=n_fRT_f$ $\quad P_f(0.280V_i)=n_iR(40.0+273.15)\ \text{K}$

Dividing these equations, $\quad\dfrac{0.280P_f}{1.00\ \text{atm}}=\dfrac{313.15\ \text{K}}{283.15\ \text{K}}$

giving $\quad P_f=3.95\ \text{atm}$

or $\quad P_f=\boxed{4.00\times10^5\ \text{Pa(abs.)}}$

(b) After being driven $\quad P_d(1.02)(0.280V_i)=n_iR(85.0+273.15)\ \text{K}$

$$P_d=1.121P_f=\boxed{4.49\times10^5\ \text{Pa}}$$

P16.18 $PV=NP'V'=\frac{4}{3}\pi r^3NP'$: $\quad N=\dfrac{3PV}{4\pi r^3P'}=\dfrac{3(150)(0.100)}{4\pi(0.150)^3(1.20)}=884$ balloons

If we have no special means for squeezing the last 100 L of helium out of the tank, the tank will be full of helium at 1.20 atm when the last balloon is inflated. The number of balloons is then reduced to $884-\dfrac{(0.100\ \text{m}^3)3}{4\pi(0.15\ \text{m})^3}=\boxed{877}$.

P16.19 The equation of state of an ideal gas is $PV=nRT$ so we need to solve for the number of moles to find N.

$$n=\frac{PV}{RT}=\frac{(1.01\times10^5\ \text{N/m}^2)[(10.0\ \text{m})(20.0\ \text{m})(30.0\ \text{m})]}{(8.314\ \text{J/mol K})(293\ \text{K})}=2.49\times10^5\ \text{mol}$$

$$N=nN_A=2.49\times10^5\ \text{mol}(6.022\times10^{23}\ \text{molecules/mol})=\boxed{1.50\times10^{29}\ \text{molecules}}$$

*P16.20 Consider the air in the tank during one discharge process. We suppose that the process is slow enough that the temperature remains constant. Then as the pressure drops from 2.40 atm to 1.20 atm, the volume of the air doubles. During the first discharge, the air volume changes from 1 L to 2 L. Just 1 L of water is expelled and 3 L remains. In the second discharge, the air volume changes from 2 L to 4 L and 2 L of water is sprayed out. In the third discharge, only the last 1 L of water comes out. Were it not for male pattern dumbness, each person could more efficiently use his device by starting with the tank half full of water.

P16.21 $\sum F_y = 0$: $\rho_{out} gV - \rho_{in} gV - (200\text{ kg})g = 0$

$$(\rho_{out} - \rho_{in})(400\text{ m}^3) = 200\text{ kg}$$

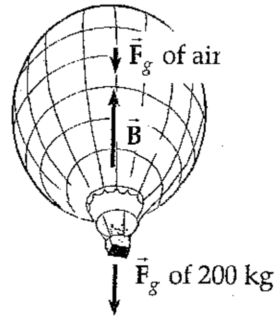

FIG. P16.21

The density of the air outside is 1.25 kg/m^3.

From $PV = nRT$, $\frac{n}{V} = \frac{P}{RT}$

The density is inversely proportional to the temperature, and the density of the hot air is

$$\rho_{in} = (1.25\text{ kg/m}^3)\left(\frac{283\text{ K}}{T_{in}}\right)$$

Then

$$(1.25\text{ kg/m}^3)\left(1 - \frac{283\text{ K}}{T_{in}}\right)(400\text{ m}^3) = 200\text{ kg}$$

$$1 - \frac{283\text{ K}}{T_{in}} = 0.400$$

$$0.600 = \frac{283\text{ K}}{T_{in}} \qquad T_{in} = \boxed{472\text{ K}}$$

P16.22 (a) $PV = nRT$ $\quad n = \frac{PV}{RT}$

$$m = nM = \frac{PVM}{RT} = \frac{1.013\times10^5\text{ Pa}(0.100\text{ m})^3(28.9\times10^{-3}\text{ kg/mol})}{(8.314\text{ J/mol K})(300\text{ K})}$$

$$m = \boxed{1.17\times10^{-3}\text{ kg}}$$

(b) $F_g = mg = 1.17\times10^{-3}\text{ kg}(9.80\text{ m/s}^2) = \boxed{11.5\text{ mN}}$

(c) $F = PA = (1.013\times10^5\text{ N/m}^2)(0.100\text{ m})^2 = \boxed{1.01\text{ kN}}$

(d) The $\boxed{\text{molecules must be moving very fast}}$ to hit the walls hard

***P16 23** (a) The air in the tube is far from liquification, so it behaves as an ideal gas. At the ocean surface it is described by $P_t V_t = nRT$ where $P_t = 1$ atm, $V_t = A(6\ 50\ \text{cm})$, and A is the cross-sectional area of the interior of the tube. At the bottom of the dive, $P_b V_b = nRT = P_b A(6.50\ \text{cm} - 2\ 70\ \text{cm})$. By division,

$$\frac{P_b(3.8\ \text{cm})}{(1\ \text{atm})(6.5\ \text{cm})} = 1$$

$$P_b = 1\ 013 \times 10^5\ \text{N/m}^2\ \frac{6.5}{3\ 8} = 1\ 73 \times 10^5\ \text{N/m}^2$$

The salt water enters the tube until the air pressure is equal to the water pressure at depth, which is described by

$$P_b = P_t + \rho g h$$

$$1\ 73 \times 10^5\ \text{N/m}^2 = 1.013 \times 10^5\ \text{N/m}^2 + \left(1\ 030\ \text{kg/m}^3\right)\left(9.8\ \text{m/s}^2\right)h$$

$$h = \frac{7.20 \times 10^4\ \text{kg m m}^2\ \text{s}^2}{1.01 \times 10^4\ \text{s}^2\ \text{m}^2\ \text{kg}} = \boxed{7\ 13\ \text{m}}$$

(b) With a very thin tube, air does not bubble out. At the bottom of the dive, the tube gives a valid reading in any orientation. The open end of the tube should be at the bottom after the bird surfaces, so that the water will drain away as the expanding air pushes it out. Students can make the tubes and dive with them in a swimming pool, to observe how dependably they work

P16.24 At depth, $P = P_0 + \rho g h$ and $PV_i = nRT_i$

At the surface, $P_0 V_f = nRT_f$: $\qquad \dfrac{P_0 V_f}{(P_0 + \rho g h)V_i} = \dfrac{T_f}{T_i}$

Therefore $\qquad V_f = V_i\left(\dfrac{T_f}{T_i}\right)\left(\dfrac{P_0 + \rho g h}{P_0}\right)$

$$V_f = 1\ 00\ \text{cm}^3\left(\frac{293\ \text{K}}{278\ \text{K}}\right)\left(\frac{1\ 013 \times 10^5\ \text{Pa} + \left(1\ 025\ \text{kg/m}^3\right)\left(9\ 80\ \text{m/s}^2\right)(25.0\ \text{m})}{1.013 \times 10^5\ \text{Pa}}\right)$$

$$V_f = \boxed{3.67\ \text{cm}^3}$$

P16 25 $PV = nRT$: $\qquad \dfrac{m_f}{m_i} = \dfrac{n_f}{n_i} = \dfrac{P_f V_f}{RT_f}\dfrac{RT_i}{P_i V_i} = \dfrac{P_f}{P_i}$

so $\qquad m_f = m_i\left(\dfrac{P_f}{P_i}\right)$

$$|\Delta m| = m_i - m_f = m_i\left(\frac{P_i - P_f}{P_i}\right) = 12\ 0\ \text{kg}\left(\frac{41.0\ \text{atm} - 26.0\ \text{atm}}{41\ 0\ \text{atm}}\right) = \boxed{4.39\ \text{kg}}$$

P16.26 My bedroom is 4 m long, 4 m wide, and 2.4 m high, enclosing air at 100 kPa and 20°C = 293 K Think of the air as 80.0% N_2 and 20.0% O_2

Avogadro's number of molecules has mass

$$(0.800)(28.0\ \text{g/mol})+(0.200)(32.0\ \text{g/mol})=0.028\,8\ \text{kg/mol}$$

Then $PV=nRT=\left(\frac{m}{M}\right)RT$

gives $m=\frac{PVM}{RT}=\frac{(1.00\times10^5\ \text{N/m}^2)(38.4\ \text{m}^3)(0.028\,8\ \text{kg/mol})}{(8.314\ \text{J/mol}\cdot\text{K})(293\ \text{K})}=45.4\ \text{kg}\ \boxed{\sim10^2\ \text{kg}}$

***P16.27** The CO_2 is far from liquefaction, so after it comes out of solution it behaves as an ideal gas Its molar mass is $M=12.0\ \text{g/mol}+2(16.0\ \text{g/mol})=44.0\ \text{g/mol}$. The quantity of gas in the cylinder is

$$n=\frac{m_{\text{sample}}}{M}=\frac{6.50\ \text{g}}{44.0\ \text{g/mol}}=0.148\ \text{mol}$$

Then $PV=nRT$

gives $V=\frac{nRT}{P}=\frac{0.148\ \text{mol}(8.314\ \text{J/mol}\cdot\text{K})(273\ \text{K}+20\ \text{K})}{1.013\times10^5\ \text{N/m}^2}\left(\frac{1\ \text{N}\cdot\text{m}}{1\ \text{J}}\right)\left(\frac{10^3\ \text{L}}{1\ \text{m}^3}\right)=\boxed{3.55\ \text{L}}$

P16.28 $N=\frac{PVN_A}{RT}=\frac{(10^{-9}\ \text{Pa})(1.00\ \text{m}^3)(6.02\times10^{23}\ \text{molecules/mol})}{(8.314\ \text{J/K}\cdot\text{mol})(300\ \text{K})}=\boxed{2.41\times10^{11}\ \text{molecules}}$

P16.29 $P_0V=n_1RT_1=\left(\frac{m_1}{M}\right)RT_1$

$P_0V=n_2RT_2=\left(\frac{m_2}{M}\right)RT_2$

$$\boxed{m_1-m_2=\frac{P_0VM}{R}\left(\frac{1}{T_1}-\frac{1}{T_2}\right)}$$

Section 16.5 The Kinetic Theory of Gases

P16.30 Consider the x axis to be perpendicular to the plane of the window Then, the average force exerted on the window by the hailstones is in magnitude

$$\overline{F}=Nm\left(\frac{\Delta v}{\Delta t}\right)=Nm\left(\frac{v_{xf}-v_{xi}}{t}\right)=Nm\left(\frac{v\sin\theta-(-v\sin\theta)}{t}\right)=\boxed{Nm\left(\frac{2v\sin\theta}{t}\right)}$$

Thus, the pressure on the windowpane is $P=\frac{\overline{F}}{A}=\boxed{Nm\left(\frac{2v\sin\theta}{At}\right)}$

P16.31 $\overline{F}=\frac{(5.00\times10^{23})[2(4.68\times10^{-26}\ \text{kg})(300\ \text{m/s})]}{1.00\ \text{s}}=14.0\ \text{N}$

and $P=\frac{\overline{F}}{A}=\frac{14.0\ \text{N}}{8.00\times10^{-4}\ \text{m}^2}=\boxed{17.6\ \text{kPa}}$

P16.32 (a) $PV = nRT = \frac{Nm_0v^2}{3}$

The total translational kinetic energy is $\frac{Nm_0v^2}{2} = E_{\text{trans}}$:

$$E_{\text{trans}} = \frac{3}{2}PV = \frac{3}{2}\left(3.00\times 1.013\times 10^5\right)\left(5.00\times 10^{-3}\right) = \boxed{2.28\text{ kJ}}$$

(b) $$\frac{m_0v^2}{2} = \frac{3k_BT}{2} = \frac{3RT}{2N_A} = \frac{3(8.314)(300)}{2\left(6.02\times 10^{23}\right)} = \boxed{6.21\times 10^{-21}\text{ J}}$$

P16.33 (a) $$PV = Nk_BT: \quad N = \frac{PV}{k_BT} = \frac{1.013\times 10^5\text{ Pa}\left[\frac{4}{3}\pi(0.150\text{ m})^3\right]}{\left(1.38\times 10^{-23}\text{ J/K}\right)(293\text{ K})} = \boxed{3.54\times 10^{23}\text{ atoms}}$$

(b) $$\overline{K} = \frac{3}{2}k_BT = \frac{3}{2}\left(1.38\times 10^{-23}\right)(293)\text{ J} = \boxed{6.07\times 10^{-21}\text{ J}}$$

(c) For helium, the atomic mass is $m_0 = \frac{4.00\text{ g/mol}}{6.02\times 10^{23}\text{ molecules/mol}} = 6.64\times 10^{-24}\text{ g/molecule}$

$$m_0 = 6.64\times 10^{-27}\text{ kg/molecule}$$

$$\frac{1}{2}m_0\overline{v^2} = \frac{3}{2}k_BT: \qquad \therefore v_{\text{rms}} = \sqrt{\frac{3k_BT}{m_0}} = \boxed{1.35\text{ km/s}}$$

***P16.34** Let $d = 2r$ represent the diameter of the particle. Its mass is $m = \rho V = \rho\frac{4}{3}\pi r^3 = \rho\frac{4}{3}\pi\left(\frac{d}{2}\right)^3 = \frac{\rho\pi d^3}{6}$.

Then $\frac{1}{2}mv_{\text{rms}}^2 = \frac{3}{2}kT$ gives $\frac{\rho\pi d^3}{6}v_{\text{rms}}^2 = 3kT$

(a) $$v_{\text{rms}} = \left(\frac{18kT}{\rho\pi d^3}\right)^{1/2} = \left(\frac{18\left(1.38\times 10^{-23}\text{ J/K}\right)(293\text{ K})}{\left(1\,000\text{ kg/m}^3\right)\pi\left(3\times 10^{-6}\text{ m}\right)^3}\right)^{1/2} = \boxed{9.26\times 10^{-4}\text{ m/s}}$$

(b) $$v = \frac{x}{t} \qquad t = \frac{x}{v} = \frac{3\times 10^{-6}\text{ m}}{9.26\times 10^{-4}\text{ m/s}} = \boxed{3.24\text{ ms}}$$

(c) $$70\text{ kg} = 1\,000\text{ kg/m}^3\frac{\pi d^3}{6} \qquad d = 0.511\text{ m}$$

$$v_{\text{rms}} = \left(\frac{18kT}{\rho\pi d^3}\right)^{1/2} = \left(\frac{18\left(1.38\times 10^{-23}\text{ J/K}\right)(293\text{ K})}{\left(1\,000\text{ kg/m}^3\right)\pi(0.511\text{ m})^3}\right)^{1/2} = \boxed{1.32\times 10^{-11}\text{ m/s}}$$

$$t = \frac{0.511\text{ m}}{1.32\times 10^{-11}\text{ m/s}} = \boxed{3.88\times 10^{10}\text{ s}} = 1\,230\text{ yr}$$

This motion is too slow to observe.

continued on next page

(d) $\left(\frac{18kT}{\rho\pi d^3}\right)^{1/2}=\frac{d}{1\text{ s}}$ $\frac{18kT}{\rho\pi}=\frac{d^5}{1\text{ s}^2}$

$$d=\left(\frac{18(1.38\times10^{-23}\text{ J/K})(293\text{ K})(1\text{ s}^2)}{(1\,000\text{ kg/m}^3)\pi}\right)^{1/5}=\boxed{2.97\times10^{-5}\text{ m}}$$

Brownian motion is best observed with pollen grains, smoke particles, or latex spheres smaller than this 29.7-μm size. Then they can jitter about convincingly, showing relatively large accelerations several times per second.

P16.35 (a) $\overline{K}=\frac{3}{2}k_BT=\frac{3}{2}(1.38\times10^{-23}\text{ J/K})(423\text{ K})=\boxed{8.76\times10^{-21}\text{ J}}$

(b) $\overline{K}=\frac{1}{2}m_0v_{\text{rms}}^2=8.76\times10^{-21}\text{ J}$

so $$v_{rms}=\sqrt{\frac{1.75\times10^{-20}\text{ J}}{m_0}} \qquad (1)$$

For helium, $m_0=\frac{4.00\text{ g/mol}}{6.02\times10^{23}\text{ molecules/mol}}=6.64\times10^{-24}\text{ g/molecule}$

$m_0=6.64\times10^{-27}\text{ kg/molecule}$

Similarly for argon, $m_0=\frac{39.9\text{ g/mol}}{6.02\times10^{23}\text{ molecules/mol}}=6.63\times10^{-23}\text{ g/molecule}$

$m_0=6.63\times10^{-26}\text{ kg/molecule}$

Substituting in (1) above,

we find for helium, $\boxed{v_{rms}=1.62\text{ km/s}}$

and for argon, $\boxed{v_{rms}=514\text{ m/s}}$

Section 16.6 Distribution of Molecular Speeds

P16.36 (a) $v_{\text{av}}=\frac{\sum n_iv_i}{N}=\frac{1}{15}[1(2)+2(3)+3(5)+4(7)+3(9)+2(12)]=\boxed{6.80\text{ m/s}}$

(b) $\left(v^2\right)_{\text{av}}=\frac{\sum n_iv_i^2}{N}=54.9\text{ m}^2/\text{s}^2$

so $v_{\text{rms}}=\sqrt{\left(v^2\right)_{\text{av}}}=\sqrt{54.9}=\boxed{7.41\text{ m/s}}$

(c) $v_{\text{mp}}=\boxed{7.00\text{ m/s}}$

P16.37 In the Maxwell Boltzmann speed distribution function take $\frac{dN_v}{dv}=0$ to find

$$4\pi N\left(\frac{m_0}{2\pi k_B T}\right)^{3/2}\exp\left(-\frac{m_0 v^2}{2k_B T}\right)\left(2v-\frac{2m_0 v^3}{2k_B T}\right)=0$$

and solve for v to find the most probable speed

Reject as solutions $v=0$ and $v=\infty$

Retain only $2-\frac{m_0 v^2}{k_B T}=0$

Then $v_{\text{mp}}=\sqrt{\frac{2k_B T}{m_0}}$

P16.38 The most probable speed is $v_{\text{mp}}=\sqrt{\frac{2k_B T}{m_0}}=\sqrt{\frac{2\left(1.38\times10^{-23}\text{ J/K}\right)(4.20\text{ K})}{6.64\times10^{-27}\text{ kg}}}=\boxed{132\text{ m/s}}$.

P16.39 (a) From $v_{\text{av}}=\sqrt{\frac{8k_B T}{\pi m_0}}$

we find the temperature as $T=\frac{\pi\left(6.64\times10^{-27}\text{ kg}\right)\left(1.12\times10^4\text{ m/s}\right)^2}{8\left(1.38\times10^{-23}\text{ J/mol K}\right)}=\boxed{2.37\times10^4\text{ K}}$

(b) $T=\frac{\pi\left(6.64\times10^{-27}\text{ kg}\right)\left(2.37\times10^3\text{ m/s}\right)^2}{8\left(1.38\times10^{-23}\text{ J/mol K}\right)}=\boxed{1.06\times10^3\text{ K}}$

Section 16.7 Context Connection—The Atmospheric Lapse Rate

P16.40 For a uniform lapse rate, the identity $\frac{\Delta T}{\Delta y}=\frac{T_f-T_i}{\Delta y}$ implies

$$T_f=T_i+\frac{\Delta T}{\Delta y}\Delta y=30°\text{C}-(6.5°\text{C/km})(3.66\text{ km})=\boxed{6.2°\text{C}}$$

P16.41 (a) $$\frac{dT}{dy}=-\frac{\gamma-1}{\gamma}\frac{gM}{R}=-\frac{0.40}{1.40}\frac{\left(9.8\text{ m/s}^2\right)(28.9\text{ g/mol})}{(8.314\text{ J/mol}\cdot\text{K})}\left(\frac{1\text{ kg}}{1\,000\text{ g}}\right)\left(\frac{1\text{ J}}{1\text{ kg}\cdot\text{m}^2/\text{s}^2}\right)$$
$$=-9.73\times10^{-3}\text{ K/m}=\boxed{-9.73°\text{C/km}}$$

(b) Air contains water vapor. Air does not behave as an ideal gas. As a parcel of air rises in the atmosphere and its temperature drops, its ability to contain water vapor decreases, so water will likely condense out as liquid drops or as ice crystals. (The condensate may or may not be visible as clouds.) The condensate releases its heat of vaporization, raising the air temperature above the value that would be expected according to part (a).

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(c) For an object of mass m on Mars, weight = force of planet's gravity: $mg = \frac{GM_{\text{Mars}}m}{r_{\text{Mars}}^2}$ or

$g = \frac{GM_{\text{Mars}}}{r_{\text{Mars}}^2}$

$$g = \frac{(6.67\times10^{-11}\text{ N m}^2/\text{kg}^2)(6.42\times10^{23}\text{ kg})}{(3.37\times10^6\text{ m})^2} = 3.77\text{ m/s}^2$$

$$\frac{dT}{dy} = -\frac{\gamma-1}{\gamma}\frac{gM}{R} = -\frac{0.30}{1.30}\frac{(3.77\text{ m/s}^2)(0.0440\text{ kg/mol})}{(8.314\text{ J/mol K})} = -4.60\times10^{-3}\text{ K/m} = \boxed{-4.60°\text{C/km}}$$

(d) $$\frac{\Delta T}{\Delta y} = \frac{dT}{dy}: \quad \Delta y = \frac{\Delta T}{\frac{dT}{dy}} = \frac{-60°\text{C}-(-40°\text{C})}{-4.60°\text{C/km}} = \boxed{4.34\text{ km}}$$

(e) The dust in the atmosphere absorbs and scatters energy from the electromagnetic radiation coming through the atmosphere from the sun. The dust contributes energy to the gas molecules high in the atmosphere, resulting in an increase in the internal energy of the atmosphere aloft and a smaller decrease in temperature with height, than in the case where there is no absorption of sunlight. The larger the amount of dust, the more the lapse rate will deviate from the theoretical value in part (c). Thus it was dustier during the *Mariner* flights in 1969.

Additional Problems

P16.42 The excess expansion of the brass is $\Delta L_{\text{rod}} - \Delta L_{\text{tape}} = (\alpha_{\text{brass}} - \alpha_{\text{steel}})L_i\Delta T$

$$\Delta(\Delta L) = (19.0-11.0)\times10^{-6}\ (°\text{C})^{-1}(0.950\text{ m})(35.0°\text{C})$$

$$\Delta(\Delta L) = 2.66\times10^{-4}\text{ m}$$

(a) The rod contracts more than tape to

a length reading $0.950\,0\text{ m} - 0.000\,266\text{ m} = \boxed{0.949\,7\text{ m}}$

(b) $0.950\,0\text{ m} + 0.000\,266\text{ m} = \boxed{0.950\,3\text{ m}}$

P16.43 At 0°C, 10.0 gallons of gasoline has mass,

from $\rho = \frac{m}{V}$

$$m = \rho V = (730\text{ kg/m}^3)(10.0\text{ gal})\left(\frac{0.003\,80\text{ m}^3}{1.00\text{ gal}}\right) = 27.7\text{ kg}$$

The gasoline will expand in volume by

$$\Delta V = \beta V_i\Delta T = 9.60\times10^{-4}\,°\text{C}^{-1}(10.0\text{ gal})(20.0°\text{C} - 0.0°\text{C}) = 0.192\text{ gal}$$

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At 20.0°C, $10.192 \text{ gal} = 27.7 \text{ kg}$

$$10.0 \text{ gal} = 27.7 \text{ kg}\left(\frac{10.0 \text{ gal}}{10.192 \text{ gal}}\right) = 27.2 \text{ kg}$$

The extra mass contained in 10.0 gallons at 0.0°C is

$$27.7 \text{ kg} - 27.2 \text{ kg} = \boxed{0.523 \text{ kg}}$$

P16.44 (a) The volume of the liquid increases as $\Delta V_\ell = V_i \beta \Delta T$. The volume of the flask increases as $\Delta V_g = 3\alpha V_i \Delta T$. Therefore, the overflow in the capillary is $V_c = V_i \Delta T(\beta - 3\alpha)$; and in the capillary $V_c = A\Delta h$.

Therefore, $\boxed{\Delta h = \frac{V_i}{A}(\beta - 3\alpha)\Delta T}$.

(b) For a mercury thermometer $\beta(\text{Hg}) = 1.82 \times 10^{-4} \, ^\circ\text{C}^{-1}$

and for glass, $3\alpha = 3 \times 3.20 \times 10^{-6} \, ^\circ\text{C}^{-1}$

Thus $\beta - 3\alpha \approx \beta$ within 5%

or $\boxed{\alpha << \beta}$

P16.45 (a) $\rho = \frac{m}{V}$ and $d\rho = -\frac{m}{V^2} dV$

For very small changes in V and ρ, this can be expressed as

$$\Delta\rho = -\frac{m}{V}\frac{\Delta V}{V} = -\rho\beta\Delta T.$$

The negative sign means that any increase in temperature causes the density to decrease and vice versa.

(b) For water we have $\beta = \left|\frac{\Delta\rho}{\rho\Delta T}\right| = \left|\frac{1.0000 \text{ g/cm}^3 - 0.9997 \text{ g/cm}^3}{(1.0000 \text{ g/cm}^3)(10.0^\circ\text{C} - 4.0^\circ\text{C})}\right| = \boxed{5 \times 10^{-5} \, ^\circ\text{C}^{-1}}$.

P16.46 The astronauts exhale this much CO_2:

$$n = \frac{m_{\text{sample}}}{M} = \frac{1.09 \text{ kg}}{\text{astronaut day}}\left(\frac{1\,000 \text{ g}}{1 \text{ kg}}\right)(3 \text{ astronauts})(7 \text{ days})\left(\frac{1 \text{ mol}}{44.0 \text{ g}}\right) = 520 \text{ mol}.$$

Then 520 mol of methane is generated. It is far from liquefaction and behaves as an ideal gas.

$$P = \frac{nRT}{V} = \frac{520 \text{ mol}(8.314 \text{ J/mol K})(273 \text{ K} - 45 \text{ K})}{150 \times 10^{-3} \text{ m}^3} = \boxed{6.57 \times 10^6 \text{ Pa}}$$

P16.47 (a) We assume that air at atmospheric pressure is above the piston

In equilibrium $P_{gas} = \frac{mg}{A} + P_0$

Therefore, $\frac{nRT}{hA} = \frac{mg}{A} + P_0$

or $\boxed{h = \frac{nRT}{mg + P_0 A}}$

where we have used $V = hA$ as the volume of the gas

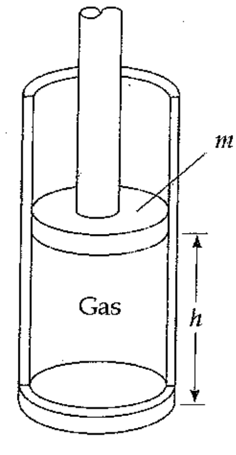

FIG. P16.47

(b) From the data given,

$$h = \frac{0.200\text{ mol}(8.314\text{ J/K mol})(400\text{ K})}{20.0\text{ kg}(9.80\text{ m/s}^2) + (1.013\times10^5\text{ N/m}^2)(0.008\,00\text{ m}^2)}$$
$$= \boxed{0.661\text{ m}}$$

P16.48 The angle of bending θ, between tangents to the two ends of the strip, is equal to the angle the strip subtends at its center of curvature. (The angles are equal because their sides are perpendicular, right side to the right side and left side to left side.)

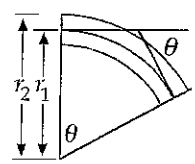

FIG. P16.48

(a) The definition of radian measure gives $L_i + \Delta L_1 = \theta r_1$

and $L_i + \Delta L_2 = \theta r_2$

By subtraction, $\Delta L_2 - \Delta L_1 = \theta(r_2 - r_1)$

$$\alpha_2 L_i \Delta T - \alpha_1 L_i \Delta T = \theta \Delta r$$

$$\boxed{\theta = \frac{(\alpha_2 - \alpha_1)L_i \Delta T}{\Delta r}}$$

(b) In the expression from part (a), θ is directly proportional to ΔT and also to $(\alpha_2 - \alpha_1)$. Therefore θ is zero when either of these quantities becomes zero.

(c) The material that expands more when heated contracts more when cooled, so the bimetallic strip bends the other way. It is fun to demonstrate this with liquid nitrogen.

(d) $$\theta = \frac{2(\alpha_2 - \alpha_1)L_i \Delta T}{2\Delta r} = \frac{2\left((19\times10^{-6} - 0.9\times10^{-6})^\circ\text{C}^{-1}\right)(200\text{ mm})(1^\circ\text{C})}{0.500\text{ mm}}$$
$$= 1.45\times10^{-2} = 1.45\times10^{-2}\text{ rad}\left(\frac{180^\circ}{\pi\text{ rad}}\right) = \boxed{0.830^\circ}$$

P16.49 From the diagram we see that the change in area is

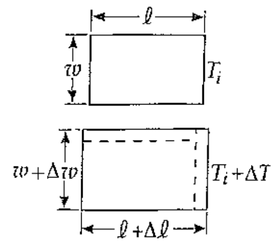

FIG. P16.49

$$\Delta A = \ell\Delta w + w\Delta\ell + \Delta w\Delta\ell$$

Since $\Delta\ell$ and Δw are each small quantities, the product $\Delta w\Delta\ell$ will be very small. Therefore, we assume $\Delta w\Delta\ell \approx 0$

Since $\Delta w = w\alpha\Delta T$ and $\Delta\ell = \ell\alpha\Delta T$,

we then have $\Delta A = \ell w\alpha\Delta T + \ell w\alpha\Delta T$

and since $A = \ell w$, $\boxed{\Delta A = 2\alpha A\Delta T}$.

The approximation assumes $\Delta w\Delta\ell \approx 0$, or $\alpha\Delta T \approx 0$. Another way of stating this is $\boxed{\alpha\Delta T << 1}$.

P16.50 (a) $T_i = 2\pi\sqrt{\dfrac{L_i}{g}}$ so $L_i = \dfrac{T_i^2 g}{4\pi^2} = \dfrac{(1.000\text{ s})^2(9.80\text{ m/s}^2)}{4\pi^2} = 0.248\,2\text{ m}$

$$\Delta L = \alpha L_i\Delta T = 19.0\times10^{-6}\,{}^\circ\text{C}^{-1}(0.248\,2\text{ m})(10.0^\circ\text{C}) = 4.72\times10^{-5}\text{ m}$$

$$T_f = 2\pi\sqrt{\frac{L_i+\Delta L}{g}} = 2\pi\sqrt{\frac{0.248\,3\text{ m}}{9.80\text{ m/s}^2}} = 1.000\,095\,0\text{ s}$$

$$\Delta T = \boxed{9.50\times10^{-5}\text{ s}}$$

(b) In one week, the time lost is time lost $= 1\text{ week}\left(9.50\times10^{-5}\text{ s lost per second}\right)$

$$\text{time lost} = (7.00\text{ d/week})\left(\frac{86\,400\text{ s}}{1.00\text{ d}}\right)\left(9.50\times10^{-5}\ \frac{\text{s lost}}{\text{s}}\right)$$

$$\text{time lost} = \boxed{57.5\text{ s lost}}$$

P16.51 $I = \int r^2 dm$ and since $r(T) = r(T_i)(1+\alpha\Delta T)$

for $\alpha\Delta T << 1$ we find $\dfrac{I(T)}{I(T_i)} = (1+\alpha\Delta T)^2 \approx 1 + 2\alpha\Delta T$

thus $\dfrac{I(T)-I(T_i)}{I(T_i)} \approx 2\alpha\Delta T$

(a) With $\alpha = 17.0\times10^{-6}\,{}^\circ\text{C}^{-1}$ and $\Delta T = 100^\circ\text{C}$

we find for Cu: $\dfrac{\Delta I}{I} = 2\left(17.0\times10^{-6}\,{}^\circ\text{C}^{-1}\right)(100^\circ\text{C}) = \boxed{0.340\%}$

(b) With $\alpha = 24.0\times10^{-6}\,{}^\circ\text{C}^{-1}$

and $\Delta T = 100^\circ\text{C}$

we find for Al: $\dfrac{\Delta I}{I} = 2\left(24.0\times10^{-6}\,{}^\circ\text{C}^{-1}\right)(100^\circ\text{C}) = \boxed{0.480\%}$

*P16.52 (a) $B = \rho g V'$ $\qquad P' = P_0 + \rho g d \qquad P'V' = P_0 V_i$

$$B = \frac{\rho g P_0 V_i}{P'} = \boxed{\frac{\rho g P_0 V_i}{(P_0 + \rho g d)}}$$

(b) Since d is in the denominator, B must $\boxed{\text{decrease}}$ as the depth increases.

(The volume of the balloon becomes smaller with increasing pressure.)

(c) $$\frac{1}{2} = \frac{B(d)}{B(0)} = \frac{\rho g P_0 V_i/(P_0 + \rho g d)}{\rho g P_0 V_i / P_0} = \frac{P_0}{P_0 + \rho g d}$$

$$P_0 + \rho g d = 2P_0$$

$$d = \frac{P_0}{\rho g} = \frac{1.013 \times 10^5 \text{ N/m}^2}{(1.00 \times 10^3 \text{ kg/m}^3)(9.80 \text{ m/s}^2)} = \boxed{10.3 \text{ m}}$$

P16.53 After expansion, the length of one of the spans is

$$L_f = L_i(1 + \alpha \Delta T) = 125 \text{ m}\left[1 + 12 \times 10^{-6}\ {}^\circ\text{C}^{-1}(20.0^\circ\text{C})\right] = 125.03 \text{ m}$$

L_f, y, and the original 125 m length of this span form a right triangle with y as the altitude. Using the Pythagorean theorem gives:

$$(125.03 \text{ m})^2 = y^2 + (125 \text{ m})^2$$

yielding $y = \boxed{2.74 \text{ m}}$.

P16.54 After expansion, the length of one of the spans is $L_f = L(1 + \alpha \Delta T)$. L_f, y, and the original length L of this span form a right triangle with y as the altitude. Using the Pythagorean theorem gives

$$L_f^2 = L^2 + y^2, \quad \text{or} \quad y = \sqrt{L_f^2 - L^2} = L\sqrt{(1 + \alpha \Delta T)^2 - 1} = L\sqrt{2\alpha \Delta T + (\alpha \Delta T)^2}$$

Since $\alpha \Delta T << 1$, $\qquad y \approx \boxed{L\sqrt{2\alpha \Delta T}}$

The height of the center of the buckling bridge is directly proportional to the length. A small bridge is geometrically similar to a large one. The height is proportional to the square root of the temperature increase. Doubling ΔT makes y increase by only 41%. A small value of ΔT can have a surprisingly large effect. In units, the equation reads $\text{m} = \text{m}({}^\circ\text{C}/{}^\circ\text{C})^{1/2}$, so it is dimensionally correct.

*P16.55 (a) No torque acts on the disk so its angular momentum is constant. Its moment of inertia decreases as it contracts so its angular speed must $\boxed{\text{increase}}$.

(b) $$I_i \omega_i = I_f \omega_f = \frac{1}{2} M R_i^2 \omega_i = \frac{1}{2} M R_f^2 \omega_f = \frac{1}{2} M [R_i + R_i \alpha \Delta T]^2 \omega_f = \frac{1}{2} M R_i^2 [1 - \alpha|\Delta T|]^2 \omega_f$$

$$\omega_f = \omega_i [1 - \alpha|\Delta T|]^{-2} = \frac{25.0 \text{ rad/s}}{\left(1 - (17 \times 10^{-6} \text{ 1/C}^\circ)830^\circ\text{C}\right)^2} = \frac{25.0 \text{ rad/s}}{0.972} = \boxed{25.7 \text{ rad/s}}$$

P16.56 (a) From $PV = nRT$, the volume is: $V = \left(\frac{nR}{P}\right)T$

Therefore, when pressure is held constant, $\frac{\partial V}{\partial T} = \frac{nR}{P} = \frac{V}{T}$

Thus, $\beta \equiv \left(\frac{1}{V}\right)\frac{\partial V}{\partial T} = \left(\frac{1}{V}\right)\frac{V}{T}$, or $\beta = \boxed{\frac{1}{T}}$

(b) At $T = 0°\text{C} = 273\text{ K}$, this predicts $\beta = \frac{1}{273\text{ K}} = \boxed{3.66 \times 10^{-3}\text{ K}^{-1}}$

Experimental values are: $\beta_{He} = 3.665 \times 10^{-3}\text{ K}^{-1}$ and $\beta_{air} = 3.67 \times 10^{-3}\text{ K}^{-1}$

They agree within 0.06% and 0.2%, respectively.

P16.57 (a) Let m represent the sample mass. The number of moles is $n = \frac{m}{M}$ and the density is $\rho = \frac{m}{V}$.

So $PV = nRT$ becomes $PV = \frac{m}{M}RT$ or $PM = \frac{m}{V}RT$

Then, $\rho = \frac{m}{V} = \boxed{\frac{PM}{RT}}$.

(b) $$\rho = \frac{PM}{RT} = \frac{(1.013 \times 10^5\text{ N/m}^2)(0.0320\text{ kg/mol})}{(8.314\text{ J/mol}\cdot\text{K})(293\text{ K})} = \boxed{1.33\text{ kg/m}^3}$$

P16.58 (a) With piston alone: $T = \text{constant}$, so $PV = P_0V_0$

or $P(Ah_i) = P_0(Ah_0)$

With $A = \text{constant}$, $P = P_0\left(\frac{h_0}{h_i}\right)$

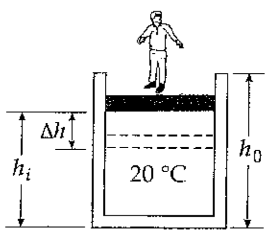

FIG. P16.58

But, $P = P_0 + \frac{m_p g}{A}$

where m_p is the mass of the piston

Thus, $P_0 + \frac{m_p g}{A} = P_0\left(\frac{h_0}{h_i}\right)$

which reduces to $$h_i = \frac{h_0}{1 + \frac{m_p g}{P_0 A}} = \frac{50.0\text{ cm}}{1 + \frac{20.0\text{ kg}(9.80\text{ m/s}^2)}{1.013\times10^5\text{ Pa}\left[\pi(0.400\text{ m})^2\right]}} = 49.81\text{ cm}$$

With the man of mass M on the piston, a very similar calculation (replacing m_p by $m_p + M$) gives:

$$h' = \frac{h_0}{1 + \frac{(m_p+M)g}{P_0 A}} = \frac{50.0\text{ cm}}{1 + \frac{95.0\text{ kg}(9.80\text{ m/s}^2)}{1.013\times10^5\text{ Pa}\left[\pi(0.400\text{ m})^2\right]}} = 49.10\text{ cm}$$

continued on next page

Thus, when the man steps on the piston, it moves downward by

$$\Delta h = h_i - h' = 49.81\text{ cm} - 49.10\text{ cm} = 0.706\text{ cm} = \boxed{7.06\text{ mm}}.$$

(b) $P = \text{const}$, so $\dfrac{V}{T} = \dfrac{V'}{T_i}$ or $\dfrac{Ah_i}{T} = \dfrac{Ah'}{T_i}$

giving $T = T_i\left(\dfrac{h_i}{h'}\right) = 293\text{ K}\left(\dfrac{49.81}{49.10}\right) = \boxed{297\text{ K}}$ (or 24°C)

P16.59 Some gas will pass through the porous plug from the reaction chamber 1 to the reservoir 2 as the reaction chamber is heated, but the net quantity of gas stays constant according to $n_{i1} + n_{i2} = n_{f1} + n_{f2}$. Assuming the gas is ideal, we apply $n = \dfrac{PV}{RT}$ to each term:

$$\frac{P_iV_0}{(300\text{ K})R} + \frac{P_i(4V_0)}{(300\text{ K})R} = \frac{P_fV_0}{(673\text{ K})R} + \frac{P_f(4V_0)}{(300\text{ K})R}$$

$$1\text{ atm}\left(\frac{5}{300\text{ K}}\right) = P_f\left(\frac{1}{673\text{ K}} + \frac{4}{300\text{ K}}\right) \qquad \boxed{P_f = 1.12\text{ atm}}$$

P16.60 Let 2θ represent the angle the curved rail subtends. We have

$$L_i + \Delta L = 2\theta R = L_i(1 + \alpha\Delta T)$$

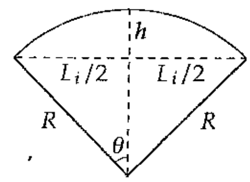

FIG. P16.60

and $\sin\theta = \dfrac{\frac{L_i}{2}}{R} = \dfrac{L_i}{2R}$

Thus, $\theta = \dfrac{L_i}{2R}(1 + \alpha\Delta T) = (1 + \alpha\Delta T)\sin\theta$

and we must solve the transcendental equation $\theta = (1 + \alpha\Delta T)\sin\theta = (1.000\,005\,5)\sin\theta$

Homing in on the non-zero solution gives, to four digits, $\theta = 0.018\,16\text{ rad} = 1.040\,5°$

Now, $h = R - R\cos\theta = \dfrac{L_i(1 - \cos\theta)}{2\sin\theta}$

This yields $\boxed{h = 4.54\text{ m}}$, a remarkably large value compared to $\Delta L = 5.50\text{ cm}$.

***P16.61** (a) Let xL represent the distance of the stationary line below the top edge of the plate. The normal force on the lower part of the plate is $mg(1-x)\cos\theta$ and the force of kinetic friction on it is $\mu_k mg(1-x)\cos\theta$ up the roof. Again, $\mu_k mgx\cos\theta$ acts down the roof on the upper part of the plate. The near-equilibrium of the plate requires $\sum F_x = 0$

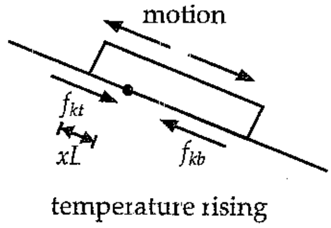

FIG. P16.61(a)

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$$-\mu_k mgx\cos\theta + \mu_k mg(1-x)\cos\theta - mg\sin\theta = 0$$
$$-2\mu_k mgx\cos\theta = mg\sin\theta - \mu_k mg\cos\theta$$
$$2\mu_k x = \mu_k - \tan\theta$$
$$x = \frac{1}{2} - \frac{\tan\theta}{2\mu_k}$$

and the stationary line is indeed below the top edge by $xL = \frac{L}{2}\left(1 - \frac{\tan\theta}{\mu_k}\right)$

(b) With the temperature falling, the plate contracts faster than the roof. The upper part slides down and feels an upward frictional force $\mu_k mg(1-x)\cos\theta$. The lower part slides up and feels downward frictional force $\mu_k mgx\cos\theta$. The equation $\sum F_x = 0$ is then the same as in part (a) and the stationary line is above the bottom edge by $xL = \frac{L}{2}\left(1 - \frac{\tan\theta}{\mu_k}\right)$

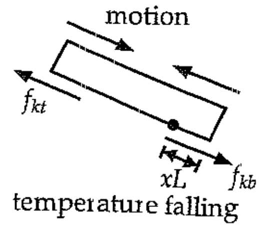

FIG. P16.61(b)

(c) Start thinking about the plate at dawn, as the temperature starts to rise. As in part (a), a line at distance xL below the top edge of the plate stays stationary relative to the roof as long as the temperature rises. The point P on the plate at distance xL above the bottom edge is destined to become the fixed point when the temperature starts falling. As the temperature rises, this point moves down the roof because of the expansion of the central part of the plate. Its displacement for the day is

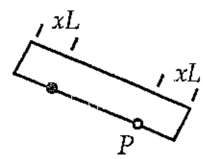

FIG. P16.61(c)

$$\Delta L = (\alpha_2 - \alpha_1)(L - xL - xL)\Delta T$$
$$= (\alpha_2 - \alpha_1)\left[L - 2\frac{L}{2}\left(1 - \frac{\tan\theta}{\mu_k}\right)\right](T_h - T_c)$$
$$= (\alpha_2 - \alpha_1)\left(\frac{L\tan\theta}{\mu_k}\right)(T_h - T_c)$$

At dawn the next day the point P is farther down the roof by the distance ΔL. It represents the displacement of every other point on the plate.

(d) $$(\alpha_2 - \alpha_1)\left(\frac{L\tan\theta}{\mu_k}\right)(T_h - T_c) = \left(24\times10^{-6}\frac{1}{\text{C}^\circ} - 15\times10^{-6}\frac{1}{\text{C}^\circ}\right)\frac{1.20\text{ m}\tan 18.5^\circ}{0.42}32^\circ\text{C} = \boxed{0.275\text{ mm}}$$

(e) If $\alpha_2 < \alpha_1$, the diagram in part (a) applies to temperature falling and the diagram in part (b) applies to temperature rising. The weight of the plate still pulls it step by step down the roof. The same expression describes how far it moves each day.

P16.62 The pressure of the gas in the lungs of the diver must be the same as the absolute pressure of the water at this depth of 50.0 meters. This is:

$$P = P_0 + \rho g h = 1.00 \text{ atm} + (1.03 \times 10^3 \text{ kg/m}^3)(9.80 \text{ m/s}^2)(50.0 \text{ m})$$

or $$P = 1.00 \text{ atm} + 5.05 \times 10^5 \text{ Pa}\left(\frac{1.00 \text{ atm}}{1.013 \times 10^5 \text{ Pa}}\right) = 5.98 \text{ atm}$$

If the partial pressure due to the oxygen in the gas mixture is to be 1.00 atmosphere (or the fraction $\frac{1}{5.98}$ of the total pressure) oxygen molecules should make up only $\frac{1}{5.98}$ of the total number of molecules. This will be true if 1.00 mole of oxygen is used for every 4.98 mole of helium. The ratio by weight is then

$$\frac{(4.98 \text{ mol He})(4.003 \text{ g/mol He})g}{(1.00 \text{ mol } O_2)(2 \times 15.999 \text{ g/mol } O_2)g} = \boxed{0.623}.$$

P16.63 $$N_v(v) = 4\pi N\left(\frac{m_0}{2\pi k_B T}\right)^{3/2} v^2 \exp\left(\frac{-m_0 v^2}{2k_B T}\right)$$

Note that $$v_{\text{mp}} = \left(\frac{2k_B T}{m_0}\right)^{1/2}$$

Thus, $$N_v(v) = 4\pi N\left(\frac{m_0}{2\pi k_B T}\right)^{3/2} v^2 e^{\left(-v^2/v_{\text{mp}}^2\right)}$$

And $$\frac{N_v(v)}{N_v(v_{\text{mp}})} = \left(\frac{v}{v_{\text{mp}}}\right)^2 e^{\left(1-v^2/v_{\text{mp}}^2\right)}$$

For $$v = \frac{v_{\text{mp}}}{50}$$

$$\frac{N_v(v)}{N_v(v_{\text{mp}})} = \left(\frac{1}{50}\right)^2 e^{\left[1-(1/50)^2\right]} = 1.09 \times 10^{-3}$$

The other values are computed similarly, with the following results:

$\frac{v}{v_{\text{mp}}}$	$\frac{N_v(v)}{N_v(v_{\text{mp}})}$
$\frac{1}{50}$	1.09×10^{-3}
$\frac{1}{10}$	2.69×10^{-2}
$\frac{1}{2}$	0.529
1	1.00
2	0.199
10	1.01×10^{-41}
50	$1.25 \times 10^{-1\,082}$

To find the last value, note:

$$(50)^2 e^{1-2\,500} = 2\,500 e^{-2\,499}$$

$$10^{\log 2\,500} e^{(\ln 10)(-2\,499/\ln 10)} = 10^{\log 2\,500} 10^{-2\,499/\ln 10} = 10^{\log 2\,500 - 2\,499/\ln 10} = 10^{-1\,081.904}$$

P16.64 (a) Maxwell's speed distribution function is

$$N_v = 4\pi N\left(\frac{m_0}{2\pi k_B T}\right)^{3/2} v^2 e^{-m_0 v^2/2k_B T}$$

With $N = 1.00\times10^4$,

$$m_0 = \frac{M}{N_A} = \frac{0.032 \text{ kg}}{6.02\times10^{23}} = 5.32\times10^{-26} \text{ kg}$$

$$T = 500 \text{ K}$$

and $k_B = 1.38\times10^{-23}$ J/molecule K

this becomes $N_v = \left(1.71\times10^{-4}\right)v^2 e^{-\left(3.85\times10^{-6}\right)v^2}$

To the right is a plot of this function for the range $0 \le v \le 1\,500$ m/s.

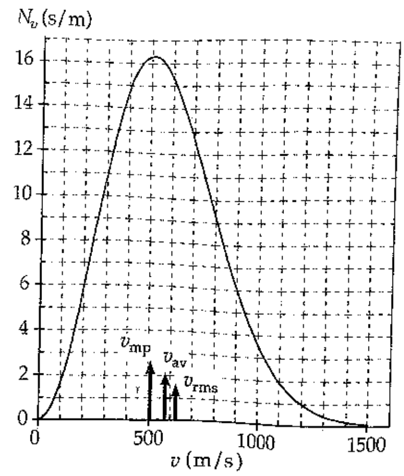

FIG. P16.64(a)

(b) The most probable speed occurs where N_v is a maximum.

From the graph, $\boxed{v_{mp} \approx 510 \text{ m/s}}$

(c) $$v_{av} = \sqrt{\frac{8k_BT}{\pi m_0}} = \sqrt{\frac{8\left(1.38\times10^{-23}\right)(500)}{\pi\left(5.32\times10^{-26}\right)}} = \boxed{575 \text{ m/s}}$$

Also,

$$v_{rms} = \sqrt{\frac{3k_BT}{m_0}} = \sqrt{\frac{3\left(1.38\times10^{-23}\right)(500)}{5.32\times10^{-26}}} = \boxed{624 \text{ m/s}}$$

(d) The fraction of particles in the range $300 \text{ m/s} \le v \le 600 \text{ m/s}$

is $$\frac{\int_{300}^{600} N_v dv}{N}$$

where $N = 10^4$

and the integral of N_v is read from the graph as the area under the curve.

This is approximately 4 400 and the fraction is 0.44 or $\boxed{44\%}$.

ANSWERS TO EVEN PROBLEMS

P16.2 (a) 37.0°C, 310 K; (b) −20.6°C, 253 K

P16.4 (a) 810°F; (b) 450 K

P16.6 55.0°C

P16.8 2.17×10^{-5} $(°\text{C})^{-1}$