P20.20 27.4 fm

P20.22 (a) 0; (b) $\frac{k_e Q}{r^2}$

P20.24 $-0.553\frac{k_e Q}{R}$

P20.26 $V=-\left(\frac{k_e \alpha L}{2}\right)\ln\left[\frac{\sqrt{(L^2/4)+b^2}-L/2}{\sqrt{(L^2/4)+b^2}+L/2}\right]$

P20.28 1.56×10^{12} electrons removed

P20.30 (a) 135 kV;
(b) 2.25 MV/m away for the large sphere and 6.74 MV/m away for the small sphere

P20.32 (a) 1.00 μF; (b) 100 V

P20.34 $\frac{(2N-1)(\pi-\theta)R^4 \in_0}{d}$

P20.36 (a) 2.68 nF; (b) 3.02 kV

P20.38 see the solution

P20.40 (a) 3.53 μF; (b) 6.35 V and 2.65 V;
(c) 31.8 μC on each

P20.42 $\frac{1}{2}C_p+\sqrt{\frac{1}{4}C_p^2-C_pC_s}$

P20.44 (a) $2C$; (b) $Q_1>Q_3>Q_2$;
(c) $\Delta V_1>\Delta V_2=\Delta V_3$;
(d) Q_3 and Q_1 increase, Q_2 decreases

P20.46 12.9 μF

P20.48 4.47 kV

P20.50 (a) 1.50 μC; (b) 1.83 kV

P20.52 $2.51\times10^{-3}\text{ m}^3=2.51\text{ L}$

P20.54 (a) 13.3 nC; (b) 272 nC

P20.56 $\sim10^{-6}$ F and $\sim10^2$ V for two 40-cm by 100-cm sheets of aluminum foil sandwiching a thin sheet of plastic

P20.58 (a) 450 kV; (b) 7.51 μC

P20.60 (a) The velocity of one particle relative to the other is first a velocity of approach, then zero at closest approach, and then a velocity of recession;
(b) $6.00\hat{\mathbf{i}}$ m/s; (c) 3.64 m;
(d) $-9.00\hat{\mathbf{i}}$ m/s for the incident particle and $12.0\hat{\mathbf{i}}$ m/s for the target particle.

P20.62 (a) -27.2 eV; (b) -6.80 eV; (c) 0

P20.64 see the solution

P20.66 (a) 1.42 mm; (b) 9.20 kV/m

P20.68 $\left(\frac{k_e q^2}{3am}\right)^{1/2}$

P20.70 (a) see the solution;
(b) $E_r=\frac{2k_e p\cos\theta}{r^3}$, $E_\theta=\frac{k_e p\sin\theta}{r^3}$, yes, no;
(c) $V=k_e py\left(x^2+y^2\right)^{-3/2}$

$$\vec{\mathbf{E}}=\frac{3k_e pxy\hat{\mathbf{i}}}{\left(x^2+y^2\right)^{5/2}}+\frac{k_e p\left(2y^2-x^2\right)\hat{\mathbf{j}}}{\left(x^2+y^2\right)^{5/2}}$$

P20.72 (a) 40.0 μJ; (b) 500 V

P20.74 23.3 V; 26.7 V

P20.76 (a) $\frac{Q_0^2 d(\ell-x)}{2\ell^3 \in_0}$; (b) $\frac{Q_0^2 d}{2\ell^3 \in_0}$ to the right;
(c) $\frac{Q_0^2}{2\ell^4 \in_0}$; (d) $\frac{Q_0^2}{2\ell^4 \in_0}$

Chapter 21

Current and Direct Current Circuits

CHAPTER OUTLINE

ANSWERS TO QUESTIONS

Q21.1 Consider all of the eastbound lanes on a highway, taken together. Individual vehicles correspond to bits of charge. The number of vehicles that pass a certain milepost, divided by the time during which they go past, corresponds to the value of the current.

Q21.2 Geometry and resistivity. In turn, the resistivity of the material depends on the temperature.

Q21.3 The radius of wire B is $\sqrt{3}$ times the radius of wire A, to make its cross-sectional area 3 times larger.

Q21.4 In a normal metal, suppose that we could proceed to a limit of zero resistance by lengthening the average time between collisions. The classical model of conduction then suggests that a constant applied voltage would cause constant acceleration of the free electrons, and a current steadily increasing in time.

On the other hand, we can actually switch to zero resistance by substituting a superconducting wire for the normal metal. In this case, the drift velocity of electrons is established by vibrations of atoms in the crystal lattice; the maximum current is limited; and it becomes impossible to establish a potential difference across the superconductor.

Q21.5 The amplitude of atomic vibrations increases with temperature. Atoms can then scatter electrons more efficiently.

Q21.6 A current will continue to exist in a superconductor without voltage because there is no resistance loss.

Q21.7 Because there are so many electrons in a conductor (approximately 10^{28} electrons/m^3) the average velocity of charges is very slow. When you connect a wire to a potential difference, you establish an electric field everywhere in the wire nearly instantaneously, to make electrons start drifting everywhere all at once.

Q21.8 The 25 W bulb has a higher resistance. The 100 W bulb carries more current.

Q21.9 One ampere–hour is 3 600 coulombs. The ampere–hour rating is the quantity of charge that the battery can lift though its nominal potential difference.

Q21.10 In series, the current is the same through each resistor. Without knowing individual resistances, nothing can be determined about potential differences or power.

Q21.11 In parallel, the potential difference is the same across each resistor. Without knowing individual resistances, nothing can be determined about current or power.

Q21.12 A short circuit can develop when the last bit of insulation frays away between the two conductors in a lamp cord. Then the two conductors touch each other, opening a low-resistance branch in parallel with the lamp. The lamp will immediately go out, carrying no current and presenting no danger. A very large current exists in the power supply, the house wiring, and the rest of the lamp cord up to the contact point. Before it blows the fuse or pops the circuit breaker, the large current can quickly raise the temperature in the short-circuit path.

Q21.13 The whole wire is very nearly at one uniform potential. There is essentially zero *difference* in potential between the bird's feet. Then negligible current goes through the bird. The resistance through the bird's body between its feet is much larger than the resistance through the wire between the same two points.

Q21.14 A wire or cable in a transmission line is thick and made of material with very low resistivity. Only when its length is very large does its resistance become significant. To transmit power over a long distance it is most efficient to use low current at high voltage, minimizing the I^2R power loss in the transmission line. Alternating current, as opposed to the direct current we study first, can be stepped up in voltage and then down again, with high-efficiency transformers at both ends of the power line.

Q21.15 The bulb will light up for a while immediately after the switch is closed. As the capacitor charges, the bulb gets progressively dimmer. When the capacitor is fully charged the current in the circuit is zero and the bulb does not glow at all. If the value of RC is small, this whole process might occupy a very short time interval.

Q21.16 Car headlights are in parallel. If they were in series, both would go out when the filament of one failed. An important safety factor would be lost.

Q21.17 Kirchhoff's junction rule expresses conservation of electric charge. If the total current into a point were different from the total current out, then charge would be continuously created or annihilated at that point.

Kirchhoff's loop rule expresses conservation of energy. For a single-loop circle with two resistors, the loop rule reads $+\mathcal{E}-IR_1-IR_2=0$. This is algebraically equivalent to $q\mathcal{E}=qIR_1+qIR_2$, where $q=I\Delta t$ is the charge passing a point in the loop in the time interval Δt. The equivalent equation states that the power supply injects energy into the circuit equal in amount to that which the resistors degrade into internal energy.

Q21.18 At their normal operating temperatures, from $\mathcal{P}=\dfrac{\Delta V^2}{R}$, the bulbs present resistances

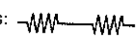
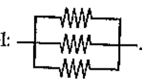
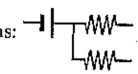
$$R=\frac{\Delta V^2}{\mathcal{P}}=\frac{(120\text{ V})^2}{60\text{ W}}=240\ \Omega,\text{ and }\frac{(120\text{ V})^2}{75\text{ W}}=190\ \Omega,\text{ and }\frac{(120\text{ V})^2}{200\text{ W}}=72\ \Omega.$$

The nominal 60 W lamp has greatest resistance. When they are connected in series, they all carry the same small current. Here the highest-resistance bulb glows most brightly and the one with lowest resistance is faintest. This is just the reverse of their order of intensity if they were connected in parallel, as they are designed to be.

Q21.19 Answer their question with a challenge. If the student is just looking at a diagram, provide the materials to build the circuit. If you are looking at a circuit where the second bulb really is fainter, get the student to unscrew them both and interchange them. But check that the student's understanding of potential has not been impaired: if you patch past the first bulb to short it out, the second gets brighter.

Q21.20 The hospital maintenance worker is right. A hospital room is full of electrical grounds, including the bed frame. If your grandmother touched the faulty knob and the bed frame at the same time, she could receive quite a jolt, as there would be a potential difference of 120 V across her. If the 120 V is DC, the shock could send her into ventricular fibrillation, and the hospital staff could use the defibrillator you read about in Example 21.10. If the 120 V is AC, which is most likely, the current could produce external and internal burns along the path of conduction. Likely no one got a shock from the radio back at home because her bedroom contained no electrical grounds—no conductors connected to zero volts. Just like the bird in Question 21.13, granny could touch the "hot" knob without getting a shock so long as there was no path to ground to supply a potential difference across her. A new appliance in the bedroom or a flood could make the radio lethal. Repair it or discard it. Enjoy the news from Lake Wobegon on the new plastic radio.

Q21.21 Suppose $\mathcal{E}=12\text{ V}$ and each lamp has $R=2\ \Omega$. Before the switch is closed the current is $\dfrac{12\text{ V}}{6\ \Omega}=2\text{ A}$. The potential difference across each lamp is $(2\text{ A})(2\ \Omega)=4\text{ V}$. The power of each lamp is $(2\text{ A})(4\text{ V})=8\text{ W}$, totaling 24 W for the circuit. Closing the switch makes the switch and the wires connected to it a zero-resistance branch. All of the current through A and B will go through the switch and (b) lamp C goes out, with zero voltage across it. With less total resistance, the (c) current in the battery $\dfrac{12\text{ V}}{4\ \Omega}=3\text{ A}$ becomes larger than before and (a) lamps A and B get brighter. (d) The voltage across each of A and B is $(3\text{ A})(2\ \Omega)=6\text{ V}$, larger than before. Each converts power $(3\text{ A})(6\text{ V})=18\text{ W}$, totaling 36 W, which is (e) an increase.

Q21.22 Two runs in series: . Three runs in parallel: . Junction of one lift and two runs: 

Gustav Robert Kirchhoff, Professor of Physics at Heidelberg and Berlin, was master of the obvious. A junction rule: The number of skiers coming into any junction must be equal to the number of skiers leaving, so that if we count the outward bound skiers as negative the net accumulation of skiers is zero. A loop rule: the total change in altitude must be zero for any skier completing a closed path.

SOLUTIONS TO PROBLEMS

Section 21.1 Electric Current

P21.1 $I=\dfrac{\Delta Q}{\Delta t}$ $\qquad \Delta Q=I\Delta t=(30.0\times10^{-6}\text{ A})(40.0\text{ s})=1.20\times10^{-3}\text{ C}$

$$N=\frac{Q}{e}=\frac{1.20\times10^{-3}\text{ C}}{1.60\times10^{-19}\text{ C/electron}}=\boxed{7.50\times10^{15}\text{ electrons}}$$

P21.2 The period of revolution for the sphere is $T=\frac{2\pi}{\omega}$, and the average current represented by this revolving charge is $I=\frac{q}{T}=\boxed{\frac{q\omega}{2\pi}}$.

P21.3 $Q(t)=\int_0^t I\,dt=I_0\tau\left(1-e^{-t/\tau}\right)$

(a) $Q(\tau)=I_0\tau\left(1-e^{-1}\right)=\boxed{(0.632)I_0\tau}$

(b) $Q(10\tau)=I_0\tau\left(1-e^{-10}\right)=\boxed{(0.999\,95)I_0\tau}$

(c) $Q(\infty)=I_0\tau\left(1-e^{-\infty}\right)=\boxed{I_0\tau}$

P21.4 $q=4t^3+5t+6$

$$A=\left(2.00\text{ cm}^2\right)\left(\frac{1.00\text{ m}}{100\text{ cm}}\right)^2=2.00\times10^{-4}\text{ m}^2$$

(a) $I(1.00\text{ s})=\left.\frac{dq}{dt}\right|_{t=1.00\text{ s}}=\left.\left(12t^2+5\right)\right|_{t=1.00\text{ s}}=\boxed{17.0\text{ A}}$

(b) $J=\frac{I}{A}=\frac{17.0\text{ A}}{2.00\times10^{-4}\text{ m}^2}=\boxed{85.0\text{ kA/m}^2}$

P21.5 We use $I=nqAv_d$. Here n is the number of charge carriers per unit volume, and is identical to the number of atoms per unit volume. We assume a contribution of 1 free electron per atom in the relationship above. For aluminum, which has a molar mass of 27, we know that Avogadro's number of atoms, N_A, has a mass of 27.0 g. Thus, the mass per atom is

$$\frac{27.0\text{ g}}{N_A}=\frac{27.0\text{ g}}{6.02\times10^{23}}=4.49\times10^{-23}\text{ g/atom}.$$

Thus, $n=\frac{\text{density of aluminum}}{\text{mass per atom}}=\frac{2.70\text{ g/cm}^3}{4.49\times10^{-23}\text{ g/atom}}$

$$n=6.02\times10^{22}\text{ atoms/cm}^3=6.02\times10^{28}\text{ atoms/m}^3$$

Therefore, $v_d=\frac{I}{nqA}=\frac{5.00\text{ A}}{\left(6.02\times10^{28}\text{ m}^{-3}\right)\left(1.60\times10^{-19}\text{ C}\right)\left(4.00\times10^{-6}\text{ m}^2\right)}=1.30\times10^{-4}\text{ m/s}$

or, $v_d=\boxed{0.130\text{ mm/s}}$.

Section 21.2 Resistance and Ohm's Law

P21.6 $I=\frac{\Delta V}{R}=\frac{120\text{ V}}{240\,\Omega}=0.500\text{ A}=\boxed{500\text{ mA}}$

P21.7 $\Delta V=IR$

and $R=\frac{\rho\ell}{A}$: $\quad A=(0.600\text{ mm})^2\left(\frac{1.00\text{ m}}{1\,000\text{ mm}}\right)^2=6.00\times10^{-7}\text{ m}^2$

$\Delta V=\frac{I\rho\ell}{A}$ $\quad I=\frac{\Delta VA}{\rho\ell}=\frac{(0.900\text{ V})\left(6.00\times10^{-7}\text{ m}^2\right)}{\left(5.60\times10^{-8}\,\Omega\cdot\text{m}\right)(1.50\text{ m})}$

$I=\boxed{6.43\text{ A}}$

P21.8 (a) Given $M=\rho_dV=\rho_dA\ell$ where $\rho_d\equiv$ mass density,

we obtain: $A=\frac{M}{\rho_d\ell}$ Taking $\rho_r\equiv$ resistivity, $R=\frac{\rho_r\ell}{A}=\frac{\rho_r\ell}{M/\rho_d\ell}=\frac{\rho_r\rho_d\ell^2}{M}$

Thus, $\ell=\sqrt{\frac{MR}{\rho_r\rho_d}}=\sqrt{\frac{\left(1.00\times10^{-3}\right)(0.500)}{\left(1.70\times10^{-8}\right)\left(8.92\times10^3\right)}}$ $\quad \ell=\boxed{1.82\text{ m}}$

(b) $V=\frac{M}{\rho_d}$, or $\pi r^2\ell=\frac{M}{\rho_d}$

Thus, $r=\sqrt{\frac{M}{\pi\rho_d\ell}}=\sqrt{\frac{1.00\times10^{-3}}{\pi\left(8.92\times10^3\right)(1.82)}}$ $\quad r=1.40\times10^{-4}\text{ m}$.

The diameter is twice this distance: diameter $=\boxed{280\,\mu\text{m}}$

P21.9 (a) $\rho=\rho_0\left[1+\alpha(T-T_0)\right]=\left(2.82\times10^{-8}\,\Omega\cdot\text{m}\right)\left[1+3.90\times10^{-3}(30.0°)\right]=\boxed{3.15\times10^{-8}\,\Omega\cdot\text{m}}$

(b) $J=\frac{E}{\rho}=\frac{0.200\text{ V/m}}{3.15\times10^{-8}\,\Omega\cdot\text{m}}=\boxed{6.35\times10^6\text{ A/m}^2}$

(c) $I=JA=J\left(\frac{\pi d^2}{4}\right)=\left(6.35\times10^6\text{ A/m}^2\right)\left[\frac{\pi\left(1.00\times10^{-4}\text{ m}\right)^2}{4}\right]=\boxed{49.9\text{ mA}}$

continued on next page

(d) $n=\dfrac{6.02\times10^{23}\text{ electrons}}{\left[26.98\text{ g}/\left(2.70\times10^{6}\text{ g/m}^3\right)\right]}=6.02\times10^{28}\text{ electrons/m}^3$

$v_d=\dfrac{J}{ne}=\dfrac{\left(6.35\times10^{6}\text{ A/m}^2\right)}{\left(6.02\times10^{28}\text{ electrons/m}^3\right)\left(1.60\times10^{-19}\text{ C}\right)}=\boxed{659\ \mu\text{m/s}}$

(e) $\Delta V=E\ell=(0.200\text{ V/m})(2.00\text{ m})=\boxed{0.400\text{ V}}$

P21.10 At the low temperature T_C we write $R_C=\dfrac{\Delta V}{I_C}=R_0\left[1+\alpha(T_C-T_0)\right]$ where $T_0=20.0°\text{C}$.

At the high temperature T_h, $R_h=\dfrac{\Delta V}{I_h}=\dfrac{\Delta V}{1\text{ A}}=R_0\left[1+\alpha(T_h-T_0)\right]$.

Then $\dfrac{(\Delta V)/(1.00\text{ A})}{(\Delta V)/I_C}=\dfrac{1+\left(3.90\times10^{-3}\right)(38.0)}{1+\left(3.90\times10^{-3}\right)(-108)}$

and $I_C=(1.00\text{ A})\left(\dfrac{1.15}{0.579}\right)=\boxed{1.98\text{ A}}$.

P21.11 For aluminum,

$\alpha_E=3.90\times10^{-3}\ °\text{C}^{-1}$ (Table 21.1)

$\alpha=24.0\times10^{-6}\ °\text{C}^{-1}$ (Table 16.1)

$$R=\frac{\rho\ell}{A}=\frac{\rho_0(1+\alpha_E\Delta T)\ell(1+\alpha\Delta T)}{A(1+\alpha\Delta T)^2}=R_0\frac{(1+\alpha_E\Delta T)}{(1+\alpha\Delta T)}=(1.234\ \Omega)\left(\frac{1.39}{1.0024}\right)=\boxed{1.71\ \Omega}$$

Section 21.3 Superconductors

No problems in this section

Section 21.4 A Structural Model for Electrical Conduction

P21.12 (a) n is $\boxed{\text{unaffected}}$

(b) $|J|=\dfrac{I}{A}\propto I$

so it $\boxed{\text{doubles}}$.

continued on next page

(c) $J=nev_d$

so v_d $\boxed{\text{doubles}}$.

(d) $\tau=\dfrac{m\sigma}{nq^2}$ is $\boxed{\text{unchanged}}$ as long as σ does not change due to a temperature change in the conductor.

P21.13 $\rho=\dfrac{m}{nq^2\tau}$

so $\tau=\dfrac{m}{\rho nq^2}=\dfrac{\left(9.11\times10^{-31}\right)}{\left(1.70\times10^{-8}\right)\left(8.49\times10^{28}\right)\left(1.60\times10^{-19}\right)}=2.47\times10^{-14}\text{ s}$

$v_d=\dfrac{qE}{m}\tau$

so $7.84\times10^{-4}=\dfrac{\left(1.60\times10^{-19}\right)E\left(2.47\times10^{-14}\right)}{9.11\times10^{-31}}$

Therefore, $E=\boxed{0.181\text{ V/m}}$. We could also compute the field from $E=\rho J$ with $J=nqv_d$.

Section 21.5 Electric Energy and Power

P21.14 $I=\dfrac{\mathcal{P}}{\Delta V}=\dfrac{600\text{ W}}{120\text{ V}}=\boxed{5.00\text{ A}}$

and $R=\dfrac{\Delta V}{I}=\dfrac{120\text{ V}}{5.00\text{ A}}=\boxed{24.0\ \Omega}$.

P21.15 $\mathcal{P}=0.800(1\,500\text{ hp})(746\text{ W/hp})=8.95\times10^{5}\text{ W}$

$\mathcal{P}=I\Delta V$ $\quad 8.95\times10^{5}=I(2\,000)$ $\quad I=\boxed{448\text{ A}}$

***P21.16** The battery takes in energy by electric transmission

$\mathcal{P}\Delta t=(\Delta V)I(\Delta t)=2.3\text{ J/C}\left(13.5\times10^{-3}\text{ C/s}\right)4.2\text{ h}\left(\dfrac{3\,600\text{ s}}{1\text{ h}}\right)=469\text{ J}$.

It puts out energy by electric transmission

$(\Delta V)I(\Delta t)=1.6\text{ J/C}\left(18\times10^{-3}\text{ C/s}\right)2.4\text{ h}\left(\dfrac{3\,600\text{ s}}{1\text{ h}}\right)=249\text{ J}$.

(a) efficiency $=\dfrac{\text{useful output}}{\text{total input}}=\dfrac{249\text{ J}}{469\text{ J}}=\boxed{0.530}$

continued on next page

(b) The only place for the missing energy to go is into internal energy:

$$469\ \text{J} = 249\ \text{J} + \Delta E_{\text{int}}$$

$$\Delta E_{\text{int}} = \boxed{221\ \text{J}}$$

(c) We imagine toasting the battery over a fire with 221 J of heat input:

$$Q = mc\Delta T$$

$$\Delta T = \frac{Q}{mc} = \frac{221\ \text{J}}{0.015\ \text{kg}}\ \frac{\text{kg}\,^\circ\text{C}}{975\ \text{J}} = \boxed{15.1^\circ\text{C}}$$

P21.17 $$\frac{\mathcal{P}}{\mathcal{P}_0} = \frac{(\Delta V)^2/R}{(\Delta V_0)^2/R} = \left(\frac{\Delta V}{\Delta V_0}\right)^2 = \left(\frac{140}{120}\right)^2 = 1.361$$

$$\Delta\% = \left(\frac{\mathcal{P}-\mathcal{P}_0}{\mathcal{P}_0}\right)(100\%) = \left(\frac{\mathcal{P}}{\mathcal{P}_0} - 1\right)(100\%) = (1.361-1)100\% = \boxed{36.1\%}$$

***P21.18** The energy taken in by electric transmission for the fluorescent lamp is

$$\mathcal{P}\Delta t = 11\ \text{J/s}(100\ \text{h})\left(\frac{3\,600\ \text{s}}{1\ \text{h}}\right) = 3.96\times10^6\ \text{J}$$

$$\text{cost} = 3.96\times10^6\ \text{J}\left(\frac{\$0.08}{\text{kWh}}\right)\left(\frac{\text{k}}{1\,000}\right)\left(\frac{\text{W}\cdot\text{s}}{\text{J}}\right)\left(\frac{\text{h}}{3\,600\ \text{s}}\right) = \$0.088$$

For the incandescent bulb,

$$\mathcal{P}\Delta t = 40\ \text{W}(100\ \text{h})\left(\frac{3\,600\ \text{s}}{1\ \text{h}}\right) = 1.44\times10^7\ \text{J}$$

$$\text{cost} = 1.44\times10^7\ \text{J}\left(\frac{\$0.08}{3.6\times10^6\ \text{J}}\right) = \$0.32$$

$$\text{saving} = \$0.32 - \$0.088 = \boxed{\$0.232}$$

P21.19 At operating temperature,

(a) $$\mathcal{P} = I\Delta V = (1.53\ \text{A})(120\ \text{V}) = \boxed{184\ \text{W}}$$

(b) Use the change in resistance to find the final operating temperature of the toaster.

$$R = R_0(1+\alpha\Delta T) \qquad \frac{120}{1.53} = \frac{120}{1.80}\left[1+\left(0.400\times10^{-3}\right)\Delta T\right]$$

$$\Delta T = 441^\circ\text{C} \qquad T = 20.0^\circ\text{C} + 441^\circ\text{C} = \boxed{461^\circ\text{C}}$$

P21.20 The total clock power is

$$\left(270\times10^6\ \text{clocks}\right)\left(2.50\ \frac{\text{J/s}}{\text{clock}}\right)\left(\frac{3\,600\ \text{s}}{1\ \text{h}}\right) = 2.43\times10^{12}\ \text{J/h}.$$

From $e = \frac{W_{\text{out}}}{Q_{\text{in}}}$, the power input to the generating plants must be:

$$\frac{Q_{\text{in}}}{\Delta t} = \frac{W_{\text{out}}/\Delta t}{e} = \frac{2.43\times10^{12}\ \text{J/h}}{0.250} = 9.72\times10^{12}\ \text{J/h}$$

and the rate of coal consumption is

$$\text{Rate} = \left(9.72\times10^{12}\ \text{J/h}\right)\left(\frac{1.00\ \text{kg coal}}{33.0\times10^6\ \text{J}}\right) = 2.95\times10^5\ \text{kg coal/h} = \boxed{295\ \text{metric ton/h}}.$$

P21.21 You pay the electric company for energy transferred in the amount $E = \mathcal{P}\Delta t$

(a) $$\mathcal{P}\Delta t = 40\ \text{W}(2\ \text{weeks})\left(\frac{7\ \text{d}}{1\ \text{week}}\right)\left(\frac{86\,400\ \text{s}}{1\ \text{d}}\right)\left(\frac{1\ \text{J}}{1\ \text{W}\cdot\text{s}}\right) = 48.4\ \text{MJ}$$

$$\mathcal{P}\Delta t = 40\ \text{W}(2\ \text{weeks})\left(\frac{7\ \text{d}}{1\ \text{week}}\right)\left(\frac{24\ \text{h}}{1\ \text{d}}\right)\left(\frac{\text{k}}{1\,000}\right) = 13.4\ \text{kWh}$$

$$\mathcal{P}\Delta t = 40\ \text{W}(2\ \text{weeks})\left(\frac{7\ \text{d}}{1\ \text{week}}\right)\left(\frac{24\ \text{h}}{1\ \text{d}}\right)\left(\frac{\text{k}}{1\,000}\right)\left(\frac{0.12\ \$}{\text{kWh}}\right) = \boxed{\$1.61}$$

(b) $$\mathcal{P}\Delta t = 970\ \text{W}(3\ \text{min})\left(\frac{1\ \text{h}}{60\ \text{min}}\right)\left(\frac{\text{k}}{1\,000}\right)\left(\frac{0.12\ \$}{\text{kWh}}\right) = \boxed{\$0.005\,82} = 0.582¢$$

(c) $$\mathcal{P}\Delta t = 5\,200\ \text{W}(40\ \text{min})\left(\frac{1\ \text{h}}{60\ \text{min}}\right)\left(\frac{\text{k}}{1\,000}\right)\left(\frac{0.12\ \$}{\text{kWh}}\right) = \boxed{\$0.416}$$

***P21.22** The heater should put out constant power

$$\mathcal{P} = \frac{Q}{\Delta t} = \frac{mc(T_f - T_i)}{\Delta t} = \frac{(0.250\ \text{kg})(4\,186\ \text{J})(100^\circ\text{C} - 20^\circ\text{C})}{\text{kg}\cdot{}^\circ\text{C}(4\ \text{min})}\left(\frac{1\ \text{min}}{60\ \text{s}}\right) = 349\ \text{J/s}$$

Then its resistance should be described by

$$\mathcal{P} = (\Delta V)I = \frac{(\Delta V)(\Delta V)}{R}$$

$$R = \frac{(\Delta V)^2}{\mathcal{P}} = \frac{(120\ \text{J/C})^2}{349\ \text{J/s}} = 41.3\ \Omega$$

continued on next page

Its resistivity at 100°C is given by

$$\rho = \rho_0\left[1+\alpha(T-T_0)\right] = \left(1.50\times10^{-6}\ \Omega\cdot\text{m}\right)\left[1+0.4\times10^{-3}(80)\right] = 1.55\times10^{-6}\ \Omega\cdot\text{m}$$

Then for a wire of circular cross section

$$R = \rho\frac{\ell}{A} = \rho\frac{\ell}{\pi r^2} = \rho\frac{\ell 4}{\pi d^2}$$

$$41.3\ \Omega = \left(1.55\times10^{-6}\ \Omega\cdot\text{m}\right)\frac{4\ell}{\pi d^2}$$

$$\frac{\ell}{d^2} = 2.09\times10^{+7}/\text{m} \qquad \text{or} \qquad d^2 = \left(4.77\times10^{-8}\ \text{m}\right)\ell$$

One possible choice is $\ell = 0.900$ m and $d = 2.07\times10^{-4}$ m. If ℓ and d are made too small, the surface area will be inadequate to transfer heat into the water fast enough to prevent overheating of the filament. To make the volume less than 0.5 cm^3, we want ℓ and d less than those described by $\frac{\pi d^2}{4}\ell = 0.5\times10^{-6}\ \text{m}^3$. Substituting $d^2 = \left(4.77\times10^{-8}\ \text{m}\right)\ell$ gives $\frac{\pi}{4}\left(4.77\times10^{-8}\ \text{m}\right)\ell^2 = 0.5\times10^{-6}\ \text{m}^3$, $\ell = 3.65$ m and $d = 4.18\times10^{-4}$ m. Thus our answer is: Any diameter d and length ℓ related by $d^2 = \left(4.77\times10^{-8}\ \text{m}\right)\ell$ would have the right resistance. One possibility is length 0.900 m and diameter 0.207 mm, but such a small wire might overheat rapidly if it were not surrounded by water. The volume can be less than 0.5 cm^3.

P21.23 (a) $\mathcal{P} = I\Delta V$

so $I = \frac{\mathcal{P}}{\Delta V} = \frac{8.00\times10^3\ \text{W}}{12.0\ \text{V}} = \boxed{667\ \text{A}}$.

(b) $\Delta t = \frac{\Delta U}{\mathcal{P}} = \frac{2.00\times10^7\ \text{J}}{8.00\times10^3\ \text{W}} = 2.50\times10^3\ \text{s}$

and $\Delta x = v\Delta t = (20.0\ \text{m/s})\left(2.50\times10^3\ \text{s}\right) = \boxed{50.0\ \text{km}}$.

P21.24 Consider a 400-W blow dryer used for ten minutes daily for a year. The energy transferred to the dryer is

$$\mathcal{P}\Delta t = (400\ \text{J/s})(600\ \text{s/d})(365\ \text{d}) \approx 9\times10^7\ \text{J}\left(\frac{1\ \text{kWh}}{3.6\times10^6\ \text{J}}\right) \approx 20\ \text{kWh}.$$

We suppose that electrically transmitted energy costs on the order of ten cents per kilowatt-hour. Then the cost of using the dryer for a year is on the order of

$$\text{Cost} \approx (20\ \text{kWh})(\$0.10/\text{kWh}) = \$2\ \boxed{\sim \$1}.$$

Section 21.6 Sources of emf

P21.25 (a) $\mathcal{P} = \frac{(\Delta V)^2}{R}$

becomes $20.0\ \text{W} = \frac{(11.6\ \text{V})^2}{R}$

so $R = \boxed{6.73\ \Omega}$.

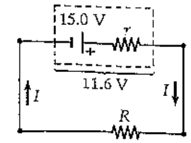


FIG. P21.25

(b) $\Delta V = IR$

so $11.6\ \text{V} = I(6.73\ \Omega)$

and $I = 1.72$ A

$\mathcal{E} = IR + Ir$

so $15.0\ \text{V} = 11.6\ \text{V} + (1.72\ \text{A})r$

$r = \boxed{1.97\ \Omega}$.

P21.26 The total resistance is $R = \frac{3.00\ \text{V}}{0.600\ \text{A}} = 5.00\ \Omega$.

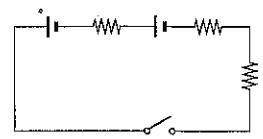


FIG. P21.26

(a) $R_{\text{lamp}} = R - r_{\text{batteries}} = 5.00\ \Omega - 0.408\ \Omega = \boxed{4.59\ \Omega}$

(b) $\frac{\mathcal{P}_{\text{batteries}}}{\mathcal{P}_{\text{total}}} = \frac{(0.408\ \Omega)I^2}{(5.00\ \Omega)I^2} = 0.081\,6 = \boxed{8.16\%}$

Section 21.7 Resistors in Series and in Parallel

P21.27 (a) $R_p = \frac{1}{(1/7.00\ \Omega)+(1/10.0\ \Omega)} = 4.12\ \Omega$

$R_s = R_1 + R_2 + R_3 = 4.00 + 4.12 + 9.00 = \boxed{17.1\ \Omega}$

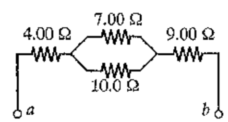


FIG. P21.27

(b) $\Delta V = IR$

$34.0\ \text{V} = I(17.1\ \Omega)$

$I = \boxed{1.99\ \text{A}}$ for 4.00 Ω, 9.00 Ω resistors.

Applying $\Delta V = IR$, $(1.99\ \text{A})(4.12\ \Omega) = 8.18\ \text{V}$

$8.18\ \text{V} = I(7.00\ \Omega)$

so $I = \boxed{1.17\ \text{A}}$ for 7.00 Ω resistor

$8.18\ \text{V} = I(10.0\ \Omega)$

so $I = \boxed{0.818\ \text{A}}$ for 10.0 Ω resistor.

P21.28 We assume that the metal wand makes low-resistance contact with the person's hand and that the resistance through the person's body is negligible compared to the resistance R_{shoes} of the shoe soles. The equivalent resistance seen by the power supply is $1.00\ \text{M}\Omega + R_{shoes}$. The current through both resistors is $\dfrac{50.0\ \text{V}}{1.00\ \text{M}\Omega + R_{shoes}}$. The voltmeter displays

$$\Delta V = I(1.00\ \text{M}\Omega) = \frac{50.0\ \text{V}(1.00\ \text{M}\Omega)}{1.00\ \text{M}\Omega + R_{shoes}} = \Delta V.$$

(a) We solve to obtain $50.0\ \text{V}(1.00\ \text{M}\Omega) = \Delta V(1.00\ \text{M}\Omega) + \Delta V(R_{shoes})$

$$R_{shoes} = \frac{1.00\ \text{M}\Omega(50.0 - \Delta V)}{\Delta V}.$$

(b) With $R_{shoes} \to 0$, the current through the person's body is

$\dfrac{50.0\ \text{V}}{1.00\ \text{M}\Omega} = 50.0\ \mu\text{A}$ $\boxed{\text{The current will never exceed } 50\ \mu\text{A}}$.

P21.29 If we turn the given diagram on its side, we find that it is the same as figure (a). The 20.0 Ω and 5.00 Ω resistors are in series, so the first reduction is shown in (b). In addition, since the 10.0 Ω, 5.00 Ω, and 25.0 Ω resistors are then in parallel, we can solve for their equivalent resistance as:

$$R_{eq} = \frac{1}{\left(\frac{1}{10.0\ \Omega} + \frac{1}{5.00\ \Omega} + \frac{1}{25.0\ \Omega}\right)} = 2.94\ \Omega.$$

This is shown in figure (c), which in turn reduces to the circuit shown in figure (d).

Next, we work backwards through the diagrams applying $I = \dfrac{\Delta V}{R}$ and $\Delta V = IR$ alternately to every resistor, real and equivalent. The 12.94 Ω resistor is connected across 25.0 V, so the current through the battery in every diagram is

$$I = \frac{\Delta V}{R} = \frac{25.0\ \text{V}}{12.94\ \Omega} = 1.93\ \text{A}.$$

In figure (c), this 1.93 A goes through the 2.94 Ω equivalent resistor to give a potential difference of:

$$\Delta V = IR = (1.93\ \text{A})(2.94\ \Omega) = 5.68\ \text{V}.$$

From figure (b), we see that this potential difference is the same across ΔV_{ab}, the 10 Ω resistor, and the 5.00 Ω resistor.

(b) Therefore, $\Delta V_{ab} = \boxed{5.68\ \text{V}}$.

(a) Since the current through the 20.0 Ω resistor is also the current through the 25.0 Ω line ab,

$$I = \frac{\Delta V_{ab}}{R_{ab}} = \frac{5.68\ \text{V}}{25.0\ \Omega} = 0.227\ \text{A} = \boxed{227\ \text{mA}}.$$

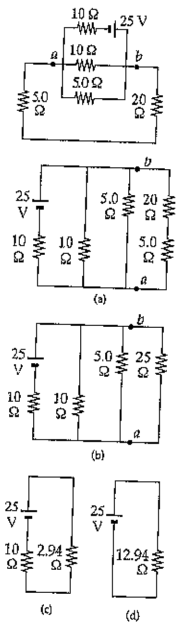


FIG. P21.29

P21.30 (a) Since all the current in the circuit must pass through the series 100 Ω resistor, $\mathcal{P} = I^2R$

$$\mathcal{P}_{max} = RI_{max}^2$$

so $I_{max} = \sqrt{\dfrac{\mathcal{P}}{R}} = \sqrt{\dfrac{25.0\ \text{W}}{100\ \Omega}} = 0.500\ \text{A}$

$$R_{eq} = 100\ \Omega + \left(\frac{1}{100} + \frac{1}{100}\right)^{-1}\ \Omega = 150\ \Omega$$

$$\Delta V_{max} = R_{eq}I_{max} = \boxed{75.0\ \text{V}}$$

(b) $\mathcal{P} = I\Delta V = (0.500\ \text{A})(75.0\ \text{V}) = \boxed{37.5\ \text{W}}$ total power

$\mathcal{P}_1 = \boxed{25.0\ \text{W}}$

$\mathcal{P}_2 = \mathcal{P}_3 = RI^2 = (100\ \Omega)(0.250\ \text{A})^2 = \boxed{6.25\ \text{W}}$

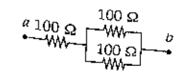


FIG. P21.30

P21.31 $R_p = \left(\dfrac{1}{3.00} + \dfrac{1}{1.00}\right)^{-1} = 0.750\ \Omega$

$R_s = (2.00 + 0.750 + 4.00)\ \Omega = 6.75\ \Omega$

$I_{battery} = \dfrac{\Delta V}{R_s} = \dfrac{18.0\ \text{V}}{6.75\ \Omega} = 2.67\ \text{A}$

$\mathcal{P} = I^2R$: $\mathcal{P}_2 = (2.67\ \text{A})^2(2.00\ \Omega)$

$\mathcal{P}_2 = \boxed{14.2\ \text{W}}$ in 2.00 Ω

$\mathcal{P}_4 = (2.67\ \text{A})^2(4.00\ \text{A}) = \boxed{28.4\ \text{W}}$ in 4.00 Ω

$\Delta V_2 = (2.67\ \text{A})(2.00\ \Omega) = 5.33\ \text{V},$

$\Delta V_4 = (2.67\ \text{A})(4.00\ \Omega) = 10.67\ \text{V}$

$\Delta V_p = 18.0\ \text{V} - \Delta V_2 - \Delta V_4 = 2.00\ \text{V} (= \Delta V_3 = \Delta V_1)$

$\mathcal{P}_3 = \dfrac{(\Delta V_3)^2}{R_3} = \dfrac{(2.00\ \text{V})^2}{3.00\ \Omega} = \boxed{1.33\ \text{W}}$ in 3.00 Ω

$\mathcal{P}_1 = \dfrac{(\Delta V_1)^2}{R_1} = \dfrac{(2.00\ \text{V})^2}{1.00\ \Omega} = \boxed{4.00\ \text{W}}$ in 1.00 Ω

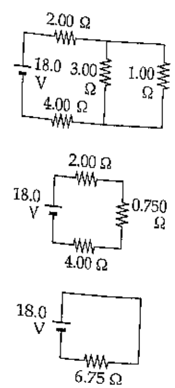


FIG. P21.31

***P21.32** (a) The resistors 2, 3, and 4 can be combined to a single $2R$ resistor. This is in series with resistor 1, with resistance R, so the equivalent resistance of the whole circuit is $3R$. In series, potential difference is shared in proportion to the resistance, so resistor 1 gets $\frac{1}{3}$ of the battery voltage and the 2-3-4 parallel combination get $\frac{2}{3}$ of the battery voltage. This is the potential difference across resistor 4, but resistors 2 and 3 must share this voltage. $\frac{1}{3}$ goes to 2 and $\frac{2}{3}$ to 3. The ranking by potential difference is $\boxed{\Delta V_4 > \Delta V_3 > \Delta V_1 > \Delta V_2}$.

continued on next page

(b) Based on the reasoning above the potential differences are

$$\boxed{\Delta V_1 = \frac{\mathcal{E}}{3},\ \Delta V_2 = \frac{2\mathcal{E}}{9},\ \Delta V_3 = \frac{4\mathcal{E}}{9},\ \Delta V_4 = \frac{2\mathcal{E}}{3}}.$$

(c) All the current goes through resistor 1, so it gets the most. The current then splits at the parallel combination. Resistor 4 gets more than half, because the resistance in that branch is less than in the other branch. Resistors 2 and 3 have equal currents because they are in series. The ranking by current is $\boxed{I_1 > I_4 > I_2 = I_3}$.

(d) Resistor 1 has a current of I. Because the resistance of 2 and 3 in series is twice that of resistor 4, twice as much current goes through 4 as through 2 and 3. The current through the resistors are $\boxed{I_1 = I,\ I_2 = I_3 = \frac{I}{3},\ I_4 = \frac{2I}{3}}$.

(e) Increasing resistor 3 increases the equivalent resistance of the entire circuit. The current in the circuit, which is the current through resistor 1, decreases. This decreases the potential difference across resistor 1, increasing the potential difference across the parallel combination. With a larger potential difference the current through resistor 4 is increased. With more current through 4, and less in the circuit to start with, the current through resistors 2 and 3 must decrease. To summarize, $\boxed{I_4 \text{ increases and } I_1,\ I_2, \text{ and } I_3 \text{ decrease}}$.

(f) If resistor 3 has an infinite resistance it blocks any current from passing through that branch, and the circuit effectively is just resistor 1 and resistor 4 in series with the battery. The circuit now has an equivalent resistance of $4R$. The current in the circuit drops to $\frac{3}{4}$ of the original current because the resistance has increased by $\frac{4}{3}$. All this current passes through resistors 1 and 4, and none passes through 2 or 3. Therefore $\boxed{I_1 = \frac{3I}{4},\ I_2 = I_3 = 0,\ I_4 = \frac{3I}{4}}$.

*P21.33 (a) We find the resistance intrinsic to the vacuum cleaner:

$$\mathcal{P} = I\Delta V = \frac{(\Delta V)^2}{R}$$

$$R = \frac{(\Delta V)^2}{\mathcal{P}} = \frac{(120\text{ V})^2}{535\text{ W}} = 26.9\ \Omega$$

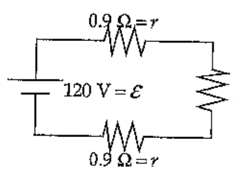


FIG. P21.33

with the inexpensive cord, the equivalent resistance is $0.9\ \Omega + 26.9\ \Omega + 0.9\ \Omega = 28.7\ \Omega$ so the current throughout the circuit is

$$I = \frac{\mathcal{E}}{R_{\text{Tot}}} = \frac{120\text{ V}}{28.7\ \Omega} = 4.18\text{ A}$$

continued on next page

and the cleaner power is

$$\mathcal{P}_{\text{cleaner}} = I(\Delta V)_{\text{cleaner}} = I^2 R = (4.18\text{ A})^2(26.9\ \Omega) = \boxed{470\text{ W}}.$$

In symbols, $R_{\text{Tot}} = R + 2r$, $I = \frac{\mathcal{E}}{R+2r}$ and $\mathcal{P}_{\text{cleaner}} = I^2 R = \frac{\mathcal{E}^2 R}{(R+2r)^2}$.

(b)

$$R + 2r = \left(\frac{\mathcal{E}^2 R}{\mathcal{P}_{\text{cleaner}}}\right)^{1/2} = 120\text{ V}\left(\frac{26.9\ \Omega}{525\text{ W}}\right)^{1/2} = 27.2\ \Omega$$

$$r = \frac{27.2\ \Omega - 26.9\ \Omega}{2} = 0.128\ \Omega = \frac{\rho\ell}{A} = \frac{\rho\ell 4}{\pi d^2}$$

$$d = \left(\frac{4\rho\ell}{\pi r}\right)^{1/2} = \left(\frac{4(1.7\times10^{-8}\ \Omega\cdot\text{m})(15\text{ m})}{\pi(0.128\ \Omega)}\right)^{1/2} = \boxed{1.60\text{ mm or more}}$$

(c) Unless the extension cord is a superconductor, it is impossible to attain cleaner power 535 W. To move from 525 W to 532 W will require a lot more copper, as we show here:

$$r = \frac{\mathcal{E}}{2}\left(\frac{R}{\mathcal{P}_{\text{cleaner}}}\right)^{1/2} - \frac{R}{2} = \frac{120\text{ V}}{2}\left(\frac{26.9\ \Omega}{532\text{ W}}\right)^{1/2} - \frac{26.9\ \Omega}{2} = 0.037\,9\ \Omega$$

$$d = \left(\frac{4\rho\ell}{\pi r}\right)^{1/2} = \left(\frac{4(1.7\times10^{-8}\ \Omega\cdot\text{m})(15\text{ m})}{\pi(0.037\,9\ \Omega)}\right)^{1/2} = \boxed{2.93\text{ mm or more}}$$

Section 21.8 Kirchhoff's Rules

P21.34 $+15.0 - (7.00)I_1 - (2.00)(5.00) = 0$

$5.00 = 7.00 I_1$ so $\boxed{I_1 = 0.714\text{ A}}$

$I_1 + I_2 - 2.00\text{ A} = 0$

$0.714 + I_2 = 2.00$ so $\boxed{I_2 = 1.29\text{ A}}$

$+\mathcal{E} - 2.00(1.29) - 5.00(2.00) = 0$ $\boxed{\mathcal{E} = 12.6\text{ V}}$

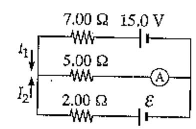


FIG. P21.34

P21.35 We name currents I_1, I_2, and I_3 as shown.

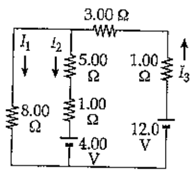


FIG. P21.35

From Kirchhoff's current rule, $I_3 - I_1 - I_2 = 0$.

Applying Kirchhoff's voltage rule to the loop containing I_2 and I_3,

$$12.0\text{ V} - (4.00)I_3 - (6.00)I_2 - 4.00\text{ V} = 0$$
$$8.00 = (4.00)I_3 + (6.00)I_2$$

Applying Kirchhoff's voltage rule to the loop containing I_1 and I_2,

$$-(6.00)I_2 - 4.00\text{ V} + (8.00)I_1 = 0 \qquad (8.00)I_1 = 4.00 + (6.00)I_2.$$

Solving the above linear system, we proceed to the pair of simultaneous equations:

$$\begin{cases} 8 = 4I_1 + 4I_2 + 6I_2 \\ 8I_1 = 4 + 6I_2 \end{cases} \quad \text{or} \quad \begin{cases} 8 = 4I_1 + 10I_2 \\ I_2 = 1.33I_1 - 0.667 \end{cases}$$

and to the single equation $8 = 4I_1 + 13.3I_1 - 6.67$

$$I_1 = \frac{14.7\text{ V}}{17.3\ \Omega} = 0.846\text{ A}. \qquad \text{Then} \qquad I_2 = 1.33(0.846\text{ A}) - 0.667$$

and $\quad I_3 = I_1 + I_2 \quad$ give $\quad \boxed{I_1 = 846\text{ mA},\ I_2 = 462\text{ mA},\ I_3 = 1.31\text{ A}}$

All currents are in the directions indicated by the arrows in the circuit diagram.

P21.36 See Figure P21.36 for solution to this problem.

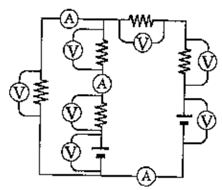


FIG. P21.36

P21.37 We use the results of Problem 21.35.

(a) By the 4.00-V battery: $\Delta U = (\Delta V)I\Delta t = (4.00\text{ V})(-0.462\text{ A})120\text{ s} = \boxed{-222\text{ J}}$.

By the 12.0-V battery: $(12.0\text{ V})(1.31\text{ A})120\text{ s} = \boxed{1.88\text{ kJ}}$

(b) By the 8.00-Ω resistor: $I^2R\Delta t = (0.846\text{ A})^2(8.00\ \Omega)120\text{ s} = \boxed{687\text{ J}}$.

By the 5.00-Ω resistor: $(0.462\text{ A})^2(5.00\ \Omega)120\text{ s} = \boxed{128\text{ J}}$.

By the 1.00-Ω resistor: $(0.462\text{ A})^2(1.00\ \Omega)120\text{ s} = \boxed{25.6\text{ J}}$.

By the 3.00-Ω resistor: $(1.31\text{ A})^2(3.00\ \Omega)120\text{ s} = \boxed{616\text{ J}}$.

By the 1.00-Ω resistor: $(1.31\text{ A})^2(1.00\ \Omega)120\text{ s} = \boxed{205\text{ J}}$.

continued on next page

(c) We can count the energy twice, like a child counting his lunch money at ten o'clock and again at eleven.

$-222\text{ J} + 1.88\text{ kJ} = \boxed{1.66\text{ kJ}}$ from chemical to electrically transmitted.

$687\text{ J} + 128\text{ J} + 25.6\text{ J} + 616\text{ J} + 205\text{ J} = 1.66\text{ kJ}$ from electrically transmitted to internal.

***P21.38** (a) The first equation represents Kirchhoff's loop theorem. We choose to think of it as describing a clockwise trip around the left-hand loop in a circuit, see Figure (a). For the right-hand loop see Figure (b). The junctions must be between the 5.8 V and the 370 Ω and between the 370 Ω and the 150 Ω. Then we have Figure (c). This is consistent with the third equation,

$$I_1 + I_3 - I_2 = 0$$
$$I_2 = I_1 + I_3$$

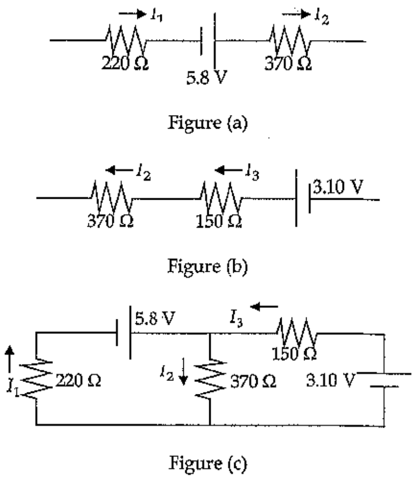


FIG. P21.38

(b) We substitute:

$$-220I_1 + 5.8 - 370I_1 - 370I_3 = 0$$
$$+370I_1 + 370I_3 + 150I_3 - 3.1 = 0$$

Next

$$I_3 = \frac{5.8 - 590I_1}{370}$$
$$370I_1 + \frac{520}{370}(5.8 - 590I_1) - 3.1 = 0$$
$$370I_1 + 8.15 - 829I_1 - 3.1 = 0$$
$$I_1 = \frac{5.05\text{ V}}{459\ \Omega} = \boxed{\text{11.0 mA in the 220-Ω resistor and out of the positive pole of the 5.8-V battery}}$$
$$I_3 = \frac{5.8 - 590(0.011\,0)}{370} = -1.87\text{ mA}$$

$\boxed{\text{The current is 1.87 mA in the 150-Ω resistor and out of the negative pole of the 3.1-V battery}}$

$I_2 = 11.0 - 1.87 = \boxed{\text{9.13 mA in the 370-Ω resistor}}$

P21.39 Label the currents in the branches as shown in the first figure. Reduce the circuit by combining the two parallel resistors as shown in the second figure.

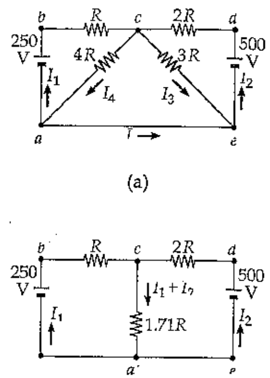


FIG. P21.39

Apply Kirchhoff's loop rule to both loops in Figure (b) to obtain:

$$(2.71R)I_1 + (1.71R)I_2 = 250$$

and $$(1.71R)I_1 + (3.71R)I_2 = 500.$$

With $R = 1\,000\ \Omega$, simultaneous solution of these equations yields:

$$I_1 = 10.0 \text{ mA}$$

and $$I_2 = 130.0 \text{ mA}.$$

From Figure (b), $V_c - V_a = (I_1 + I_2)(1.71R) = 240 \text{ V}.$

Thus, from Figure (a), $I_4 = \dfrac{V_c - V_a}{4R} = \dfrac{240 \text{ V}}{4\,000\ \Omega} = 60.0 \text{ mA}.$

Finally, applying Kirchhoff's point rule at point a in Figure (a) gives:

$$I_4 - I_1 - I = 0$$
$$I = I_4 - I_1 = 60.0 \text{ mA} - 10.0 \text{ mA} = +50.0 \text{ mA}$$

or $I = \boxed{50.0 \text{ mA from point } a \text{ to point } e}$

P21.40 Using Kirchhoff's rules,

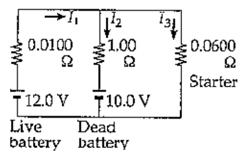


FIG. P21.40

$$12.0 - (0.010\,0)I_1 - (0.060\,0)I_3 = 0$$
$$10.0 + (1.00)I_2 - (0.060\,0)I_3 = 0$$

and $I_1 - I_2 - I_3 = 0$ or $I_1 = I_2 + I_3$

then $12.0 - (0.010\,0)I_2 - (0.070\,0)I_3 = 0$

and $I_2 = 0.06I_3 - 10$

Solving simultaneously, $12 - (0.01)(0.06I_3 - 10) - 0.07I_3 = 0$

$I_2 = \boxed{0.283 \text{ A downward}}$ in the dead battery

and $I_3 = \boxed{171 \text{ A downward}}$ in the starter.

The currents are forward in the live battery and in the starter, relative to normal starting operation. The current is backward in the dead battery, tending to charge it up.

Section 21.9 RC Circuits

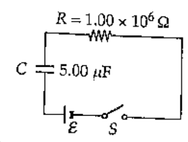


FIG. P21.41

P21.41 (a) $RC = (1.00\times10^6\ \Omega)(5.00\times10^{-6}\text{ F}) = \boxed{5.00 \text{ s}}$

(b) $Q = C\mathcal{E} = (5.00\times10^{-6}\text{ C})(30.0\text{ V}) = \boxed{150\ \mu\text{C}}$

(c) $I(t) = \dfrac{\mathcal{E}}{R}e^{-t/RC} = \left(\dfrac{30.0}{1.00\times10^6}\right)\exp\left[\dfrac{-10.0}{(1.00\times10^6)(5.00\times10^{-6})}\right] = \boxed{4.06\ \mu\text{A}}$

P21.42 (a) $I(t) = -I_0 e^{-t/RC}$

$$I_0 = \frac{Q}{RC} = \frac{5.10\times10^{-6}\text{ C}}{(1\,300\ \Omega)(2.00\times10^{-9}\text{ F})} = 1.96 \text{ A}$$

$$I(t) = -(1.96\text{ A})\exp\left[\frac{-9.00\times10^{-6}\text{ s}}{(1\,300\ \Omega)(2.00\times10^{-9}\text{ F})}\right] = \boxed{-61.6 \text{ mA}}$$

(b) $q(t) = Qe^{-t/RC} = (5.10\ \mu\text{C})\exp\left[\dfrac{-8.00\times10^{-6}\text{ s}}{(1\,300\ \Omega)(2.00\times10^{-9}\text{ F})}\right] = \boxed{0.235\ \mu\text{C}}$

(c) The magnitude of the maximum current is $I_0 = \boxed{1.96 \text{ A}}$.

P21.43 (a) $\tau = RC = (1.50\times10^5\ \Omega)(10.0\times10^{-6}\text{ F}) = \boxed{1.50 \text{ s}}$

(b) $\tau = (1.00\times10^5\ \Omega)(10.0\times10^{-6}\text{ F}) = \boxed{1.00 \text{ s}}$

(c) The battery carries current $\dfrac{10.0 \text{ V}}{50.0\times10^3\ \Omega} = 200\ \mu\text{A}.$

The 100 kΩ carries current of magnitude $I = I_0 e^{-t/RC} = \left(\dfrac{10.0 \text{ V}}{100\times10^3\ \Omega}\right)e^{-t/1.00\text{ s}}$

So the switch carries downward current $\boxed{200\ \mu\text{A} + (100\ \mu\text{A})e^{-t/1.00\text{ s}}}$.

***P21.44** (a) We model the person's body and street shoes as shown. For the discharge to reach 100 V,

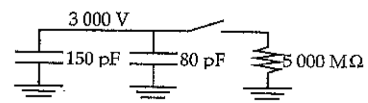


FIG. P21.44(a)

$$q(t) = Qe^{-t/RC} = C\Delta V(t) = C\Delta V_0 e^{-t/RC}$$

$$\frac{\Delta V}{\Delta V_0} = e^{-t/RC} \qquad \frac{\Delta V_0}{\Delta V} = e^{+t/RC} \qquad \frac{t}{RC} = \ln\left(\frac{\Delta V_0}{\Delta V}\right)$$

$$t = RC\ln\left(\frac{\Delta V_0}{\Delta V}\right) = 5\,000\times10^6\ \Omega(230\times10^{-12}\text{ F})\ln\left(\frac{3\,000}{100}\right) = \boxed{3.91 \text{ s}}$$

(b) $t = 1\times10^6\text{ V/A}(230\times10^{-12}\text{ C/V})\ln 30 = \boxed{782\ \mu\text{s}}$

P21.45 (a) Call the potential at the left junction V_L and at the right V_R. After a "long" time, the capacitor is fully charged.

$V_L = 8.00$ V because of voltage divider:

$$I_L = \frac{10.0\text{ V}}{5.00\ \Omega} = 2.00\text{ A}$$

$$V_L = 10.0\text{ V} - (2.00\text{ A})(1.00\ \Omega) = 8.00\text{ V}$$

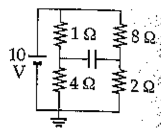


FIG. P21.45(a)

Likewise, $$V_R = \left(\frac{2.00\ \Omega}{2.00\ \Omega + 8.00\ \Omega}\right)(10.0\text{ V}) = 2.00\text{ V}$$

or $$I_R = \frac{10.0\text{ V}}{10.0\ \Omega} = 1.00\text{ A}$$

$$V_R = (10.0\text{ V}) - (8.00\ \Omega)(1.00\text{ A}) = 2.00\text{ V}.$$

Therefore, $$\Delta V = V_L - V_R = 8.00 - 2.00 = \boxed{6.00\text{ V}}.$$

(b) Redraw the circuit $$R = \frac{1}{(1/9.00\ \Omega) + (1/6.00\ \Omega)} = 3.60\ \Omega$$

$$RC = 3.60 \times 10^{-6}\text{ s}$$

and $$e^{-t/RC} = \frac{1}{10}$$

so $$t = RC\ln 10 = \boxed{8.29\ \mu\text{s}}$$

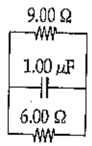


FIG. P21.45(b)

P21.46 The potential difference across the capacitor $\Delta V(t) = \Delta V_{\max}\left(1 - e^{-t/RC}\right)$.

Using 1 Farad = 1 s/Ω, $$4.00\text{ V} = (10.0\text{ V})\left[1 - e^{-(3.00\text{ s})/\left[R\left(10.0\times10^{-6}\text{ s}/\Omega\right)\right]}\right].$$

Therefore, $$0.400 = 1.00 - e^{-(3.00\times10^5\ \Omega)/R}$$

or $$e^{-(3.00\times10^5\ \Omega)/R} = 0.600.$$

Taking the natural logarithm of both sides, $$-\frac{3.00\times10^5\ \Omega}{R} = \ln(0.600)$$

and $$R = -\frac{3.00\times10^5\ \Omega}{\ln(0.600)} = +5.87\times10^5\ \Omega = \boxed{587\text{ k}\Omega}.$$

Section 21.10 Context Connection—The Atmosphere as a Conductor

P21.47 $J = \sigma E$ so $$\sigma = \frac{J}{E} = \frac{6.00\times10^{-13}\text{ A/m}^2}{100\text{ V/m}} = \boxed{6.00\times10^{-15}\ (\Omega\cdot\text{m})^{-1}}$$

P21.48 (a) $$\mathcal{P} = \Delta VI = \left(300\times10^3\text{ J/C}\right)\left(1.00\times10^3\text{ C/s}\right) = \boxed{3.00\times10^8\text{ W}}$$

A large electric generating station, fed by a trainload of coal each day, converts energy faster.

(b) $$I = \frac{\mathcal{P}}{A} = \frac{\mathcal{P}}{\pi r^2} \qquad \mathcal{P} = I\left(\pi r^2\right) = \left(1\,370\text{ W/m}^2\right)\left[\pi\left(6.37\times10^6\text{ m}\right)^2\right] = \boxed{1.75\times10^{17}\text{ W}}$$

Terrestrial solar power is immense compared to lightning and compared to all human energy conversions.

Additional Problems

P21.49 (a) $$I = \frac{\Delta V}{R} \quad\text{so}\quad \mathcal{P} = I\Delta V = \frac{(\Delta V)^2}{R}$$

$$R = \frac{(\Delta V)^2}{\mathcal{P}} = \frac{(120\text{ V})^2}{25.0\text{ W}} = \boxed{576\ \Omega} \quad\text{and}\quad R = \frac{(\Delta V)^2}{\mathcal{P}} = \frac{(120\text{ V})^2}{100\text{ W}} = \boxed{144\ \Omega}$$

(b) $$I = \frac{\mathcal{P}}{\Delta V} = \frac{25.0\text{ W}}{120\text{ V}} = 0.208\text{ A} = \frac{Q}{\Delta t} = \frac{1.00\text{ C}}{\Delta t}$$

$$\Delta t = \frac{1.00\text{ C}}{0.208\text{ A}} = \boxed{4.80\text{ s}}$$

The bulb takes in charge at high potential and puts out the same amount of charge at low potential. The charge in itself is identical.

(c) $$\mathcal{P} = 25.0\text{ W} = \frac{\Delta U}{\Delta t} = \frac{1.00\text{ J}}{\Delta t} \qquad \Delta t = \frac{1.00\text{ J}}{25.0\text{ W}} = \boxed{0.040\,0\text{ s}}$$

The bulb takes in energy by electrical transmission and puts out the same amount of energy by heat and light.

(d) $$\Delta U = \mathcal{P}\Delta t = (25.0\text{ J/s})(86\,400\text{ s/d})(30.0\text{ d}) = 64.8\times10^8\text{ J}$$

The electric company sells $\boxed{\text{energy}}$

$$\text{Cost} = 64.8\times10^6\text{ J}\left(\frac{\$0.070\,0}{\text{kWh}}\right)\left(\frac{\text{k}}{1\,000}\right)\left(\frac{\text{W}\cdot\text{s}}{\text{J}}\right)\left(\frac{\text{h}}{3\,600\text{ s}}\right) = \boxed{\$1.26}$$

$$\text{Cost per joule} = \frac{\$0.070\,0}{\text{kWh}}\left(\frac{\text{kWh}}{3.60\times10^6\text{ J}}\right) = \boxed{\$1.94\times10^{-8}/\text{J}}$$

P21.50 $$\rho = \frac{RA}{\ell} = \frac{(\Delta V)}{I}\frac{A}{\ell}$$

ℓ (m)	R (Ω)	ρ (Ω·m)
0.540	10.4	1.41×10^{-6}
1.028	21.1	1.50×10^{-6}
1.543	31.8	1.50×10^{-6}

$\bar{\rho} = \boxed{1.47\times10^{-6}\ \Omega\cdot\text{m}}$ (in agreement with tabulated value of $1.50\times10^{-6}\ \Omega\cdot\text{m}$ in Table 21.1)

P21.51 (a) $\vec{E} = -\frac{dV}{dx}\hat{i} = -\frac{(0-4.00)\text{ V}}{(0.500-0)\text{ m}} = \boxed{8.00\hat{i}\text{ V/m}}$

(b) $R = \frac{\rho\ell}{A} = \frac{(4.00\times10^{-8}\ \Omega\cdot\text{m})(0.500\text{ m})}{\pi(1.00\times10^{-4}\text{ m})^2} = \boxed{0.637\ \Omega}$

(c) $I = \frac{\Delta V}{R} = \frac{4.00\text{ V}}{0.637\ \Omega} = \boxed{6.28\text{ A}}$

(d) $\vec{J} = \frac{I}{A}\hat{i} = \frac{6.28\text{ A}}{\pi(1.00\times10^{-4}\text{ m})^2} = 2.00\times10^8\hat{i}\text{ A/m}^2 = \boxed{200\hat{i}\text{ MA/m}^2}$

(e) $\rho\vec{J} = (4.00\times10^{-8}\ \Omega\cdot\text{m})(2.00\times10^8\hat{i}\text{ A/m}^2) = 8.00\hat{i}\text{ V/m} = \vec{E}$

P21.52 (a) $\vec{E} = -\frac{dV(x)}{dx}\hat{i} = \boxed{\frac{V}{L}\hat{i}}$ Note that the field is independent of the material and the area.

(b) $R = \frac{\rho\ell}{A} = \boxed{\frac{4\rho L}{\pi d^2}}$ Independent of voltage, field, and current.

(c) $I = \frac{\Delta V}{R} = \boxed{\frac{V\pi d^2}{4\rho L}}$ This is how Georg Simon Ohm thought of his experimental results on all the things current depends on.

(d) $\vec{J} = \frac{I}{A}\hat{i} = \boxed{\frac{V}{\rho L}\hat{i}}$ Independent of area.

(e) $\rho\vec{J} = \frac{V}{L}\hat{i} = \boxed{\vec{E}}$ Independent of length and area. At each point within the material, vector field causes vector current density according to this relation. This is how an atom thinks of Ohm's law.

P21.53 (a) $\mathscr{P} = I\Delta V$: So for the Heater, $I = \frac{\mathscr{P}}{\Delta V} = \frac{1500\text{ W}}{120\text{ V}} = \boxed{12.5\text{ A}}$.

For the Toaster, $I = \frac{750\text{ W}}{120\text{ V}} = \boxed{6.25\text{ A}}$.

And for the Grill, $I = \frac{1000\text{ W}}{120\text{ V}} = \boxed{8.33\text{ A}}$.

(b) $12.5 + 6.25 + 8.33 = \boxed{27.1\text{ A}}$

The current draw is greater than 25.0 amps, so this circuit would not be sufficient.

P21.54 (a) A thin cylindrical shell of radius r, thickness dr, and length L contributes resistance

$$dR = \frac{\rho\, d\ell}{A} = \frac{\rho\, dr}{(2\pi r)L} = \left(\frac{\rho}{2\pi L}\right)\frac{dr}{r}$$

The resistance of the whole annulus is the series summation of the contributions of the thin shells:

$$R = \frac{\rho}{2\pi L}\int_{r_a}^{r_b}\frac{dr}{r} = \boxed{\frac{\rho}{2\pi L}\ln\left(\frac{r_b}{r_a}\right)}.$$

(b) In this equation $\frac{\Delta V}{I} = \frac{\rho}{2\pi L}\ln\left(\frac{r_b}{r_a}\right)$

we solve for $\boxed{\rho = \frac{2\pi L\Delta V}{I\ln(r_b/r_a)}}$.

***P21.55** The set of four batteries boosts the electric potential of each bit of charge that goes through them by $4\times1.50\text{ V} = 6.00\text{ V}$. The chemical energy they store is

$$\Delta U = q\Delta V = (240\text{ C})(6.00\text{ J/C}) = 1\,440\text{ J}.$$

The radio draws current $I = \frac{\Delta V}{R} = \frac{6.00\text{ V}}{200\ \Omega} = 0.030\,0\text{ A}$.

So, its power is $\mathscr{P} = (\Delta V)I = (6.00\text{ V})(0.030\,0\text{ A}) = 0.180\text{ W} = 0.180\text{ J/s}$.

Then for the time the energy lasts, we have $\mathscr{P} = \frac{E}{\Delta t}$ $\Delta t = \frac{E}{\mathscr{P}} = \frac{1\,440\text{ J}}{0.180\text{ J/s}} = 8.00\times10^3\text{ s}$.

We could also compute this from $I = \frac{Q}{\Delta t}$ $\Delta t = \frac{Q}{I} = \frac{240\text{ C}}{0.030\,0\text{ A}} = 8.00\times10^3\text{ s} = \boxed{2.22\text{ h}}$.

P21.56 $I = \frac{\mathcal{E}}{R+r}$, so $\mathscr{P} = I^2R = \frac{\mathcal{E}^2R}{(R+r)^2}$ or $(R+r)^2 = \left(\frac{\mathcal{E}^2}{\mathscr{P}}\right)R$.

Let $x \equiv \frac{\mathcal{E}^2}{\mathscr{P}}$, then $(R+r)^2 = xR$ or $R^2 + (2r-x)R - r^2 = 0$.

With $r = 1.20\ \Omega$, this becomes $R^2 + (2.40-x)R - 1.44 = 0$,

which has solutions of $R = \frac{-(2.40-x)\pm\sqrt{(2.40-x)^2-5.76}}{2}$

(a) With $\mathcal{E} = 9.20\text{ V}$ and $\mathscr{P} = 12.8\text{ W}$, $x = 6.61$:

$$R = \frac{+4.21\pm\sqrt{(4.21)^2-5.76}}{2} = \boxed{3.84\ \Omega}\text{ or }\boxed{0.375\ \Omega}.$$

continued on next page

(b) For $\mathcal{E} = 9.20$ V and $\mathcal{P} = 21.2$ W, $x \equiv \frac{\mathcal{E}^2}{\mathcal{P}} = 3.99$

$$R = \frac{+1.59 \pm \sqrt{(1.59)^2 - 5.76}}{2} = \frac{1.59 \pm \sqrt{-3.22}}{2}$$

The equation for the load resistance yields a complex number, so $\boxed{\text{there is no resistance}}$ that will extract 21.2 W from this battery. The maximum power output occurs when $R = r = 1.20\ \Omega$, and that maximum is: $\mathcal{P}_{max} = \frac{\mathcal{E}^2}{4r} = 17.6$ W.

P21.57 The current in the simple loop circuit will be $I = \frac{\mathcal{E}}{R + r}$.

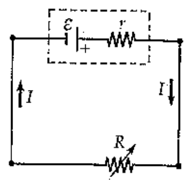


FIG. P21.57

(a) $\Delta V_{ter} = \mathcal{E} - Ir = \frac{\mathcal{E}R}{R + r}$ and $\Delta V_{ter} \to \mathcal{E}$ as $\boxed{R \to \infty}$.

(b) $I = \frac{\mathcal{E}}{R + r}$ and $I \to \frac{\mathcal{E}}{r}$ as $\boxed{R \to 0}$.

(c) $\mathcal{P} = I^2 R = \mathcal{E}^2 \frac{R}{(R + r)^2}$ $\qquad \frac{d\mathcal{P}}{dR} = \frac{-2\mathcal{E}^2 R}{(R + r)^3} + \frac{\mathcal{E}^2}{(R + r)^2} = 0$

Then $2R = R + r$ and $\boxed{R = r}$.

P21.58 The battery supplies energy at a changing rate $\frac{dE}{dt} = \mathcal{P} = \mathcal{E}I = \mathcal{E}\left(\frac{\mathcal{E}}{R}e^{-1/RC}\right)$.

Then the total energy put out by the battery is $\int dE = \int_{t=0}^{\infty} \frac{\mathcal{E}^2}{R} \exp\left(-\frac{t}{RC}\right)dt$

$$\int dE = \frac{\mathcal{E}^2}{R}(-RC)\int_0^{\infty} \exp\left(-\frac{t}{RC}\right)\left(-\frac{dt}{RC}\right) = -\mathcal{E}^2 C \exp\left(-\frac{t}{RC}\right)\Big|_0^{\infty} = -\mathcal{E}^2 C[0 - 1] = \mathcal{E}^2 C.$$

The power delivered to the resistor is $\frac{dE}{dt} = \mathcal{P} = \Delta V_R I = I^2 R = R\frac{\mathcal{E}^2}{R^2}\exp\left(-\frac{2t}{RC}\right)$

So the total internal energy appearing in the resistor is $\int dE = \int_0^{\infty} \frac{\mathcal{E}^2}{R}\exp\left(-\frac{2t}{RC}\right)dt$

$$\int dE = \frac{\mathcal{E}^2}{R}\left(-\frac{RC}{2}\right)\int_0^{\infty} \exp\left(-\frac{2t}{RC}\right)\left(-\frac{2dt}{RC}\right) = -\frac{\mathcal{E}^2 C}{2}\exp\left(-\frac{2t}{RC}\right)\Big|_0^{\infty} = -\frac{\mathcal{E}^2 C}{2}[0 - 1] = \frac{\mathcal{E}^2 C}{2}$$

The energy finally stored in the capacitor is $U = \frac{1}{2}C(\Delta V)^2 = \frac{1}{2}C\mathcal{E}^2$. Thus, energy of the circuit is conserved $\mathcal{E}^2 C = \frac{1}{2}\mathcal{E}^2 C + \frac{1}{2}\mathcal{E}^2 C$ and resistor and capacitor share equally in the energy from the battery.

P21.59 (a) $q = C\Delta V\left(1 - e^{-t/RC}\right)$

$$q = (1.00 \times 10^{-6}\ \text{F})(10.0\ \text{V})\left[1 - e^{-10.0/[(2.00\times10^6)(1.00\times10^{-6})]}\right] = \boxed{9.93\ \mu\text{C}}$$

(b) $I = \frac{dq}{dt} = \left(\frac{\Delta V}{R}\right)e^{-t/RC}$

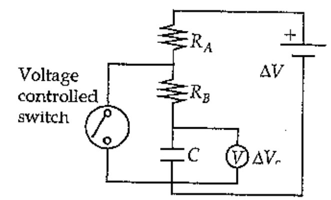
$$I = \left(\frac{10.0\ \text{V}}{2.00 \times 10^6\ \Omega}\right)e^{-5.00} = 3.37 \times 10^{-8}\ \text{A} = \boxed{33.7\ \text{nA}}$$

(c) $\frac{dU}{dt} = \frac{d}{dt}\left(\frac{1}{2}\frac{q^2}{C}\right) = \left(\frac{q}{C}\right)\frac{dq}{dt} = \left(\frac{q}{C}\right)I$

$$\frac{dU}{dt} = \left(\frac{9.93 \times 10^{-6}\ \text{C}}{1.00 \times 10^{-6}\ \text{C/V}}\right)(3.37 \times 10^{-8}\ \text{A}) = 3.34 \times 10^{-7}\ \text{W} = \boxed{334\ \text{nW}}$$

(d) $\mathcal{P}_{battery} = I\mathcal{E} = (3.37 \times 10^{-8}\ \text{A})(10.0\ \text{V}) = 3.37 \times 10^{-7}\ \text{W} = \boxed{337\ \text{nW}}$

P21.60 Start at the point when the voltage has just reached $\frac{2}{3}\Delta V$ and the switch has just closed. The voltage is $\frac{2}{3}\Delta V$ and is decaying towards 0 V with a time constant $R_B C$



$$\Delta V_C(t) = \left[\frac{2}{3}\Delta V\right]e^{-t/R_B C}$$

We want to know when $\Delta V_C(t)$ will reach $\frac{1}{3}\Delta V$.

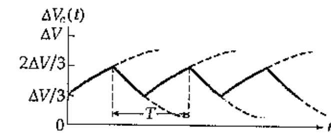


FIG. P21.60

Therefore, $\frac{1}{3}\Delta V = \left[\frac{2}{3}\Delta V\right]e^{-t/R_B C}$

or $e^{-t/R_B C} = \frac{1}{2}$

or $t_1 = R_B C \ln 2$.

After the switch opens, the voltage is $\frac{1}{3}\Delta V$, increasing toward ΔV with time constant $(R_A + R_B)C$.

$$\Delta V_C(t) = \Delta V - \left[\frac{2}{3}\Delta V\right]e^{-t/(R_A + R_B)C}$$

When $\Delta V_C(t) = \frac{2}{3}\Delta V$

$\frac{2}{3}\Delta V = \Delta V - \frac{2}{3}\Delta V e^{-t/(R_A + R_B)C}$ or $e^{-t/(R_A + R_B)C} = \frac{1}{2}$

So $t_2 = (R_A + R_B)C \ln 2$ and $T = t_1 + t_2 = \boxed{(R_A + 2R_B)C \ln 2}$.

P21.61 (a) With the switch closed, current exists in a simple series circuit as shown. The capacitors carry no current. For R_2 we have

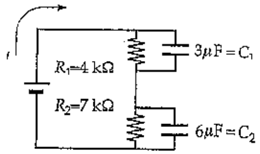


FIG. P21.61(a)

$$\mathcal{P}=I^2R_2 \qquad I=\sqrt{\frac{\mathcal{P}}{R_2}}=\sqrt{\frac{2.40\text{ V}\cdot\text{A}}{7\,000\text{ V/A}}}=18.5\text{ mA}.$$

The potential difference across R_1 and C_1 is

$$\Delta V=IR_1=\left(1.85\times10^{-4}\text{ A}\right)(4\,000\text{ V/A})=74.1\text{ V}.$$

The charge on C_1 is

$$Q=C_1\Delta V=\left(3.00\times10^{-6}\text{ C/V}\right)(74.1\text{ V})=\boxed{222\ \mu\text{C}}.$$

The potential difference across R_2 and C_2 is

$$\Delta V=IR_2=\left(1.85\times10^{-4}\text{ A}\right)(7\,000\ \Omega)=130\text{ V}.$$

The charge on C_2 is

$$Q=C_2\Delta V=\left(6.00\times10^{-6}\text{ C/V}\right)(130\text{ V})=778\ \mu\text{C}.$$

The battery emf is

$$IR_{eq}=I(R_1+R_2)=1.85\times10^{-4}\text{ A}(4\,000+7\,000)\text{ V/A}=204\text{ V}.$$

(b) In equilibrium after the switch has been opened, no current exists. The potential difference across each resistor is zero. The full 204 V appears across both capacitors. The new charge on C_2 is

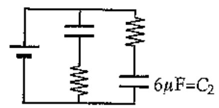


FIG. P21.61(b)

$$Q=C_2\Delta V=\left(6.00\times10^{-6}\text{ C/V}\right)(204\text{ V})=1\,222\ \mu\text{C}$$

for a change of $1\,222\ \mu\text{C}-778\ \mu\text{C}=\boxed{444\ \mu\text{C}}$.

P21.62 $\Delta V=\mathcal{E}e^{-t/RC}$

so $\ln\left(\frac{\mathcal{E}}{\Delta V}\right)=\left(\frac{1}{RC}\right)t.$

A plot of $\ln\left(\frac{\mathcal{E}}{\Delta V}\right)$ versus t should be a straight line with slope equal to $\frac{1}{RC}$.

Using the given data values:

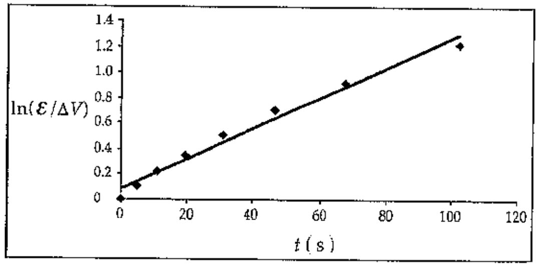


FIG. P21.62

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t(s)	ΔV(V)	$\ln(\mathcal{E}/\Delta V)$
0	6.19	0
4.87	5.55	0.109
11.1	4.93	0.228
19.4	4.34	0.355
30.8	3.72	0.509
46.6	3.09	0.695
67.3	2.47	0.919
102.2	1.83	1.219

(a) A least-square fit to this data yields the graph above.

$\sum x_i=282,\qquad \sum x_i^2=1.86\times10^4,$

$\sum x_iy_i=244,\qquad \sum y_i=4.03,\qquad N=8$

$$\text{Slope}=\frac{N\left(\sum x_iy_i\right)-\left(\sum x_i\right)\left(\sum y_i\right)}{N\left(\sum x_i^2\right)-\left(\sum x_i\right)^2}=0.011\,8$$

$$\text{Intercept}=\frac{\left(\sum x_i^2\right)\left(\sum y_i\right)-\left(\sum x_i\right)\left(\sum x_iy_i\right)}{N\left(\sum x_i^2\right)-\left(\sum x_i\right)^2}=0.088\,2$$

The equation of the best fit line is: $\boxed{\ln\left(\frac{\mathcal{E}}{\Delta V}\right)=(0.011\,8)t+0.088\,2}$.

(b) Thus, the time constant is $\tau=RC=\frac{1}{\text{slope}}=\frac{1}{0.011\,8}=\boxed{84.7\text{ s}}$

and the capacitance is $C=\frac{\tau}{R}=\frac{84.7\text{ s}}{10.0\times10^6\ \Omega}=\boxed{8.47\ \mu\text{F}}$.

P21.63 **An editorial in *The Physics Teacher* suggested the idea for this problem.**

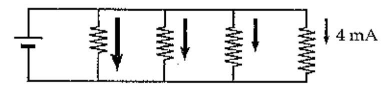

FIG. P21.63

The battery current is

$$(150+45+14+4)\text{ mA}=213\text{ mA}.$$

(a) The resistor with highest resistance is that carrying 4 mA. Doubling its resistance will reduce the current it carries to 2 mA. Then the total current is

$(150+45+14+2)\text{ mA}=211\text{ mA}$, nearly the same as before. The ratio is $\frac{211}{213}=\boxed{0.991}$.

(b) The resistor with least resistance carries 150 mA. Doubling its resistance changes this current to 75 mA and changes the total to

$(75+45+14+4)\text{ mA}=138\text{ mA}$. The ratio is $\frac{138}{213}=\boxed{0.648}$, representing a much larger reduction (35.2% instead of 0.9%).

(c) This problem is precisely analogous. As a battery maintained a potential difference in parts (a) and (b), a furnace maintains a temperature difference here. Energy flow by heat is analogous to current and takes place through thermal resistances in parallel. Each resistance can have its "R-value" increased by adding insulation. Doubling the thermal resistance of the attic door will produce only a negligible (0.9%) saving in fuel. Doubling the thermal resistance of $\boxed{\text{the ceiling}}$ will produce a much larger saving. The ceiling originally has the smallest thermal resistance.

*P21.64 (a) For the first measurement, the equivalent circuit is as shown in Figure 1.

$$R_{ab} = R_1 = R_y + R_y = 2R_y$$

so $R_y = \frac{1}{2}R_1$. (1)

For the second measurement, the equivalent circuit is shown in Figure 2.

Thus, $R_{ac} = R_2 = \frac{1}{2}R_y + R_x$. (2)

Substitute (1) into (2) to obtain:

$$R_2 = \frac{1}{2}\left(\frac{1}{2}R_1\right) + R_x, \text{ or } \boxed{R_x = R_2 - \frac{1}{4}R_1}.$$

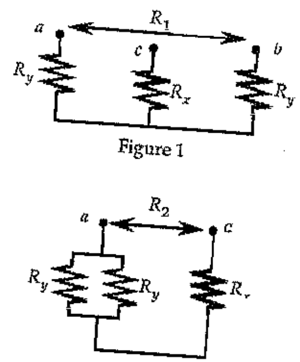

FIG. P21.64

(b) If $R_1 = 13.0\ \Omega$ and $R_2 = 6.00\ \Omega$, then $\boxed{R_x = 2.75\ \Omega}$.

$\boxed{\text{The antenna is inadequately grounded}}$ since this exceeds the limit of 2.00 Ω.

ANSWERS TO EVEN PROBLEMS

P21.2 $\frac{q\omega}{2\pi}$

P21.4 (a) 17.0 A; (b) 85.0 kA/m^2

P21.6 500 mA

P21.8 (a) 1.82 m; (b) 280 μm

P21.10 1.98 A

P21.12 (a) no change; (b) doubles; (c) doubles; (d) no change

P21.14 5.00 A; 24.0 Ω

P21.16 (a) 0.530; (b) 221 J; (c) 15.1°C

P21.18 $0.232

P21.20 295 metric ton/h

P21.22 Any diameter d and length ℓ related by $d^2 = (4.77\times10^{-8}\ \text{m})\ell$, such as length 0.900 m and diameter 0.207 mm. Yes.

P21.24 ~$1

P21.26 (a) 4.59 Ω; (b) 8.16%

P21.28 (a) see the solution; (b) no

P21.30 (a) 75.0 V; (b) 25.0 W, 6.25 W, and 6.25 W; 37.5 W

P21.32 (a) $\Delta V_4 > \Delta V_3 > \Delta V_1 > \Delta V_2$,
(b) $\Delta V_1 = \frac{\mathcal{E}}{3}$, $\Delta V_2 = \frac{2\mathcal{E}}{9}$, $\Delta V_3 = \frac{4\mathcal{E}}{9}$, $\Delta V_4 = \frac{2\mathcal{E}}{3}$;
(c) $I_1 > I_4 > I_2 = I_3$,
(d) $I_1 = I$, $I_2 = I_3 = \frac{I}{3}$, $I_4 = \frac{2I}{3}$;
(e) I_4 increases while I_1, I_2, and I_3 decrease;
(f) $I_1 = \frac{3I}{4}$, $I_2 = I_3 = 0$, $I_4 = \frac{3I}{4}$

P21.34 0.714 A, 1.29 A, 12.6 V

P21.36 see the solution

P21.38 (a) see the solution;
(b) The current in the 220-Ω resistor and the 5.80-V battery is 11.0 mA out of the positive battery pole. The current in the 370-Ω resistor is 9.13 mA. The current in the 150-Ω resistor and the 3.10-V battery is 1.87 mA out of the negative battery pole.

P21.40 starter 171 A; battery 0.283 A

P21.42 (a) −61.6 mA; (b) 0.235 μC; (c) 1.96 A

P21.44 (a) 3.91 s; (b) 0.782 ms

P21.46 587 kΩ

P21.48 (a) 300 MW; (b) 175 PW

P21.50 Experimental resistivity = 1.47 μΩ·m ± 4%, in agreement with 1.50 μΩ·m

P21.52 (a) $\frac{V}{L}\hat{i}$; (b) $\frac{4\rho L}{\pi d^2}$; (c) $\frac{V\pi d^2}{4\rho L}$; (d) $\frac{V}{\rho L}\hat{i}$;
(e) see the solution

P21.54 (a) $R = \frac{\rho}{2\pi L}\ln\left(\frac{r_b}{r_a}\right)$; (b) $\rho = \frac{2\pi L\Delta V}{I\ln\left(\frac{r_b}{r_a}\right)}$

P21.56 (a) either 3.84 Ω or 0.375 Ω; (b) impossible

P21.58 see the solution

P21.60 $(R_A + 2R_B)C\ln 2$

P21.62 (a) $\ln\left(\frac{\mathcal{E}}{\Delta V}\right) = 0.011\,8t + 0.088\,2$;
(b) 84.7 s, 8.47 μF

P21.64 (a) $R_x = R_2 - \frac{R_1}{4}$;
(b) $R_x = 2.75\ \Omega$, the station is inadequately grounded

CONTEXT 6 CONCLUSION SOLUTIONS TO PROBLEMS

CC6.1 The resistance of the spherical shell of air carrying radial electric current is

$$R = \frac{\rho L}{A} = \frac{(2\times10^{13}\ \Omega\cdot\text{m})(4\,000\ \text{m})}{4\pi(6.37\times10^{6}\ \text{m})^2} = 157\ \Omega.$$

The time constant for discharge is $RC = (157\ \text{V/A})(0.8\ \text{C/V}) = 126\ \text{s}$.

(a) $q = Qe^{-t/RC}$ describes the discharge

$$\frac{q}{Q} = e^{-t/RC}$$

$$e^{+t/RC} = \frac{Q}{q}$$

$$\frac{t}{RC} = \ln\left(\frac{Q}{q}\right)$$

$$t = RC\ln\left(\frac{Q}{q}\right)$$

$$t = 126\ \text{s}\left(\ln\left(\frac{4\times10^4\ \text{C}}{2\times10^4\ \text{C}}\right)\right) = \boxed{87.0\ \text{s}}$$

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