CC5.3 (a) The power balance equation for the planet as a whole is

$$\mathscr{P}_{\text{absorbed}} = \mathscr{P}_{\text{emitted}}: \quad e_{vis} I_s A_{\text{absorbing}} = e_{ir} \sigma A_{\text{emitting}} T_1^4$$

$$0.70(1\ 370\ \text{W/m}^2)\pi R_E^2 = 1(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)(4\pi R_E^2)T_1^4$$

$$T_1 = \sqrt[4]{\frac{0.70(1370\ \text{W/m}^2)}{4(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)}} = \boxed{255\ \text{K}}$$

(b) The power balance equation for layer 1, required for the constancy of its temperature, is $e_{ir}\sigma(8\pi R_E^2)T_1^4 = e_{ir}\sigma(4\pi R_E^2)T_2^4$ or $2T_1^4 = T_2^4$. Similarly, for layer 2, $2T_2^4 = T_1^4 + T_3^4$. Again, $2T_3^4 = T_2^4 + T_4^4, \ldots\ 2T_N^4 = T_{N-1}^4 + T_S^4$. Thus, $T_2 = \sqrt[4]{2}\,T_1$.

By substitution, $4T_1^4 = T_1^4 + T_3^4$ $\quad T_3 = \sqrt[4]{3}\,T_1$

$6T_1^4 = 2T_1^4 + T_4^4$ $\quad T_4 = \sqrt[4]{4}\,T_1, \ldots\ T_N = \sqrt[4]{N}\,T_1$

and $T_S = (N+1)^{1/4} T_1$

(c) From part (b), $T_S = (2+1)^{1/4}(255\ \text{K}) = \boxed{336\ \text{K}}$.

(d) The troposphere and stratosphere are too thick to be accurately modeled as having uniform temperatures.

(e) Venus is at 1.08×10^{11} m from the Sun, instead of 1.496×10^{11} m like the Earth. The intensity of solar radiation incident on Venus is then $(1\ 370\ \text{W/m}^2)\left(\frac{1.496}{1.08}\right)^2 = 2.63\times10^3\ \text{W/m}^2$. The overall power balance equation for Venus is $e_{vis} I_s A_{\text{absorbing}} = e_{ir}\sigma A_{\text{emitting}} T_1^4$

$$(1-0.77)(2.63\times10^3\ \text{W/m}^2)(\pi R_V^2) = 1(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)(4\pi R_V^2)T_1^4$$

$$T_1 = \sqrt[4]{\frac{0.23(2.63\times10^3\ \text{W/m}^2)}{4(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)}} = \boxed{227\ \text{K}}$$

(f) $T_S = (N+1)^{1/4} T_1$. $\quad N = \left(\frac{T_S}{T_1}\right)^4 - 1 = \left(\frac{732}{227}\right)^4 - 1 = \boxed{107}$

(g) The multi-layer model should be better for Venus than for Earth. There are many layers, so the temperature of each can be reasonably uniform.

ANSWERS TO EVEN CONTEXT 5 CONCLUSION PROBLEMS

CC5.2 60 km

Chapter 19 Electric Forces and Electric Fields

CHAPTER OUTLINE

19.1 Historical Overview
19.2 Properties of Electric Charges
19.3 Insulators and Conductors
19.4 Coulomb's Law
19.5 Electric Fields
19.6 Electric Field Lines
19.7 Motion of Charged Particles in a Uniform Electric Field
19.8 Electric Flux
19.9 Gauss's Law
19.10 Application of Gauss's Law to Symmetric Charge Distributions
19.11 Conductors in Electrostatic Equilibrium
19.12 Context Connection—The Atmospheric Electric Field

ANSWERS TO QUESTIONS

Q19.1 A neutral atom is one that has no net charge. This means that it has the same number of electrons orbiting the nucleus as it has protons in the nucleus. A negatively charged atom has one or more excess electrons.

Q19.2 The clothes dryer rubs dissimilar materials together as it tumbles the clothes. Electrons are transferred from one kind of molecule to another. The charges on pieces of cloth, or on nearby objects charged by induction, can produce strong electric fields that promote the ionization process in the surrounding air that is necessary for a spark to occur. Then you hear or see the sparks.

Q19.3 To avoid making a spark. Rubber-soled shoes acquire a charge by friction with the floor and could discharge with a spark, possibly causing an explosion of any flammable material in the oxygen-enriched atmosphere.

Q19.4 Similarities: A force of gravity is proportional to the product of the intrinsic properties (masses) of two particles, and inversely proportional to the square of the separation distance. An electrical force exhibits the same proportionalities, with charge as the intrinsic property.

Differences: The electrical force can either attract or repel, while the gravitational force as described by Newton's law can only attract. The electrical force between elementary particles is vastly stronger than the gravitational force.

Q19.5 No. The balloon induces polarization of the molecules in the wall, so that a layer of positive charge exists near the balloon. This is just like the situation in Figure 19.5a, except that the signs of the charges are reversed. The attraction between these charges and the negative charges on the balloon is stronger than the repulsion between the negative charges on the balloon and the negative charges in the polarized molecules (because they are farther from the balloon), so that there is a net attractive force toward the wall. Ionization processes in the air surrounding the balloon provide ions to which excess electrons in the balloon can transfer, reducing the charge on the balloon and eventually causing the attractive force to be insufficient to support the weight of the balloon.

Q19.6 An electric field once established by a positive or negative charge extends in all directions from the charge. Thus, it can exist in empty space if that is what surrounds the charge. There is no material at point A in Figure 19.18(a), so there is no charge, nor is there a force. There would be a force if a charge were present at point A, however. A field does exist at point A.

Q19.7 If a charge distribution is small compared to the distance of a field point from it, the charge distribution can be modeled as a single particle with charge equal to the net charge of the distribution. Further, if a charge distribution is spherically symmetric, it will create a field at exterior points just as if all of its charge were a point charge at its center.

Q19.8 Both figures are drawn correctly. $\vec{E}_1$ and $\vec{E}_2$ are the electric fields separately created by the point charges q and $-q$ in Figure 19.11. The net electric field is the vector sum of $\vec{E}_1$ and $\vec{E}_2$, shown as $\vec{E}$. Figure 19.16 shows only one electric field line at each point away from the charge. At the point location of an object modeled as a point charge, the direction of the field is undefined, and so is its magnitude.

Q19.9 No. Life would be no different if electrons were + charged and protons were − charged. Opposite charges would still attract, and like charges would repel. The naming of + and − charge is merely a convention.

Q19.10 At a point exactly midway between the two charges.

Q19.11 In special orientations the force between two dipoles can be zero or a force of repulsion. In general each dipole will exert a torque on the other, tending to align its axis with the field created by the first dipole. After this alignment, each dipole exerts a force of attraction on the other.

Q19.12 The negative charge will be drawn to the center of the positively charged ring. Since it will then have velocity, it will continue on, to an equidistant point on the opposite side of the ring. It will then start moving back and arrive again at point P. This periodic motion will continue. If x is much less than a, the motion can be shown from the solution for the electric field to be simple harmonic.

Q19.13 The surface must enclose a positive total charge.

Q19.14 The net flux through any gaussian surface is zero. We can argue it two ways. Any surface contains zero charge so Gauss's law says the total flux is zero. The field is uniform, so the field lines entering one side of the closed surface come out the other side and the net flux is zero.

Q19.15 Gauss's law cannot tell the different values of the electric field at different points on the surface. When E is an unknown number, then we can say $\int E\cos\theta\, dA = E\int \cos\theta\, dA$. When $E(x, y, z)$ is an unknown function, then there is no such simplification.

Q19.16 Inject some charge at arbitrary places within a conducting object. Every bit of the charge repels every other bit, so each bit runs away as far as it can, stopping only when it reaches the outer surface of the conductor.

Q19.17 If the person is uncharged, the electric field inside the sphere is zero. The interior wall of the shell carries no charge. The person is not harmed by touching this wall. If the person carries a (small) charge q, the electric field inside the sphere is no longer zero. Charge $-q$ is induced on the inner wall of the sphere. The person will get a (small) shock when touching the sphere, as all the charge on his body jumps to the metal.

Q19.18 The electric fields outside are identical. The electric fields inside are very different. We have $\vec{E}=0$ everywhere inside the conducting sphere while E decreases gradually as you go below the surface of the sphere with uniform volume charge density.

Q19.19 There is zero force. The huge charged sheet creates a uniform field. The field can polarize the neutral sheet, creating in effect a film of opposite charge on the near face and a film with an equal amount of like charge on the far face of the neutral sheet. Since the field is uniform, the films of charge feel equal-magnitude forces of attraction and repulsion to the charged sheet. The forces add to zero.

SOLUTIONS TO PROBLEMS

Section 19.1 Historical Overview

No problems in this section

Section 19.2 Properties of Electric Charges

***P19.1** (a) The mass of an average neutral hydrogen atom is 1.007 9u. Losing one electron reduces its mass by a negligible amount, to

$$1.007\,9\left(1.660\times10^{-27}\text{ kg}\right)-9.11\times10^{-31}\text{ kg}=\boxed{1.67\times10^{-27}\text{ kg}}.$$

Its charge, due to loss of one electron, is

$$0-1\left(-1.60\times10^{-19}\text{ C}\right)=\boxed{+1.60\times10^{-19}\text{ C}}.$$

(b) By similar logic, charge $=\boxed{+1.60\times10^{-19}\text{ C}}$

$$\text{mass}=22.99\left(1.66\times10^{-27}\text{ kg}\right)-9.11\times10^{-31}\text{ kg}=\boxed{3.82\times10^{-26}\text{ kg}}$$

(c) charge of $Cl^-=\boxed{-1.60\times10^{-19}\text{ C}}$

$$\text{mass}=35.453\left(1.66\times10^{-27}\text{ kg}\right)+9.11\times10^{-31}\text{ kg}=\boxed{5.89\times10^{-26}\text{ kg}}$$

(d) charge of $Ca^{++}=-2\left(-1.60\times10^{-19}\text{ C}\right)=\boxed{+3.20\times10^{-19}\text{ C}}$

$$\text{mass}=40.078\left(1.66\times10^{-27}\text{ kg}\right)-2\left(9.11\times10^{-31}\text{ kg}\right)=\boxed{6.65\times10^{-26}\text{ kg}}$$

(e) charge of $N^{3-}=3\left(-1.60\times10^{-19}\text{ C}\right)=\boxed{-4.80\times10^{-19}\text{ C}}$

$$\text{mass}=14.007\left(1.66\times10^{-27}\text{ kg}\right)+3\left(9.11\times10^{-31}\text{ kg}\right)=\boxed{2.33\times10^{-26}\text{ kg}}$$

continued on next page

(f) charge of $N^{4+} = 4(1.60\times10^{-19}\ \text{C}) = \boxed{+6.40\times10^{-19}\ \text{C}}$

mass $= 14.007(1.66\times10^{-27}\ \text{kg}) - 4(9.11\times10^{-31}\ \text{kg}) = \boxed{2.32\times10^{-26}\ \text{kg}}$

(g) We think of a nitrogen nucleus as a seven-times ionized nitrogen atom.

charge $= 7(1.60\times10^{-19}\ \text{C}) = \boxed{1.12\times10^{-18}\ \text{C}}$

mass $= 14.007(1.66\times10^{-27}\ \text{kg}) - 7(9.11\times10^{-31}\ \text{kg}) = \boxed{2.32\times10^{-26}\ \text{kg}}$

(h) charge $= \boxed{-1.60\times10^{-19}\ \text{C}}$

mass $= [2(1.0079)+15.999]1.66\times10^{-27}\ \text{kg} + 9.11\times10^{-31}\ \text{kg} = \boxed{2.99\times10^{-26}\ \text{kg}}$

P19.2 (a) $N = \left(\dfrac{10.0\ \text{grams}}{107.87\ \text{grams/mol}}\right)\left(6.02\times10^{23}\ \dfrac{\text{atoms}}{\text{mol}}\right)\left(47\ \dfrac{\text{electrons}}{\text{atom}}\right) = \boxed{2.62\times10^{24}}$

(b) $\#\ \text{electrons added} = \dfrac{Q}{e} = \dfrac{1.00\times10^{-3}\ \text{C}}{1.60\times10^{-19}\ \text{C/electron}} = 6.25\times10^{15}$

or $\boxed{2.38\ \text{electrons for every}\ 10^9\ \text{already present}}$.

Section 19.3 Insulators and Conductors

No problems in this section

Section 19.4 Coulomb's Law

P19.3 If each person has a mass of ≈ 70 kg and is (almost) composed of water, then each person contains

$$N = \left(\frac{70\,000\ \text{grams}}{18\ \text{grams/mol}}\right)\left(6.02\times10^{23}\ \frac{\text{molecules}}{\text{mol}}\right)\left(10\ \frac{\text{protons}}{\text{molecule}}\right) = 2.3\times10^{28}\ \text{protons}.$$

With an excess of 1% electrons over protons, each person has a charge

$$q = 0.01(1.6\times10^{-19}\ \text{C})(2.3\times10^{28}) = 3.7\times10^{7}\ \text{C}.$$

So $F = k_e\dfrac{q_1q_2}{r^2} = (9\times10^9)\dfrac{(3.7\times10^7)^2}{0.6^2}\ \text{N} = 4\times10^{25}\ \text{N}\ \boxed{\sim10^{26}\ \text{N}}$.

This force is almost enough to lift a weight equal to that of the Earth:

$$Mg = 6\times10^{24}\ \text{kg}(9.8\ \text{m/s}^2) = 6\times10^{25}\ \text{N} \sim 10^{26}\ \text{N}.$$

P19.4 The force on one proton is $\vec{\mathbf{F}} = \dfrac{k_eq_1q_2}{r^2}$ away from the other proton. Its magnitude is

$$(8.99\times10^9\ \text{N}\cdot\text{m/C}^2)\left(\frac{1.6\times10^{-19}\ \text{C}}{2\times10^{-15}\ \text{m}}\right)^2 = \boxed{57.5\ \text{N}}.$$

P19.5 $F_1 = k_e\dfrac{q_1q_2}{r^2} = \dfrac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(7.00\times10^{-6}\ \text{C})(2.00\times10^{-6}\ \text{C})}{(0.500\ \text{m})^2} = 0.503\ \text{N}$

$F_2 = k_e\dfrac{q_1q_2}{r^2} = \dfrac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(7.00\times10^{-6}\ \text{C})(4.00\times10^{-6}\ \text{C})}{(0.500\ \text{m})^2} = 1.01\ \text{N}$

$F_x = 0.503\cos60.0° + 1.01\cos60.0° = 0.755\ \text{N}$
$F_y = 0.503\sin60.0° - 1.01\sin60.0° = -0.436\ \text{N}$
$\vec{\mathbf{F}} = (0.755\ \text{N})\hat{\mathbf{i}} - (0.436\ \text{N})\hat{\mathbf{j}} = \boxed{0.872\ \text{N at an angle of } 330°}$

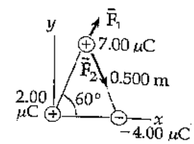

FIG. P19.5

*P19.6 In the first situation, $\vec{\mathbf{F}}_{A\text{ on }B,1} = \dfrac{k_e|q_A||q_B|}{r_1^2}\hat{\mathbf{i}}$. In the second situation, $|q_A|$ and $|q_B|$ are the same.

$$\vec{\mathbf{F}}_{B\text{ on }A,2} = -\vec{\mathbf{F}}_{A\text{ on }B} = \frac{k_e|q_A||q_B|}{r_2^2}(-\hat{\mathbf{i}})$$

$$\frac{F_2}{F_1} = \frac{k_e|q_A||q_B|}{r_2^2}\frac{r_1^2}{k_e|q_A||q_B|}$$

$$F_2 = \frac{F_1r_1^2}{r_2^2} = 2.62\ \mu\text{N}\left(\frac{13.7\ \text{mm}}{17.7\ \text{mm}}\right)^2 = 1.57\ \mu\text{N}$$

Then $\vec{\mathbf{F}}_{B\text{ on }A,2} = \boxed{1.57\ \mu\text{N to the left}}$

P19.7 (a) The force is one of $\boxed{\text{attraction}}$. The distance r in Coulomb's law is the distance between centers. The magnitude of the force is

$$F = \frac{k_eq_1q_2}{r^2} = (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{(12.0\times10^{-9}\ \text{C})(18.0\times10^{-9}\ \text{C})}{(0.300\ \text{m})^2} = \boxed{2.16\times10^{-5}\ \text{N}}.$$

(b) The net charge of -6.00×10^{-9} C will be equally split between the two spheres, or -3.00×10^{-9} C on each. The force is one of $\boxed{\text{repulsion}}$, and its magnitude is

$$F = \frac{k_eq_1q_2}{r^2} = (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{(3.00\times10^{-9}\ \text{C})(3.00\times10^{-9}\ \text{C})}{(0.300\ \text{m})^2} = \boxed{8.99\times10^{-7}\ \text{N}}.$$

*P19.8 Let the third bead have charge Q and be located distance x from the left end of the rod. This bead will experience a net force given by

$$\vec{F}=\frac{k_e(3q)Q}{x^2}\hat{i}+\frac{k_e(q)Q}{(d-x)^2}(-\hat{i}).$$

The net force will be zero if $\frac{3}{x^2}=\frac{1}{(d-x)^2}$, or $d-x=\frac{x}{\sqrt{3}}$.

This gives an equilibrium position of the third bead of $x=\boxed{0.634d}$

The equilibrium is $\boxed{\text{stable if the third bead has positive charge}}$.

P19.9 (a) $F=\frac{k_e e^2}{r^2}=\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\frac{\left(1.60\times10^{-19}\ \text{C}\right)^2}{\left(0.529\times10^{-10}\ \text{m}\right)^2}=\boxed{8.22\times10^{-8}\ \text{N}}$

(b) We have $F=\frac{mv^2}{r}$ from which

$$v=\sqrt{\frac{Fr}{m}}=\sqrt{\frac{8.22\times10^{-8}\ \text{N}\left(0.529\times10^{-10}\ \text{m}\right)}{9.11\times10^{-31}\ \text{kg}}}=\boxed{2.19\times10^6\ \text{m/s}}.$$

Section 19.5 Electric Fields

P19.10 For equilibrium, $\vec{F}_e=-\vec{F}_g$

or $q\vec{E}=-mg(-\hat{j})$.

Thus, $\vec{E}=\frac{mg}{q}\hat{j}$.

(a) $\vec{E}=\frac{mg}{q}\hat{j}=\frac{\left(9.11\times10^{-31}\ \text{kg}\right)\left(9.80\ \text{m/s}^2\right)}{\left(-1.60\times10^{-19}\ \text{C}\right)}\hat{j}=\boxed{-\left(5.58\times10^{-11}\ \text{N/C}\right)\hat{j}}$

(b) $\vec{E}=\frac{mg}{q}\hat{j}=\frac{\left(1.67\times10^{-27}\ \text{kg}\right)\left(9.80\ \text{m/s}^2\right)}{\left(1.60\times10^{-19}\ \text{C}\right)}\hat{j}=\boxed{\left(1.02\times10^{-7}\ \text{N/C}\right)\hat{j}}$

P19.11 The point is designated in the sketch. The magnitudes of the electric fields, E_1, (due to the -2.50×10^{-6} C charge) and E_2 (due to the 6.00×10^{-6} C charge) are

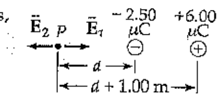

FIG. P19.11

$$E_1=\frac{k_eq}{r^2}=\frac{\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(2.50\times10^{-6}\ \text{C}\right)}{d^2}\qquad(1)$$

$$E_2=\frac{k_eq}{r^2}=\frac{\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(6.00\times10^{-6}\ \text{C}\right)}{(d+1.00\ \text{m})^2}\qquad(2)$$

Equate the right sides of (1) and (2)

to get $(d+1.00\ \text{m})^2=2.40d^2$

or $d+1.00\ \text{m}=\pm1.55d$

which yields $d=1.82$ m

or $d=-0.392$ m.

The negative value for d is unsatisfactory because that locates a point between the charges where both fields are in the same direction.

Thus, $d=\boxed{1.82\ \text{m to the left of the }-2.50\ \mu\text{C charge}}$.

*P19.12 The first charge creates at the origin field $\frac{k_eQ}{a^2}$ to the right.

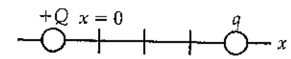

FIG. P19.12

Suppose the total field at the origin is to the right. Then q must be negative:

$$\frac{k_eQ}{a^2}\hat{i}+\frac{k_eq}{(3a)^2}(-\hat{i})=\frac{2k_eQ}{a^2}\hat{i}\qquad\boxed{q=-9Q}.$$

In the alternative, the total field at the origin is to the left:

$$\frac{k_eQ}{a^2}\hat{i}+\frac{k_eq}{9a^2}(-\hat{i})=\frac{2k_eQ}{a^2}(-\hat{i})\qquad\boxed{q=+27Q}.$$

P19.13 (a) $\vec{E}_1=\frac{k_e|q_1|}{r_1^2}(-\hat{j})=\frac{\left(8.99\times10^9\right)\left(3.00\times10^{-9}\right)}{(0.100)^2}(-\hat{j})=-\left(2.70\times10^3\ \text{N/C}\right)\hat{j}$

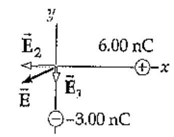

FIG. P19.13

$\vec{E}_2=\frac{k_e|q_2|}{r_2^2}(-\hat{i})=\frac{\left(8.99\times10^9\right)\left(6.00\times10^{-9}\right)}{(0.300)^2}(-\hat{i})=-\left(5.99\times10^2\ \text{N/C}\right)\hat{i}$

$\vec{E}=\vec{E}_2+\vec{E}_1=\boxed{-\left(5.99\times10^2\ \text{N/C}\right)\hat{i}-\left(2.70\times10^3\ \text{N/C}\right)\hat{j}}$

(b) $\vec{F}=q\vec{E}=\left(5.00\times10^{-9}\ \text{C}\right)\left(-599\hat{i}-2\,700\hat{j}\right)$ N/C

$\vec{F}=\left(-3.00\times10^{-6}\hat{i}-13.5\times10^{-6}\hat{j}\right)\text{N}=\boxed{\left(-3.00\hat{i}-13.5\hat{j}\right)\mu\text{N}}$

P19.14 (a) $E=\frac{k_e q}{r^2}=\frac{(8.99\times10^9)(2.00\times10^{-6})}{(1.12)^2}=14\,400\text{ N/C}$

$E_x=0$ and $E_y=2(14\,400)\sin 26.6°=1.29\times10^4\text{ N/C}$

so $\boxed{\vec{\mathbf{E}}=1.29\times10^4\,\hat{\mathbf{j}}\text{ N/C}}$.

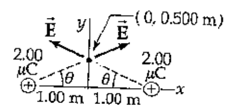

FIG. P19.14

(b) $\vec{\mathbf{F}}=q\vec{\mathbf{E}}=(-3.00\times10^{-6})(1.29\times10^4\,\hat{\mathbf{j}})=\boxed{-3.86\times10^{-2}\,\hat{\mathbf{j}}\text{ N}}$

P19.15 (a) $\vec{\mathbf{E}}=\frac{k_e q_1}{r_1^2}\hat{\mathbf{r}}_1+\frac{k_e q_2}{r_2^2}\hat{\mathbf{r}}_2+\frac{k_e q_3}{r_3^2}\hat{\mathbf{r}}_3=\frac{k_e(2q)}{a^2}\hat{\mathbf{i}}+\frac{k_e(3q)}{2a^2}(\hat{\mathbf{i}}\cos 45.0°+\hat{\mathbf{j}}\sin 45.0°)+\frac{k_e(4q)}{a^2}\hat{\mathbf{j}}$

$\vec{\mathbf{E}}=3.06\frac{k_e q}{a^2}\hat{\mathbf{i}}+5.06\frac{k_e q}{a^2}\hat{\mathbf{j}}=\boxed{5.91\frac{k_e q}{a^2}\text{ at }58.8°}$

(b) $\vec{\mathbf{F}}=q\vec{\mathbf{E}}=\boxed{5.91\frac{k_e q^2}{a^2}\text{ at }58.8°}$

P19.16 The electric field at any point x is

$$E=\frac{k_e q}{(x-a)^2}-\frac{k_e q}{(x-(-a))^2}=\frac{k_e q(4ax)}{(x^2-a^2)^2}.$$

When x is much, much greater than a, we find $E\approx\boxed{\frac{4a(k_e q)}{x^3}}$.

P19.17 $E=\frac{k_e\lambda\ell}{d(\ell+d)}=\frac{k_e(Q/\ell)\ell}{d(\ell+d)}=\frac{k_e Q}{d(\ell+d)}=\frac{(8.99\times10^9)(22.0\times10^{-6})}{(0.290)(0.140+0.290)}$

$\vec{\mathbf{E}}=\boxed{1.59\times10^6\text{ N/C, directed toward the rod.}}$

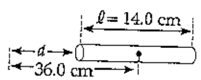

FIG. P19.17

P19.18 $E=\int\frac{k_e\,dq}{x^2}$, where $dq=\lambda_0\,dx$

$E=k_e\lambda_0\int_{x_0}^{\infty}\frac{dx}{x^2}=k_e\lambda_0\left(-\frac{1}{x}\right)\Bigg|_{x_0}^{\infty}=\boxed{\frac{k_e\lambda_0}{x_0}}$ $\boxed{\text{The direction is }-\hat{\mathbf{i}}\text{ or left for }\lambda_0>0}$

P19.19 $E=\frac{k_e xQ}{(x^2+a^2)^{3/2}}=\frac{(8.99\times10^9)(75.0\times10^{-6})x}{(x^2+0.100^2)^{3/2}}=\frac{6.74\times10^5 x}{(x^2+0.010\,0)^{3/2}}$

(a) At $x=0.010\,0$ m, $\vec{\mathbf{E}}=6.64\times10^6\,\hat{\mathbf{i}}\text{ N/C}=\boxed{6.64\hat{\mathbf{i}}\text{ MN/C}}$

(b) At $x=0.050\,0$ m, $\vec{\mathbf{E}}=2.41\times10^7\,\hat{\mathbf{i}}\text{ N/C}=\boxed{24.1\hat{\mathbf{i}}\text{ MN/C}}$

continued on next page

(c) At $x=0.300$ m, $\vec{\mathbf{E}}=6.40\times10^6\,\hat{\mathbf{i}}\text{ N/C}=\boxed{6.40\hat{\mathbf{i}}\text{ MN/C}}$

(d) At $x=1.00$ m, $\vec{\mathbf{E}}=6.64\times10^5\,\hat{\mathbf{i}}\text{ N/C}=\boxed{0.664\hat{\mathbf{i}}\text{ MN/C}}$

***P19.20** $E=\frac{k_e Qx}{(x^2+a^2)^{3/2}}$

For a maximum, $\frac{dE}{dx}=Qk_e\left[\frac{1}{(x^2+a^2)^{3/2}}-\frac{3x^2}{(x^2+a^2)^{5/2}}\right]=0$

$x^2+a^2-3x^2=0$ or $x=\frac{a}{\sqrt{2}}$.

Substituting into the expression for E gives

$$E=\frac{k_e Qa}{\sqrt{2}\left(\frac{3}{2}a^2\right)^{3/2}}=\frac{k_e Q}{3\frac{\sqrt{3}}{2}a^2}=\boxed{\frac{2k_e Q}{3\sqrt{3}a^2}}=\boxed{\frac{Q}{6\sqrt{3}\pi\in_0 a^2}}.$$

P19.21 Due to symmetry $E_y=\int dE_y=0$, and $E_x=\int dE\sin\theta=k_e\int\frac{dq\sin\theta}{r^2}$

where $dq=\lambda ds=\lambda r\,d\theta$,

so that, $E_x=\frac{k_e\lambda}{r}\int_0^{\pi}\sin\theta\,d\theta=\frac{k_e\lambda}{r}(-\cos\theta)\Big|_0^{\pi}=\frac{2k_e\lambda}{r}$

where $\lambda=\frac{q}{L}$ and $r=\frac{L}{\pi}$

Thus, $E_x=\frac{2k_e q\pi}{L^2}=\frac{2(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(7.50\times10^{-6}\text{ C})\pi}{(0.140\text{ m})^2}$

Solving, $E_x=2.16\times10^7$ N/C.

Since the rod has a negative charge, $\vec{\mathbf{E}}=(-2.16\times10^7\,\hat{\mathbf{i}})\text{ N/C}=\boxed{-21.6\hat{\mathbf{i}}\text{ MN/C}}$.

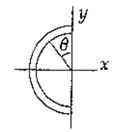

FIG. P19.21

P19.22 (a) The electric field at point P due to each element of length dx is $dE = \frac{k_e dq}{x^2 + y^2}$ and is directed along the line joining the element to point P. By symmetry,

$E_x = \int dE_x = 0$ and since $dq = \lambda dx$,

$E = E_y = \int dE_y = \int dE \cos\theta$ where $\cos\theta = \frac{y}{\sqrt{x^2+y^2}}$.

Therefore, $E = 2k_e \lambda y \int_0^{\ell/2} \frac{dx}{\left(x^2+y^2\right)^{3/2}} = \boxed{\frac{2k_e \lambda \sin\theta_0}{y}}$.

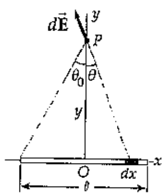

FIG. P19.22

(b) For a bar of infinite length, $\theta_0 = 90°$ and $E_y = \boxed{\frac{2k_e \lambda}{y}}$.

P19.23 (a) The whole surface area of the cylinder is $A = 2\pi r^2 + 2\pi rL = 2\pi r(r+L)$.

$Q = \sigma A = \left(15.0 \times 10^{-9}\ \text{C/m}^2\right) 2\pi (0.025\,0\ \text{m})\left[0.025\,0\ \text{m} + 0.060\,0\ \text{m}\right] = \boxed{2.00 \times 10^{-10}\ \text{C}}$

(b) For the curved lateral surface only, $A = 2\pi rL$.

$Q = \sigma A = \left(15.0 \times 10^{-9}\ \text{C/m}^2\right)\left[2\pi (0.025\,0\ \text{m})(0.060\,0\ \text{m})\right] = \boxed{1.41 \times 10^{-10}\ \text{C}}$

(c) $Q = \rho V = \rho \pi r^2 L = \left(500 \times 10^{-9}\ \text{C/m}^3\right)\left[\pi (0.025\,0\ \text{m})^2 (0.060\,0\ \text{m})\right] = \boxed{5.89 \times 10^{-11}\ \text{C}}$

Section 19.6 Electric Field Lines

***P19.24** (a) $\frac{q_1}{q_2} = \frac{-6}{18} = \boxed{-\frac{1}{3}}$

(b) $\boxed{q_1 \text{ is negative, } q_2 \text{ is positive}}$

P19.25

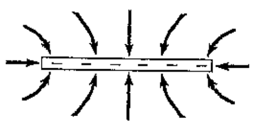

FIG. P19.25

P19.26 (a) The electric field has the general appearance shown. It is zero $\boxed{\text{at the center}}$, where (by symmetry) one can see that the three charges individually produce fields that cancel out.

In addition to the center of the triangle, the electric field lines in the second figure to the right indicate three other points near the middle of each leg of the triangle where $E = 0$, but they are more difficult to find mathematically.

(b) You may need to review vector addition in Chapter One. The electric field at point P can be found by adding the electric field vectors due to each of the two lower point charges: $\vec{E} = \vec{E}_1 + \vec{E}_2$.

The electric field from a point charge is $\vec{E} = k_e \frac{q}{r^2}\hat{r}$.

As shown in the solution figure at right,

$\vec{E}_1 = k_e \frac{q}{a^2}$ to the right and upward at 60°

$\vec{E}_2 = k_e \frac{q}{a^2}$ to the left and upward at 60°

$\vec{E} = \vec{E}_1 + \vec{E}_2 = k_e \frac{q}{a^2}\left[\left(\cos 60°\,\hat{i} + \sin 60°\,\hat{j}\right) + \left(-\cos 60°\,\hat{i} + \sin 60°\,\hat{j}\right)\right] = k_e \frac{q}{a^2}\left[2\left(\sin 60°\,\hat{j}\right)\right]$

$= \boxed{1.73 k_e \frac{q}{a^2}\hat{j}}$

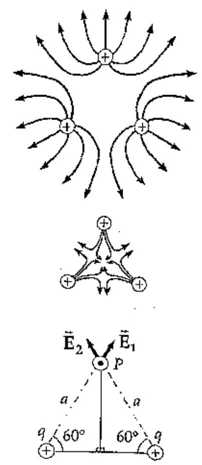

FIG. P19.26

Section 19.7 Motion of Charged Particles in a Uniform Electric Field

P19.27 (a) $a = \frac{qE}{m} = \frac{1.60 \times 10^{-19}(640)}{1.67 \times 10^{-27}} = \boxed{6.13 \times 10^{10}\ \text{m/s}^2}$

(b) $v_f = v_i + at$ $\quad 1.20 \times 10^6 = \left(6.13 \times 10^{10}\right)t$ $\quad t = \boxed{1.95 \times 10^{-5}\ \text{s}}$

(c) $x_f - x_i = \frac{1}{2}\left(v_i + v_f\right)t$ $\quad x_f = \frac{1}{2}\left(1.20 \times 10^6\right)\left(1.95 \times 10^{-5}\right) = \boxed{11.7\ \text{m}}$

(d) $K = \frac{1}{2}mv^2 = \frac{1}{2}\left(1.67 \times 10^{-27}\ \text{kg}\right)\left(1.20 \times 10^6\ \text{m/s}\right)^2 = \boxed{1.20 \times 10^{-15}\ \text{J}}$

P19.28 The required electric field will be $\boxed{\text{in the direction of motion}}$.

Work done $= \Delta K$

so, $-Fd = -\frac{1}{2}mv_i^2$ (since the final velocity $= 0$)

which becomes $eEd = K$

and $E = \boxed{\frac{K}{ed}}$.

P19.29 (a) $t = \frac{x}{v_x} = \frac{0.0500}{4.50\times10^5} = 1.11\times10^{-7}\text{ s} = \boxed{111\text{ ns}}$

(b) $a_y = \frac{qE}{m} = \frac{(1.602\times10^{-19})(9.60\times10^3)}{(1.67\times10^{-27})} = 9.21\times10^{11}\text{ m/s}^2$

$y_f - y_i = v_{yi}t + \frac{1}{2}a_y t^2$ $\quad y_f = \frac{1}{2}(9.21\times10^{11})(1.11\times10^{-7})^2 = 5.68\times10^{-3}\text{ m} = \boxed{5.68\text{ mm}}$

(c) $v_x = \boxed{4.50\times10^5\text{ m/s}}$ $\quad v_{yf} = v_{yi} + a_y t = (9.21\times10^{11})(1.11\times10^{-7}) = \boxed{1.02\times10^5\text{ m/s}}$

Section 19.8 Electric Flux

P19.30 $\Phi_E = EA\cos\theta = (2.00\times10^4\text{ N/C})(18.0\text{ m}^2)\cos 10.0° = \boxed{355\text{ kN}\cdot\text{m}^2/\text{C}}$

P19.31 $\Phi_E = EA\cos\theta$ $\quad A = \pi r^2 = \pi(0.200)^2 = 0.126\text{ m}^2$

$5.20\times10^5 = E(0.126)\cos 0°$ $\quad E = 4.14\times10^6\text{ N/C} = \boxed{4.14\text{ MN/C}}$

Section 19.9 Gauss's Law

P19.32 (a) $E = \frac{k_e Q}{r^2}$ $\quad 8.90\times10^2 = \frac{(8.99\times10^9)Q}{(0.750)^2}$

But Q is negative since $\vec{E}$ points inward. $\quad Q = -5.57\times10^{-8}\text{ C} = \boxed{-55.7\text{ nC}}$

(b) The $\boxed{\text{negative}}$ charge has a $\boxed{\text{spherically symmetric}}$ charge distribution, concentric with the spherical shell.

P19.33 (a) With δ very small, all points on the hemisphere are nearly at a distance R from the charge, so the field everywhere on the curved surface is $\frac{k_e Q}{R^2}$ radially outward (normal to the surface). Therefore, the flux is this field strength times the area of half a sphere:

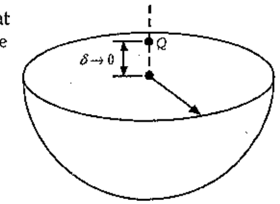

FIG. P19.33

$\Phi_{\text{curved}} = \int \vec{E}\cdot d\vec{A} = E_{\text{local}}A_{\text{hemisphere}}$

$\Phi_{\text{curved}} = \left(k_e \frac{Q}{R^2}\right)\left(\frac{1}{2}4\pi R^2\right) = \frac{1}{4\pi\epsilon_0}Q(2\pi) = \boxed{\frac{+Q}{2\epsilon_0}}$

(b) The closed surface encloses zero charge so Gauss's law gives

$\Phi_{\text{curved}} + \Phi_{\text{flat}} = 0$ or $\Phi_{\text{flat}} = -\Phi_{\text{curved}} = \boxed{\frac{-Q}{2\epsilon_0}}$.

P19.34 $\Phi_E = \frac{q_{\text{in}}}{\epsilon_0} = \frac{170\times10^{-6}\text{ C}}{8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2} = 1.92\times10^7\text{ N}\cdot\text{m}^2/\text{C}$

(a) $(\Phi_E)_{\text{one face}} = \frac{1}{6}\Phi_E = \frac{1.92\times10^7\text{ N}\cdot\text{m}^2/\text{C}}{6}$ $\quad (\Phi_E)_{\text{one face}} = \boxed{3.20\text{ MN}\cdot\text{m}^2/\text{C}}$

(b) $\Phi_E = \boxed{19.2\text{ MN}\cdot\text{m}^2/\text{C}}$

(c) The answer to (a) would change because the flux through each face of the cube would not be equal with an asymmetric charge distribution. The sides of the cube nearer the charge would have more flux and the ones further away would have less. The answer to (b) would remain the same, since the overall flux would remain the same.

Section 19.10 Application of Gauss's Law to Symmetric Charge Distributions

P19.35 (a) $E = \frac{k_e Qr}{a^3} = \boxed{0}$

(b) $E = \frac{k_e Qr}{a^3} = \frac{(8.99\times10^9)(26.0\times10^{-6})(0.100)}{(0.400)^3} = \boxed{365\text{ kN/C}}$

(c) $E = \frac{k_e Q}{r^2} = \frac{(8.99\times10^9)(26.0\times10^{-6})}{(0.400)^2} = \boxed{1.46\text{ MN/C}}$

(d) $E = \frac{k_e Q}{r^2} = \frac{(8.99\times10^9)(26.0\times10^{-6})}{(0.600)^2} = \boxed{649\text{ kN/C}}$

The direction for each electric field is $\boxed{\text{radially outward}}$.

*P19.36 $mg = qE = q\left(\frac{\sigma}{2\epsilon_0}\right) = q\left(\frac{Q/A}{2\epsilon_0}\right)$ $\quad \frac{Q}{A} = \frac{2\epsilon_0 mg}{q} = \frac{2(8.85\times10^{-12})(0.01)(9.8)}{-0.7\times10^{-6}} = \boxed{-2.48\ \mu\text{C/m}^2}$

P19.37 (a) $E = \frac{2k_e\lambda}{r}$ $\quad 3.60\times10^4 = \frac{2(8.99\times10^9)(Q/2.40)}{0.190}$

$Q = +9.13\times10^{-7}\ \text{C} = \boxed{+913\ \text{nC}}$

(b) $\vec{E} = \boxed{0}$

P19.38 (a) $\vec{E} = \boxed{0}$

(b) $E = \frac{k_eQ}{r^2} = \frac{(8.99\times10^9)(32.0\times10^{-6})}{(0.200)^2} = 7.19\ \text{MN/C}$ $\quad \vec{E} = \boxed{7.19\ \text{MN/C radially outward}}$

P19.39 If ρ is positive, the field must be radially outward. Choose as the gaussian surface a cylinder of length L and radius r, contained inside the charged rod. Its volume is $\pi r^2 L$ and it encloses charge $\rho\pi r^2 L$. Because the charge distribution is long, no electric flux passes through the circular end caps; $\vec{E}\cdot d\vec{A} = EdA\cos 90.0° = 0$. The curved surface has $\vec{E}\cdot d\vec{A} = EdA\cos 0°$, and E must be the same strength everywhere over the curved surface.

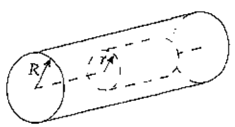

FIG. P19.39

Gauss's law, $\oint \vec{E}\cdot d\vec{A} = \frac{q}{\epsilon_0}$, becomes $E \int_{\substack{\text{Curved}\\ \text{Surface}}} dA = \frac{\rho\pi r^2 L}{\epsilon_0}$

Now the lateral surface area of the cylinder is $2\pi rL$:

$E(2\pi r)L = \frac{\rho\pi r^2 L}{\epsilon_0}$ $\quad$ Thus, $\vec{E} = \boxed{\frac{\rho r}{2\epsilon_0}\ \text{radially away from the cylinder axis}}$.

*P19.40 The charge density is determined by $Q = \frac{4}{3}\pi a^3 \rho$ $\quad \rho = \frac{3Q}{4\pi a^3}$

(a) The flux is that created by the enclosed charge within radius r:

$$\Phi_E = \frac{q_{\text{in}}}{\epsilon_0} = \frac{4\pi r^3\rho}{3\epsilon_0} = \frac{4\pi r^3 3Q}{3\epsilon_0 4\pi a^3} = \boxed{\frac{Qr^3}{\epsilon_0 a^3}}$$

(b) $\Phi_E = \boxed{\frac{Q}{\epsilon_0}}$. Note that the answers to parts (a) and (b) agree at $r = a$.

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(c)

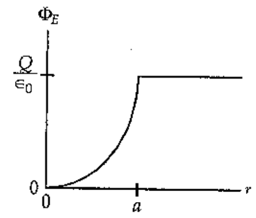

FIG. P19.40(c)

P19.41 The distance between centers is $2\times5.90\times10^{-15}$ m. Each produces a field as if it were a point charge at its center, and each feels a force as if all its charge were a point at its center.

$$F = \frac{k_e q_1 q_2}{r^2} = (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{(46)^2(1.60\times10^{-19}\ \text{C})^2}{(2\times5.90\times10^{-15}\ \text{m})^2} = 3.50\times10^3\ \text{N} = \boxed{3.50\ \text{kN}}$$

Section 19.11 Conductors in Electrostatic Equilibrium

P19.42 $\oint EdA = E(2\pi r l) = \frac{q_{\text{in}}}{\epsilon_0}$ $\quad E = \frac{q_{\text{in}}/l}{2\pi\epsilon_0 r} = \frac{\lambda}{2\pi\epsilon_0 r}$

(a) $r = 3.00$ cm $\quad \vec{E} = \boxed{0}$

(b) $r = 10.0$ cm $\quad \vec{E} = \frac{30.0\times10^{-9}}{2\pi(8.85\times10^{-12})(0.100)} = \boxed{5\,400\ \text{N/C, outward}}$

(c) $r = 100$ cm $\quad \vec{E} = \frac{30.0\times10^{-9}}{2\pi(8.85\times10^{-12})(1.00)} = \boxed{540\ \text{N/C, outward}}$

P19.43 The fields are equal. The Equation 19.25 $E = \frac{\sigma_{\text{conductor}}}{\epsilon_0}$ for the field outside the aluminum looks different from Equation 19.24 $E = \frac{\sigma_{\text{insulator}}}{2\epsilon_0}$ for the field around glass. But its charge will spread out to cover both sides of the aluminum plate, so the density is $\sigma_{\text{conductor}} = \frac{Q}{2A}$. The glass carries charge only on area A, with $\sigma_{\text{insulator}} = \frac{Q}{A}$. The two fields are $\frac{Q}{2A\epsilon_0}$ the same in magnitude, and both are perpendicular to the plates, vertically upward if Q is positive.

P19.44 (a) $E=\frac{\sigma}{\epsilon_0}$ $\quad \sigma=\left(8.00\times10^{4}\right)\left(8.85\times10^{-12}\right)=7.08\times10^{-7}\ \text{C/m}^2$

$\sigma=\boxed{708\ \text{nC/m}^2}$, positive on one face and negative on the other.

(b) $\sigma=\frac{Q}{A}$ $\quad Q=\sigma A=\left(7.08\times10^{-7}\right)(0.500)^2\ \text{C}$

$Q=1.77\times10^{-7}\ \text{C}=\boxed{177\ \text{nC}}$, positive on one face and negative on the other.

P19.45 (a) $\vec{E}=\boxed{0}$

(b) $E=\frac{k_eQ}{r^2}=\frac{\left(8.99\times10^{9}\right)\left(8.00\times10^{-6}\right)}{(0.030\,0)^2}=7.99\times10^{7}\ \text{N/C}$ $\quad \vec{E}=\boxed{79.9\ \text{MN/C radially outward}}$

(c) $\vec{E}=\boxed{0}$

(d) $E=\frac{k_eQ}{r^2}=\frac{\left(8.99\times10^{9}\right)\left(4.00\times10^{-6}\right)}{(0.070\,0)^2}=7.34\times10^{6}\ \text{N/C}$ $\quad \vec{E}=\boxed{7.34\ \text{MN/C radially outward}}$

P19.46 The electric field on the surface of a conductor varies inversely with the radius of curvature of the surface. Thus, the field is most intense where the radius of curvature is smallest and vice-versa. The local charge density and the electric field intensity are related by

$E=\frac{\sigma}{\epsilon_0}$ or $\sigma=\epsilon_0 E$.

(a) Where the radius of curvature is the greatest,

$\sigma=\epsilon_0 E_{\min}=\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)\left(2.80\times10^{4}\ \text{N/C}\right)=\boxed{248\ \text{nC/m}^2}$.

(b) Where the radius of curvature is the smallest,

$\sigma=\epsilon_0 E_{\max}=\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)\left(5.60\times10^{4}\ \text{N/C}\right)=\boxed{496\ \text{nC/m}^2}$.

P19.47 (a) Inside surface: consider a cylindrical surface within the metal. Since E inside the conducting shell is zero, the total charge inside the gaussian surface must be zero, so the inside charge/length $=-\lambda$.

$0=\lambda\ell+q_{\text{in}}$ so $\frac{q_{\text{in}}}{\ell}=\boxed{-\lambda}$

Outside surface: The total charge on the metal cylinder is $2\lambda\ell=q_{\text{in}}+q_{\text{out}}$

$q_{\text{out}}=2\lambda\ell+\lambda\ell$ so the outside charge/length is $\boxed{3\lambda}$.

(b) $E=\frac{2k_e(3\lambda)}{r}=\frac{6k_e\lambda}{r}=\boxed{\frac{3\lambda}{2\pi\epsilon_0 r}\ \text{radially outward}}$

***P19.48** (a) Consider a gaussian surface in the shape of a rectangular box with two faces perpendicular to the direction of the field. It encloses some charge, so the net flux out of the box is nonzero. The field must be stronger on one side than on the other. It cannot be uniform in magnitude.

(b) Now the volume contains no charge. The net flux out of the box is zero. The flux entering is equal to the flux exiting. The field magnitude is uniform.

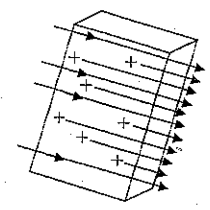

FIG. P19.48(a)

P19.49 (a) The charge density on each of the surfaces (upper and lower) of the plate is:

$\sigma=\frac{1}{2}\left(\frac{q}{A}\right)=\frac{1}{2}\frac{\left(4.00\times10^{-8}\ \text{C}\right)}{(0.500\ \text{m})^2}=8.00\times10^{-8}\ \text{C/m}^2=\boxed{80.0\ \text{nC/m}^2}$.

(b) $\vec{E}=\left(\frac{\sigma}{\epsilon_0}\right)\hat{k}=\left(\frac{8.00\times10^{-8}\ \text{C/m}^2}{8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2}\right)\hat{k}=\boxed{(9.04\ \text{kN/C})\hat{k}}$

(c) $\vec{E}=\boxed{(-9.04\ \text{kN/C})\hat{k}}$

Section 19.12 Context Connection—The Atmospheric Electric Field

P19.50 (a) $\vec{E}=\frac{\sigma}{\epsilon_0}$ toward a $\boxed{\text{negative}}$ charge.

$\sigma=E\epsilon_0=(120\ \text{N/C})\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)=\boxed{1.06\times10^{-9}\ \text{C/m}^2}$

(b) $Q=\sigma A=\sigma\left(4\pi R^2\right)=\left(-1.06\times10^{-9}\ \text{C/m}^2\right)\left[4\pi\left(6.37\times10^{6}\ \text{m}\right)^2\right]=\boxed{-5.42\times10^{5}\ \text{C}}$

$-5.42\times10^{5}\ \text{C}\left(\frac{1\ \text{electron}}{-1.6\times10^{-19}\ \text{C}}\right)=\boxed{3.38\times10^{24}\ \text{excess electrons}}$

P19.51 Consider as a gaussian surface a box with horizontal area A, lying between 500 and 600 m elevation.

$\oint\vec{E}\cdot d\vec{A}=\frac{q}{\epsilon_0}$ $\quad (+120\ \text{N/C})A+(-100\ \text{N/C})A=\frac{\rho A(100\ \text{m})}{\epsilon_0}$

$\rho=\frac{(20\ \text{N/C})\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)}{100\ \text{m}}=\boxed{1.77\times10^{-12}\ \text{C/m}^3}$

The charge is $\boxed{\text{positive}}$, to produce the net outward flux of electric field.

P19.52 (a) The Moon would feel a force $\boxed{\text{away from Earth}}$ of magnitude

$$F=\frac{k_e q_1 q_2}{r^2}=\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(\frac{\left(-5\times10^5\ \text{C}\right)\left(-1.37\times10^5\ \text{C}\right)}{\left(3.84\times10^8\ \text{m}\right)^2}\right)=\boxed{4.18\times10^3\ \text{N}}.$$

(b) The gravitational force is

$$F=\frac{Gm_1m_2}{r^2}=\frac{\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(5.98\times10^{24}\ \text{kg}\right)\left(7.36\times10^{22}\ \text{kg}\right)}{\left(3.84\times10^8\ \text{m}\right)^2}$$

$F=1.99\times10^{20}$ N toward Earth.

Thus, $\boxed{\text{the electric force is weaker by}}$

$$\frac{1.99\times10^{20}\ \text{N}}{4.18\times10^3\ \text{N}}=\boxed{4.77\times10^{16}\ \text{times and in the opposite direction}}.$$

Additional Problems

***P19.53** The two given charges exert equal-size forces of attraction on each other. If a third charge, positive or negative, were placed between them they could not be in equilibrium. If the third charge were at a point $x>15$ cm, it would exert a stronger force on the 45 μC than on the $-12\ \mu$C, and could not produce equilibrium for both. Thus the third charge must be at $x=-d<0$. Its equilibrium requires

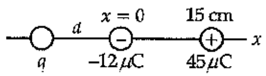

FIG. P19.53

$$\frac{k_eq(12\ \mu\text{C})}{d^2}=\frac{k_eq(45\ \mu\text{C})}{(15\ \text{cm}+d)^2}\qquad\left(\frac{15\ \text{cm}+d}{d}\right)^2=\frac{45}{12}=3.75$$

$15\ \text{cm}+d=1.94d \qquad d=16.0\ \text{cm}.$

The third charge is at $\boxed{x=-16.0\ \text{cm}}$. The equilibrium of the $-12\ \mu$C requires

$$\frac{k_eq(12\ \mu\text{C})}{(16.0\ \text{cm})^2}=\frac{k_e(45\ \mu\text{C})12\ \mu\text{C}}{(15\ \text{cm})^2}\qquad\boxed{q=51.3\ \mu\text{C}}.$$

All six individual forces are now equal in magnitude, so we have equilibrium as required, and this is the only solution.

P19.54 From the free-body diagram shown,

$\sum F_y=0:\qquad T\cos15.0°=1.96\times10^{-2}$ N.

So $\qquad T=2.03\times10^{-2}$ N.

From $\sum F_x=0$, we have $\qquad qE=T\sin15.0°$

or
$$q=\frac{T\sin15.0°}{E}=\frac{\left(2.03\times10^{-2}\ \text{N}\right)\sin15.0°}{1.00\times10^3\ \text{N/C}}=5.25\times10^{-6}\ \text{C}=\boxed{5.25\ \mu\text{C}}.$$

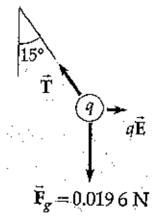

FIG. P19.54

P19.55 $F=\dfrac{k_eq_1q_2}{r^2}\qquad \tan\theta=\dfrac{15.0}{60.0}\qquad \theta=14.0°$

$$F_1=\frac{\left(8.99\times10^9\right)\left(10.0\times10^{-6}\right)^2}{(0.150)^2}=40.0\ \text{N}$$

$$F_3=\frac{\left(8.99\times10^9\right)\left(10.0\times10^{-6}\right)^2}{(0.600)^2}=2.50\ \text{N}$$

$$F_2=\frac{\left(8.99\times10^9\right)\left(10.0\times10^{-6}\right)^2}{(0.619)^2}=2.35\ \text{N}$$

$F_x=-F_3-F_2\cos14.0°=-2.50-2.35\cos14.0°=-4.78$ N

$F_y=-F_1-F_2\sin14.0°=-40.0-2.35\sin14.0°=-40.6$ N

$$F_{\text{net}}=\sqrt{F_x^2+F_y^2}=\sqrt{(4.78)^2+(40.6)^2}=\boxed{40.9\ \text{N}}$$

$$\tan\phi=\frac{F_y}{F_x}=\frac{-40.6}{-4.78}$$

$\phi=\boxed{263°}$

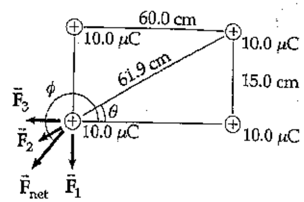

FIG. P19.55

P19.56 FIG. P19.56(a): $d\cos 30.0° = 15.0\text{ cm}$,

or $d = \dfrac{15.0\text{ cm}}{\cos 30.0°}$

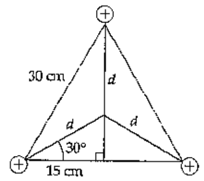

FIG. P19.56(a)

FIG. P19.56(b): $\theta = \sin^{-1}\left(\dfrac{d}{50.0\text{ cm}}\right)$

$$\theta = \sin^{-1}\left(\frac{15.0\text{ cm}}{50.0\text{ cm}(\cos 30.0°)}\right) = 20.3°$$

$$\frac{F_q}{mg} = \tan\theta$$

or $F_q = mg\tan 20.3°$ (1)

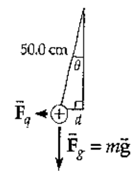

FIG. P19.56(b)

FIG. P19.56(c): $F_q = 2F\cos 30.0°$

$$F_q = 2\left[\frac{k_e q^2}{(0.300\text{ m})^2}\right]\cos 30.0° \quad (2)$$

Combining equations (1) and (2),

$$2\left[\frac{k_e q^2}{(0.300\text{ m})^2}\right]\cos 30.0° = mg\tan 20.3°$$

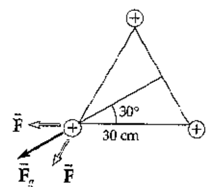

FIG. P19.56(c)

$$q^2 = \frac{mg(0.300\text{ m})^2\tan 20.3°}{2k_e\cos 30.0°}$$

$$q^2 = \frac{(2.00\times10^{-3}\text{ kg})(9.80\text{ m/s}^2)(0.300\text{ m})^2\tan 20.3°}{2(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)\cos 30.0°}$$

$$q = \sqrt{4.20\times10^{-14}\text{ C}^2} = 2.05\times10^{-7}\text{ C} = \boxed{0.205\ \mu\text{C}}$$

P19.57 Charge $\dfrac{Q}{2}$ resides on each block, which repel as point charges:

$$F = \frac{k_e\left(\frac{Q}{2}\right)\left(\frac{Q}{2}\right)}{L^2} = k(L - L_i)$$

$$Q = 2L\sqrt{\frac{k(L-L_i)}{k_e}} = 2(0.400\text{ m})\sqrt{\frac{(100\text{ N/m})(0.100\text{ m})}{(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)}} = \boxed{26.7\ \mu\text{C}}$$

P19.58 Charge $\dfrac{Q}{2}$ resides on each block, which repel as point charges: $F = \dfrac{k_e\left(\frac{Q}{2}\right)\left(\frac{Q}{2}\right)}{L^2} = k(L - L_i)$.

Solving for Q, $Q = \boxed{2L\sqrt{\dfrac{k(L-L_i)}{k_e}}}$

Observing the spring extension gives a way of measuring the charge. If L is large compared to L_i, the charge is proportional to L to the $\frac{3}{2}$ power. The charge is proportional to the spring constant to the $\frac{1}{2}$ power. We demonstrate dimensional correctness by evaluating the units of the right hand side:

$$\text{m}\left(\frac{\text{N}\cdot\text{m}\cdot\text{C}^2}{\text{m}\cdot\text{N}\cdot\text{m}^2}\right)^{1/2} = \text{C}.$$

P19.59 (a) From the $2Q$ charge we have $F_e - T_2\sin\theta_2 = 0$ and $mg - T_2\cos\theta_2 = 0$.

Combining these we find $\dfrac{F_e}{mg} = \dfrac{T_2\sin\theta_2}{T_2\cos\theta_2} = \tan\theta_2$.

From the Q charge we have $F_e = T_1\sin\theta_1 = 0$ and $mg - T_1\cos\theta_1 = 0$.

Combining these we find $\dfrac{F_e}{mg} = \dfrac{T_1\sin\theta_1}{T_1\cos\theta_1} = \tan\theta_1$ or $\boxed{\theta_2 = \theta_1}$.

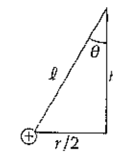

FIG. P19.59

(b) $F_e = \dfrac{k_e 2QQ}{r^2} = \dfrac{2k_e Q^2}{r^2}$

If we assume θ is small then $\tan\theta \approx \dfrac{r/2}{\ell}$

Substitute expressions for F_e and $\tan\theta$ into either equation found in part (a) and solve for r.

$\dfrac{F_e}{mg} = \tan\theta$ then $\dfrac{2k_e Q^2}{r^2}\left(\dfrac{1}{mg}\right) \approx \dfrac{r}{2\ell}$ and solving for r we find $r \approx \left(\dfrac{4k_e Q^2\ell}{mg}\right)^{1/3}$

P19.60 At an equilibrium position, the net force on the charge Q is zero. The equilibrium position can be located by determining the angle θ corresponding to equilibrium.

In terms of lengths s, $\frac{1}{2}a\sqrt{3}$, and r, shown in Figure P19.60, the charge at the origin exerts an attractive force

$$\frac{k_e Qq}{\left(s + \frac{1}{2}a\sqrt{3}\right)^2}$$

The other two charges exert equal repulsive forces of magnitude $\dfrac{k_e Qq}{r^2}$. The horizontal components of the two repulsive forces add, balancing the attractive force,

continued on next page

$$F_{\text{net}} = k_e Qq\left[\frac{2\cos\theta}{r^2} - \frac{1}{\left(s+\frac{1}{2}a\sqrt{3}\right)^2}\right] = 0$$

From Figure P19.60 $\qquad r = \dfrac{\frac{1}{2}a}{\sin\theta} \qquad s = \dfrac{1}{2}a\cot\theta$

The equilibrium condition, in terms of θ, is $\qquad F_{\text{net}} = \left(\dfrac{4}{a^2}\right)k_e Qq\left(2\cos\theta\sin^2\theta - \dfrac{1}{\left(\sqrt{3}+\cot\theta\right)^2}\right) = 0.$

Thus the equilibrium value of θ satisfies $\qquad 2\cos\theta\sin^2\theta\left(\sqrt{3}+\cot\theta\right)^2 = 1.$

One method for solving for θ is to tabulate the left side. To three significant figures a value of θ corresponding to equilibrium is 81.7°

The distance from the vertical side of the triangle to the equilibrium position is

$$s = \frac{1}{2}a\cot 81.7° = \boxed{0.0729a}.$$

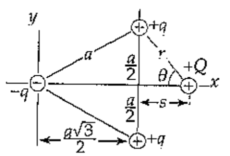

FIG. P19.60

θ	$2\cos\theta\sin^2\theta\left(\sqrt{3}+\cot\theta\right)^2$
60°	4
70°	2.654
80°	1.226
90°	0
81°	1.091
81.5°	1.024
81.7°	0.997

A second zero-field point is on the negative side of the x-axis, where $\theta = -9.16°$ and $s = -3.10a$.

P19.61 (a) Zero contribution from the same face due to symmetry, opposite face contributes

$4\left(\dfrac{k_e q}{r^2}\sin\phi\right)$ where $\quad r = \sqrt{\left(\dfrac{s}{2}\right)^2 + \left(\dfrac{s}{2}\right)^2 + s^2} = \sqrt{1.5}s = 1.22s$

$\sin\phi = \dfrac{s}{r} \qquad E = 4\dfrac{k_e qs}{r^3} = \dfrac{4}{(1.22)^3}\dfrac{k_e q}{s^2} = \boxed{2.18\dfrac{k_e q}{s^2}}$

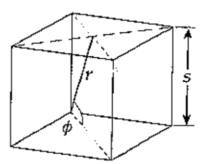

FIG. P19.61

(b) $\boxed{\text{The direction is the } \hat{\mathbf{k}} \text{ direction.}}$

P19.62 Consider the field due to a single sheet and let E_+ and E_- represent the fields due to the positive and negative sheets. The field at any distance from each sheet has a magnitude given by Equation 19.24:

$$|E_+| = |E_-| = \frac{\sigma}{2\epsilon_0}$$

(a) To the left of the positive sheet, E_+ is directed toward the left and E_- toward the right and the net field over this region is $\vec{\mathbf{E}} = \boxed{0}$.

(b) In the region between the sheets, E_+ and E_- are both directed toward the right and the net field is

$$\vec{\mathbf{E}} = \boxed{\frac{\sigma}{\epsilon_0} \text{ to the right}}.$$

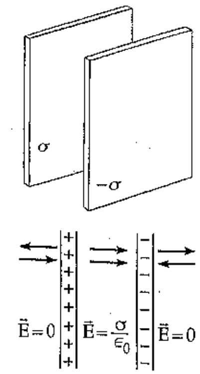

FIG. P19.62

(c) To the right of the negative sheet, E_+ and E_- are again oppositely directed and $\vec{\mathbf{E}} = \boxed{0}$.

P19.63 The magnitude of the field due to the each sheet given by Equation 19.24 is

$$E = \frac{\sigma}{2\epsilon_0} \text{ directed perpendicular to the sheet.}$$

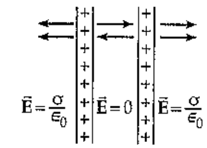

FIG. P19.63

(a) In the region to the left of the pair of sheets, both fields are directed toward the left and the net field is

$$\vec{\mathbf{E}} = \boxed{\frac{\sigma}{\epsilon_0} \text{ to the left}}.$$

(b) In the region between the sheets, the fields due to the individual sheets are oppositely directed and the net field is

$$\vec{\mathbf{E}} = \boxed{0}$$

(c) In the region to the right of the pair of sheets, both fields are directed toward the right and the net field is

$$\vec{\mathbf{E}} = \boxed{\frac{\sigma}{\epsilon_0} \text{ to the right}}.$$

P19.64 $d\vec{E} = \frac{k_e dq}{x^2+(0.150\text{ m})^2}\left(\frac{-x\hat{i}+0.150\text{ m}\hat{j}}{\sqrt{x^2+(0.150\text{ m})^2}}\right) = \frac{k_e\lambda\left(-x\hat{i}+0.150\text{ m}\hat{j}\right)dx}{\left[x^2+(0.150\text{ m})^2\right]^{3/2}}$

$$\vec{E} = \int_{\text{all charge}} d\vec{E} = k_e\lambda\int_{x=0}^{0.400\text{ m}}\frac{\left(-x\hat{i}+0.150\text{ m}\hat{j}\right)dx}{\left[x^2+(0.150\text{ m})^2\right]^{3/2}}$$

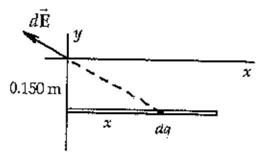

FIG. P19.64

$$\vec{E} = k_e\lambda\left[\left.\frac{+\hat{i}}{\sqrt{x^2+(0.150\text{ m})^2}}\right|_0^{0.400\text{ m}} + \left.\frac{(0.150\text{ m})\hat{j}x}{(0.150\text{ m})^2\sqrt{x^2+(0.150\text{ m})^2}}\right|_0^{0.400\text{ m}}\right]$$

$$\vec{E} = \left(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2\right)\left(35.0\times10^{-9}\text{ C/m}\right)\left[\hat{i}(2.34-6.67)\text{ m}^{-1}+\hat{j}(6.24-0)\text{ m}^{-1}\right]$$

$$\vec{E} = \left(-1.36\hat{i}+1.96\hat{j}\right)\times10^3\text{ N/C} = \boxed{\left(-1.36\hat{i}+1.96\hat{j}\right)\text{ kN/C}}$$

P19.65 (a) $\oint\vec{E}\cdot d\vec{A} = E\left(4\pi r^2\right) = \frac{q_{in}}{\epsilon_0}$

For $r<a$, $\quad q_{in} = \rho\left(\frac{4}{3}\pi r^3\right)$

so $\quad E = \boxed{\frac{\rho r}{3\epsilon_0}}$.

For $a<r<b$ and $c<r$, $\quad q_{in} = Q$.

So $\quad E = \boxed{\frac{Q}{4\pi r^2\epsilon_0}}$.

For $b\le r\le c$, $\quad E=0$, since $\boxed{E=0}$ inside a conductor.

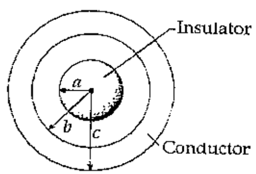

FIG. P19.65

(b) Let q_1 = induced charge on the inner surface of the hollow sphere. Since $E=0$ inside the conductor, the total charge enclosed by a spherical surface of radius $b\le r\le c$ must be zero.

Therefore, $\quad q_1+Q=0 \quad$ and $\quad \sigma_1 = \frac{q_1}{4\pi b^2} = \boxed{\frac{-Q}{4\pi b^2}}$

Let q_2 = induced charge on the outside surface of the hollow sphere. Since the hollow sphere is uncharged, we require

$$q_1+q_2=0 \quad\text{and}\quad \sigma_2 = \frac{q_1}{4\pi c^2} = \boxed{\frac{Q}{4\pi c^2}}.$$

P19.66 The field on the axis of the ring is calculated in Example 19.5, $\quad E = E_x = \frac{k_e xQ}{\left(x^2+a^2\right)^{3/2}}$

The force experienced by a charge $-q$ placed along the axis of the ring is $\quad F = -k_eQq\left[\frac{x}{\left(x^2+a^2\right)^{3/2}}\right]$

and when $x \ll a$, this becomes $\quad F = -\left(\frac{k_eQq}{a^3}\right)x$

This expression for the force is in the form of Hooke's law, with an effective spring constant of $\quad k = \frac{k_eQq}{a^3}$

Since $\omega = 2\pi f = \sqrt{\frac{k}{m}}$, we have $\quad f = \boxed{\frac{1}{2\pi}\sqrt{\frac{k_eQq}{ma^3}}}$.

P19.67 The resultant field within the cavity is the superposition of two fields, one $\vec{E}_+$ due to a uniform sphere of positive charge of radius $2a$, and the other $\vec{E}_-$ due to a sphere of negative charge of radius a centered within the cavity.

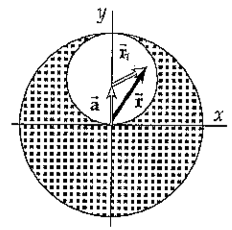

FIG. P19.67

$\frac{4}{3}\left(\frac{\pi r^3\rho}{\epsilon_0}\right) = 4\pi r^2 E_+ \quad$ so $\quad \vec{E}_+ = \frac{\rho r}{3\epsilon_0}\hat{r} = \frac{\rho\vec{r}}{3\epsilon_0}$

$-\frac{4}{3}\left(\frac{\pi r_1^3\rho}{\epsilon_0}\right) = 4\pi r_1^2 E_- \quad$ so $\quad \vec{E}_- = \frac{\rho r_1}{3\epsilon_0}\left(-\hat{r}_1\right) = \frac{-\rho}{3\epsilon_0}\vec{r}_1$.

Since $\vec{r} = \vec{a}+\vec{r}_1$, $\quad \vec{E}_- = \frac{-\rho\left(\vec{r}-\vec{a}\right)}{3\epsilon_0}$

$$\vec{E} = \vec{E}_+ + \vec{E}_- = \frac{\rho\vec{r}}{3\epsilon_0} - \frac{\rho\vec{r}}{3\epsilon_0} + \frac{\rho\vec{a}}{3\epsilon_0} = \frac{\rho\vec{a}}{3\epsilon_0} = 0\hat{i} + \frac{\rho a}{3\epsilon_0}\hat{j}.$$

Thus, $\quad \boxed{E_x = 0}$

and $\quad \boxed{E_y = \frac{\rho a}{3\epsilon_0}}$ at all points within the cavity.

ANSWERS TO EVEN PROBLEMS

P19.2 (a) 2.62×10^{24} electrons; (b) 2.38 electrons for every 10^9 already present

P19.4 57.5 N away from the other proton

P19.6 1.57 μN to the left

P19.8 $0.634d$, stable if the third bead has positive charge