P22.16 (a) 4.73 N; (b) 5.46 N; (c) 4.73 N

P22.18 *ab*: 0, *bc*: $-40.0\text{ mN}\hat{\mathbf{i}}$, *cd*: $-40.0\text{ mN}\hat{\mathbf{k}}$, *da*: $(40.0\hat{\mathbf{i}}+40.0\hat{\mathbf{k}})$ mN

P22.20 see the solution

P22.22 (a) 640 μN·m; (b) 241 mW; (c) 2.56 mJ; (d) 154 mW

P22.24 20.0 μT

P22.26 200 nT

P22.28 see the solution

P22.30 (a) along the line $(y=-0.420\text{ m},\ z=0)$; (b) $-34.7\hat{\mathbf{j}}$ mN; (c) $-17.3\hat{\mathbf{j}}$ kN/C

P22.32 (a) 53.3 μT toward the bottom of the page; (b) 20.0 μT toward the bottom of the page; (c) zero

P22.34 (a) 10.0 μT out of the page; (b) 80.0 μN toward the first wire; (c) 16.0 μT into the page; (d) 80.0 μN toward the second wire

P22.36 (a) 12.0 cm to the left of W_1; (b) 2.40 A, downward

P22.38 5.40 cm

P22.40 (a) 400 cm; (b) 7.50 nT; (c) 1.26 m; (d) zero

P22.42 (a) 0; (b) $\dfrac{\mu_0 I}{2\pi R}$ tangent to the wall in a counterclockwise sense; (c) $\dfrac{\mu_0 I^2}{(2\pi R)^2}$ inward

P22.44 226 μN away from the center of the loop, 0

P22.46 (a) $\dfrac{\mu_0 IN}{2\ell}\left(\dfrac{a}{\sqrt{a^2+R^2}}-\dfrac{a-\ell}{\sqrt{(a-\ell)^2+R^2}}\right)$; (b) see the solution

P22.48 (a) $9.27\times10^{-24}\text{ A}\cdot\text{m}^2$; (b) down

P22.50 4×10^{10}

P22.52 see the solution

P22.54 (a) 17.9 ns; (b) 35.1 eV

P22.56 39.2 mT

P22.58 (a) -8.00×10^{-21} kg·m/s; (b) 8.90°

P22.60 2.75 Mrad/s

P22.62 (a) 1.33 m/s; (b) Positive ions moving toward you in magnetic field to the right feel upward magnetic force, and migrate upward in the blood vessel. Negative ions moving toward you feel downward magnetic force and accumulate at the bottom of this section of vessel. Thus both species can participate in the generation of the emf.

P22.64 2.01 GA toward the west

P22.66 see the solution

P22.68 (a) 274 μT; (b) $-274\hat{\mathbf{j}}\ \mu$T; (c) $1.15\hat{\mathbf{i}}$ mN; (d) $0.384\hat{\mathbf{i}}\text{ m/s}^2$; (e) acceleration is constant; (f) $0.999\hat{\mathbf{i}}$ m/s

Chapter 23

Faraday's Law and Inductance

CHAPTER OUTLINE

23.1 Faraday's Law of Induction
23.2 Motional emf
23.3 Lenz's Law
23.4 Induced emfs and Electric Fields
23.5 Self-Inductance
23.6 *RL* Circuits
23.7 Energy Stored in a Magnetic Field
23.8 Context Connection—The Repulsive Model for Magnetic Levitation

ANSWERS TO QUESTIONS

Q23.1 The magnetic flux is $\Phi_B = BA\cos\theta$. Therefore the flux is maximum when $\vec{\mathbf{B}}$ is perpendicular to the loop of wire and zero when there is no component of magnetic field perpendicular to the loop. The flux is zero when the loop is turned so that the field lies in the plane of its area.

Q23.2 (a) The south pole of the magnet produces an upward magnetic field that increases as the magnet approaches. The loop opposes change by making its own downward magnetic field; it carries current clockwise, which goes to the left through the resistor.

(b) The north pole of the magnet produces an upward magnetic field. The loop sees decreasing upward flux as the magnet falls away, and tries to make an upward magnetic field of its own by carrying current counterclockwise, to the right in the resistor.

Q23.3 The force on positive charges in the bar is $\vec{\mathbf{F}}=q(\vec{\mathbf{v}}\times\vec{\mathbf{B}})$. If the bar is moving to the left, positive charge will move downward and accumulate at the bottom end of the bar, so that an electric field will be established upward.

Q23.4 No. The magnetic force acts within the bar, but has no influence on the forward motion of the bar.

Q23.5 By the magnetic force law $\vec{\mathbf{F}}=q(\vec{\mathbf{v}}\times\vec{\mathbf{B}})$: the positive charges in the moving bar will flow downward and therefore clockwise in the circuit. If the bar is moving to the left, the positive charge in the bar will flow upward and therefore counterclockwise in the circuit.

Q23.6 We ignore mechanical friction between the bar and the rails. Moving the conducting bar through the magnetic field will force charges to move around the circuit to constitute clockwise current. The downward current in the bar feels a magnetic force to the left. Then a counterbalancing applied force to the right is required to maintain the motion.

Q23.7 As water falls, it gains speed and kinetic energy. It then pushes against turbine blades, transferring its energy to the rotor coils of a large AC generator. The rotor of the generator turns within a strong magnetic field. Because the rotor is spinning, the magnetic flux through its turns changes in time as $\Phi_B = BA\cos\omega t$. Generated in the rotor is an induced emf of $\mathcal{E}=\dfrac{-Nd\Phi_B}{dt}$. This induced emf is the voltage driving the current in our electric power lines.

Q23.8 Yes. The induced eddy currents on the surface of the aluminum will slow the descent of the aluminum. It may fall very slowly.

Q23.9 The increasing counterclockwise current in the solenoid coil produces an upward magnetic field that increases rapidly. The increasing upward flux of this field through the ring induces an emf to produce clockwise current in the ring. The magnetic field of the solenoid has a radially outward component at each point on the ring. This field component exerts upward force on the current in the ring there. The whole ring feels a total upward force larger than its weight.

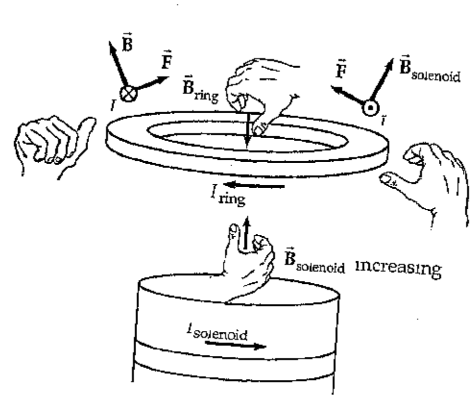

FIG. Q23.9

Q23.10 Oscillating current in the solenoid produces an always-changing magnetic field. Vertical flux through the ring, alternately increasing and decreasing, produces current in it with a direction that is alternately clockwise and counterclockwise. The current through the ring's resistance produces internal energy at the rate I^2R.

Q23.11 (a) The battery makes counterclockwise current I_1 in the primary coil, so its magnetic field $\vec{B}_1$ is to the right and increasing just after the switch is closed. The secondary coil will oppose the change with a leftward field $\vec{B}_2$, which comes from an induced clockwise current I_2 that goes to the right in the resistor.

(b) At steady state the primary magnetic field is unchanging, so no emf is induced in the secondary.

(c) The primary's field is to the right and decreasing as the switch is opened. The secondary coil opposes this decrease by making its own field to the right, carrying counterclockwise current to the left in the resistor.

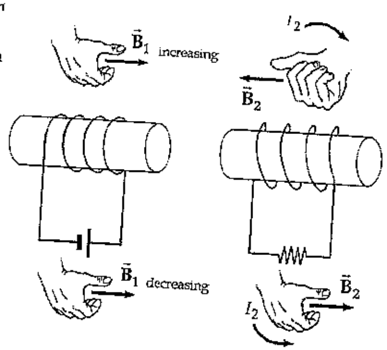

FIG. Q23.11

Q23.12 The motional emf between the wingtips cannot be used to run a light bulb. To connect the light, an extra insulated wire would have to be run out along each wing, making contact with the wing tip. The wings with the extra wires and the bulb constitute a single-loop circuit. As the plane flies through a uniform magnetic field, the magnetic flux through this loop is constant and zero emf is generated. On the other hand, if the magnetic field is not uniform, a large loop towed through it will generate pulses of positive and negative emf. This phenomenon has been demonstrated with a cable unreeled from the Space Shuttle.

Q23.13 As Section 23.4 describes, the electric field lines form closed loops for an electric field created by a changing magnetic field. If a positively charged object is carried around one of these loops in the direction of $\vec{E}$, the electric field does positive work on it every step of the way. The work adds up to a nonzero positive total. This fact proves that the force is nonconservative. The work in fact is $W=\oint \vec{F}\cdot d\vec{s}=\oint q\vec{E}\cdot d\vec{s}=-q\frac{d}{dt}\int \vec{B}\cdot d\vec{A}$. If going around once extracts energy from the changing magnetic field, going around several lines is better. This is why generators and transformers have coils with many turns.

Q23.14 The inductance of a coil is determined by (a) the geometry of the coil and (b) the "contents" of the coil. This is similar to the parameters that determine the capacitance of a capacitor and the resistance of a resistor. With an inductor, the most important factor in the geometry is the number of turns of wire, or turns per unit length. By the "contents" we refer to the material in which the inductor establishes a magnetic field, notably the magnetic properties of the core around which the wire is wrapped.

Q23.15 The current decreases not instantaneously but over some span of time. The faster the decrease in the current, the larger will be the emf generated in the inductor. A spark can appear at the switch as it is opened because the self-induced voltage is a maximum at this instant. The voltage can therefore briefly cause dielectric breakdown of the air between the contacts.

Q23.16 A physicist's list of constituents of the universe in 1829 might include matter, light, heat, the stuff of stars, charge, momentum, and several other entries. Our list today might include the quarks, electrons, muons, tauons, and neutrinos of matter; gravitons of gravitational fields; photons of electric and magnetic fields; W and Z particles; gluons; energy; momentum; angular momentum; charge; baryon number; three different lepton numbers; upness; downness; strangeness; charm; topness; and bottomness. Alternatively, the relativistic interconvertability of mass and energy, and of electric and magnetic fields, can be used to make the list look shorter. Some might think of the conserved quantities energy, momentum, … bottomness as properties of matter, rather than as things with their own existence. The idea of a field is not due to Henry, but rather to Faraday, to whom Henry personally demonstrated self-induction. Still the thesis stated in the question has an important germ of truth. Henry precipitated a basic change if he did not cause it. The biggest difference between the two lists is that the 1829 list does not include fields and today's list does.

Q23.17 The energy stored in the magnetic field of an inductor is proportional to the square of the current. Doubling I makes $U=\frac{1}{2}LI^2$ get four times larger.

Q23.18 The energy stored in a capacitor is proportional to the square of the electric field, and the energy stored in an induction coil is proportional to the square of the magnetic field. The capacitor's energy is proportional to its capacitance, which depends on its geometry and the dielectric material inside. The coil's energy is proportional to its inductance, which depends on its geometry and the core material. On the other hand, we can think of Henry's discovery of self-inductance as fundamentally new. Before a certain school vacation at the Albany Academy about 1830, one could visualize the universe as consisting of only one thing, matter. All the forms of energy then known (kinetic, gravitational, elastic, internal, electrical) belonged to chunks of matter. But the energy that temporarily maintains a current in a coil after the battery is removed is not energy that belongs to any bit of matter. This energy is vastly larger than the kinetic energy of the drifting electrons in the wires. This energy belongs to the magnetic field around the coil. Beginning in 1830, Nature has forced us to admit that the universe consists of matter and also of fields, massless and invisible, known only by their effects.

Q23.19 The inductance of the series combination of inductor L_1 and inductor L_2 is $L_1 + L_2 + M_{12}$, where M_{12} is the mutual inductance of the two coils. It can be defined as the emf induced in coil two when the current in coil one changes at one ampere per second, due to the magnetic field of coil one producing flux through coil two. The coils can be arranged to have large mutual inductance, as by winding them onto the same core. The coils can be arranged to have negligible mutual inductance, as separate toroids do.

Q23.20 An object cannot exert a net force on itself. An object cannot create momentum out of nothing. A coil can induce an emf in itself. When it does so, the actual forces acting on charges in different parts of the loop add as vectors to zero. The term electromotive force does not refer to a force, but to a voltage.

SOLUTIONS TO PROBLEMS

Section 23.1 Faraday's Law of Induction

P23.1 $$|\mathcal{E}| = \left|\frac{\Delta\Phi_B}{\Delta t}\right| = \frac{\Delta(\vec{B}\cdot\vec{A})}{\Delta t} = \frac{(2.50\text{ T} - 0.500\text{ T})(8.00\times10^{-4}\text{ m}^2)}{1.00\text{ s}}\left(\frac{1\text{ N}\cdot\text{s}}{1\text{ T}\cdot\text{C}\cdot\text{m}}\right)\left(\frac{1\text{ V}\cdot\text{C}}{1\text{ N}\cdot\text{m}}\right)$$

$$|\mathcal{E}| = 1.60\text{ mV} \text{ and } I_{\text{loop}} = \frac{\mathcal{E}}{R} = \frac{1.60\text{ mV}}{2.00\ \Omega} = \boxed{0.800\text{ mA}}$$

P23.2 $$\mathcal{E} = -N\frac{\Delta BA\cos\theta}{\Delta t} = -NB\pi r^2\left(\frac{\cos\theta_f - \cos\theta_i}{\Delta t}\right) = -25.0(50.0\times10^{-6}\text{ T})\left[\pi(0.500\text{ m})^2\right]\left(\frac{\cos 180^\circ - \cos 0^\circ}{0.200\text{ s}}\right)$$

$$\mathcal{E} = \boxed{+9.82\text{ mV}}$$

P23.3 Noting unit conversions from $\vec{F} = q\vec{v}\times\vec{B}$ and $U = qV$, the induced voltage is

$$\mathcal{E} = -N\frac{d(\vec{B}\cdot\vec{A})}{dt} = -N\left(\frac{0 - B_iA\cos\theta}{\Delta t}\right) = \frac{+200(1.60\text{ T})(0.200\text{ m}^2)\cos 0^\circ}{20.0\times10^{-3}\text{ s}}\left(\frac{1\text{ N}\cdot\text{s}}{1\text{ T}\cdot\text{C}\cdot\text{m}}\right)\left(\frac{1\text{ V}\cdot\text{C}}{\text{N}\cdot\text{m}}\right) = 3\,200\text{ V}$$

$$I = \frac{\mathcal{E}}{R} = \frac{3\,200\text{ V}}{20.0\ \Omega} = \boxed{160\text{ A}}$$

P23.4 $$|\mathcal{E}| = \frac{d(BA)}{dt} = 0.500\mu_0 nA\frac{dI}{dt} = 0.500\mu_0 n\pi r_2^2\frac{\Delta I}{\Delta t}$$

(a) $$I_{\text{ring}} = \frac{\mathcal{E}}{R} = \boxed{\frac{\mu_0 n\pi r_2^2}{2R}\frac{\Delta I}{\Delta t}}$$

(b) $$B = \frac{\mu_0 I}{2r_1} = \boxed{\frac{\mu_0^2 n\pi r_2^2}{4r_1 R}\frac{\Delta I}{\Delta t}}$$

(c) The coil's field points downward, and is increasing, so $\boxed{B_{\text{ring}}\text{ points upward}}$.

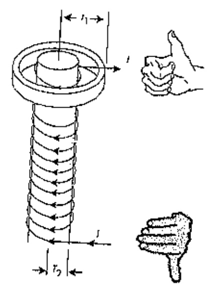

FIG. P23.4

P23.5 (a) $$d\Phi_B = \vec{B}\cdot d\vec{A} = \frac{\mu_0 I}{2\pi x}L\,dx:\ \Phi_B = \int_h^{h+w}\frac{\mu_0 IL}{2\pi}\frac{dx}{x} = \boxed{\frac{\mu_0 IL}{2\pi}\ln\left(\frac{h+w}{h}\right)}$$

(b) $$\mathcal{E} = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}\left[\frac{\mu_0 IL}{2\pi}\ln\left(\frac{h+w}{h}\right)\right] = -\left[\frac{\mu_0 L}{2\pi}\ln\left(\frac{h+w}{h}\right)\right]\frac{dI}{dt}$$

$$\mathcal{E} = -\frac{(4\pi\times10^{-7}\text{ T}\cdot\text{m/A})(1.00\text{ m})}{2\pi}\ln\left(\frac{1.00+10.0}{1.00}\right)(10.0\text{ A/s}) = \boxed{-4.80\ \mu\text{V}}$$

The long wire produces magnetic flux into the page through the rectangle, shown by the first hand in the figure to the right.

As the magnetic flux increases, the rectangle produces its own magnetic field out of the page, which it does by carrying $\boxed{\text{counterclockwise}}$ current (second hand in the figure).

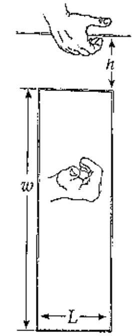

FIG. P23.5

P23.6 $$\Phi_B = (\mu_0 nI)A_{\text{solenoid}}$$

$$\mathcal{E} = -N\frac{d\Phi_B}{dt} = -N\mu_0 n\left(\pi r_{\text{solenoid}}^2\right)\frac{dI}{dt}$$

$$\mathcal{E} = -15.0(4\pi\times10^{-7}\text{ T}\cdot\text{m/A})(1.00\times10^3\text{ m}^{-1})\pi(0.020\,0\text{ m})^2(600\text{ A/s})\cos(120t)$$

$$\boxed{\mathcal{E} = -14.2\cos(120t)\text{ mV}}$$

P23.7 $$|\mathcal{E}| = \left|\frac{\Delta\Phi_B}{\Delta t}\right| = N\left(\frac{dB}{dt}\right)A = N(0.010\,0 + 0.080\,0t)A$$

At $t = 5.00$ s, $|\mathcal{E}| = 30.0(0.410\text{ T/s})\left[\pi(0.040\,0\text{ m})^2\right] = \boxed{61.8\text{ mV}}$

*P23.8 (a) Each coil has a pulse of voltage tending to produce counterclockwise current as the projectile approaches, and then a pulse of clockwise voltage as the projectile recedes.

(b) $$v = \frac{d}{t} = \frac{1.50\text{ m}}{2.40\times10^{-3}\text{ s}} = \boxed{625\text{ m/s}}$$

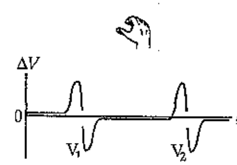

FIG. P23.8

P23.9 (a) Suppose, first, that the central wire is long and straight. The enclosed current of unknown amplitude creates a circular magnetic field around it, with the magnitude of the field given by Ampère's Law.

$$\oint \vec{B}\cdot d\vec{s} = \mu_0 I: \quad B = \frac{\mu_0 I_{max} \sin \omega t}{2\pi R}$$

at the location of the Rogowski coil, which we assume is centered on the wire. This field passes perpendicularly through each turn of the toroid, producing flux

$$\vec{B}\cdot\vec{A} = \frac{\mu_0 I_{max} A \sin \omega t}{2\pi R}$$

The toroid has $2\pi R n$ turns. As the magnetic field varies, the emf induced in it is

$$\mathcal{E} = -N\frac{d}{dt}\vec{B}\cdot\vec{A} = -2\pi R n \frac{\mu_0 I_{max} A}{2\pi R}\frac{d}{dt}\sin \omega t = -\mu_0 I_{max} n A \omega \cos \omega t.$$

This is an alternating voltage with amplitude $\mathcal{E}_{max} = \mu_0 n A \omega I_{max}$. Measuring the amplitude determines the size I_{max} of the central current. Our assumptions that the central wire is long and straight and passes perpendicularly through the center of the Rogowski coil are all unnecessary.

(b) If the wire is not centered, the coil will respond to stronger magnetic fields on one side, but to correspondingly weaker fields on the opposite side. The emf induced in the coil is proportional to the line integral of the magnetic field around the circular axis of the toroid. Ampère's Law says that this line integral depends only on the amount of current the coil encloses. It does not depend on the shape or location of the current within the coil, or on any currents outside the coil.

*P23.10 The upper loop has area $\pi(0.05\text{ m})^2 = 7.85\times10^{-3}\text{ m}^2$. The induced emf in it is

$$\mathcal{E} = -N\frac{d}{dt}BA\cos\theta = -1A\cos 0°\frac{dB}{dt} = -7.85\times10^{-3}\text{ m}^2(2\text{ T/s}) = -1.57\times10^{-2}\text{ V}.$$

The minus sign indicates that it tends to produce counterclockwise current, to make its own magnetic field out of the page. Similarly, the induced emf in the lower loop is

$$\mathcal{E} = -NA\cos\theta\frac{dB}{dt} = -\pi(0.09\text{ m})^2 2\text{ T/s} = -5.09\times10^{-2}\text{ V} = +5.09\times10^{-2}\text{ V}$$ to produce

$\boxed{\text{counterclockwise current in the lower loop, which becomes clockwise current in the upper loop}}$.

The net emf for current in this sense around the figure 8 is

$5.09\times10^{-2}\text{ V} - 1.57\times10^{-2}\text{ V} = 3.52\times10^{-2}\text{ V}.$

It pushes current in this sense through series resistance $[2\pi(0.05\text{ m}) + 2\pi(0.09\text{ m})]3\ \Omega/\text{m} = 2.64\ \Omega$.

The current is $I = \frac{\mathcal{E}}{R} = \frac{3.52\times10^{-2}\text{ V}}{2.64\ \Omega} = \boxed{13.3\text{ mA}}$.

Section 23.2 Motional emf

Section 23.3 Lenz's Law

P23.11 (a) For maximum induced emf, with positive charge at the top of the antenna,

$\vec{F}_+ = q_+(\vec{v}\times\vec{B})$, so the auto must move $\boxed{east}$, since east × north = up.

(b) $\mathcal{E} = B\ell v = (5.00\times10^{-5}\text{ T})(1.20\text{ m})\left(\frac{65.0\times10^3\text{ m}}{3\,600\text{ s}}\right)\cos 65.0° = \boxed{4.58\times10^{-4}\text{ V}}$

P23.12 $I = \frac{\mathcal{E}}{R} = \frac{B\ell v}{R}$

$\boxed{v = 1.00\text{ m/s}}$

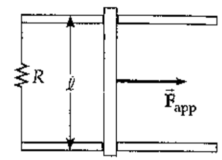

FIG. P23.12

P23.13 (a) $|\vec{F}_B| = I|\vec{\ell}\times\vec{B}| = I\ell B$

When $I = \frac{\mathcal{E}}{R}$

and $\mathcal{E} = B\ell v$

we get $F_B = \frac{B\ell v}{R}(\ell B) = \frac{B^2\ell^2 v}{R} = \frac{(2.50)^2(1.20)^2(2.00)}{6.00} = 3.00\text{ N}.$

The applied force is $\boxed{3.00\text{ N to the right}}$

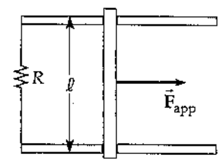

FIG. P23.13

(b) $\mathcal{P} = I^2R = \frac{B^2\ell^2v^2}{R} = 6.00\text{ W}$ or $\mathcal{P} = Fv = \boxed{6.00\text{ W}}$

P23.14 $F_B = I\ell B$ and $\mathcal{E} = B\ell v$

$I = \frac{\mathcal{E}}{R} = \frac{B\ell v}{R}$ so $B = \frac{IR}{\ell v}$

(a) $F_B = \frac{I^2\ell R}{\ell v}$ and $I = \sqrt{\frac{F_B v}{R}} = \boxed{0.500\text{ A}}$

(b) $I^2R = \boxed{2.00\text{ W}}$

(c) For constant force, $\mathcal{P} = \vec{F}\cdot\vec{v} = (1.00\text{ N})(2.00\text{ m/s}) = \boxed{2.00\text{ W}}$.

*P23.15 With v representing the initial speed of the bar, let u represent its speed at any later time. The motional emf induced in the bar is $\mathcal{E} = B\ell u$. The induced current is $I = \frac{\mathcal{E}}{R} = \frac{B\ell u}{R}$. The magnetic force on the bar is backward $F = -I\ell B = -\frac{B^2\ell^2 u}{R} = \frac{m\,du}{dt}$

Method one: To find u as a function of time, we separate variables thus:

$$-\frac{B^2\ell^2}{Rm}dt = \frac{du}{u}$$

$$\int_0^t -\frac{B^2\ell^2}{Rm}dt = \int_v^u \frac{du}{u}$$

$$-\frac{B^2\ell^2}{Rm}(t-0) = \ln u - \ln v = \ln\frac{u}{v}$$

$$e^{-B^2\ell^2 t/Rm} = \frac{u}{v} \qquad u = ve^{-B^2\ell^2 t/Rm} = \frac{dx}{dt}$$

The distance traveled is given by

$$\int_0^{x_{\max}} dx = \int_0^{\infty} ve^{-B^2\ell^2 t/Rm}dt = v\left(-\frac{Rm}{B^2\ell^2}\right)\int_0^{\infty} e^{-B^2\ell^2 t/Rm}\left(-\frac{-B^2\ell^2 dt}{Rm}\right)$$

$$x_{\max} - 0 = -\frac{Rmv}{B^2\ell^2}\left[e^{-\infty} - e^{-0}\right] = \boxed{\frac{Rmv}{B^2\ell^2}}$$

Method two: Newton's second law is

$$-\frac{B^2\ell^2 u}{R} = -\frac{B^2\ell^2}{R}\frac{dx}{dt} = m\frac{du}{dt}$$

$$m\,du = -\frac{B^2\ell^2}{R}dx.$$

Direct integration from the initial to the stopping point gives

$$\int_v^0 m\,du = \int_0^{x_{\max}} -\frac{B^2\ell^2}{R}dx$$

$$m(0-v) = -\frac{B^2\ell^2}{R}(x_{\max} - 0)$$

$$x_{\max} = \frac{mvR}{B^2\ell^2}.$$

*P23.16 Model the magnetic flux inside the metallic tube as constant as it shrinks from radius R to radius r:

$2.50\ \mathrm{T}(\pi R^2) = B_f \pi r^2$

$B_f = 2.50\ \mathrm{T}\left(\frac{R}{r}\right)^2 = 2.50\ \mathrm{T}(12)^2 = \boxed{360\ \mathrm{T}}$

P23.17 Observe that the homopolar generator has no commutator and produces a voltage constant in time: DC with no ripple. In time dt, the disk turns by angle $d\theta = \omega dt$. The outer brush slides over distance $rd\theta$.

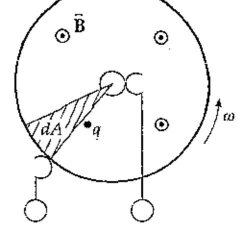

FIG. P23.17

The radial line to the outer brush sweeps over area

$$dA = \frac{1}{2}rrd\theta = \frac{1}{2}r^2\omega dt.$$

The emf generated is $\quad \mathcal{E} = -N\frac{d}{dt}\vec{\mathbf{B}}\cdot\vec{\mathbf{A}}$

$$\mathcal{E} = -(1)B\cos 0°\frac{dA}{dt} = -B\left(\frac{1}{2}r^2\omega\right).$$

(We could think of this as following from the result of Example 23.3.)

The magnitude of the emf is

$$|\mathcal{E}| = B\left(\frac{1}{2}r^2\omega\right) = (0.9\ \mathrm{N\cdot s/C\cdot m})\left[\frac{1}{2}(0.4\ \mathrm{m})^2(3\,200\ \mathrm{rev/min})\right]\left(\frac{2\pi\ \mathrm{rad/rev}}{60\ \mathrm{s/min}}\right)$$

$$|\mathcal{E}| = \boxed{24.1\ \mathrm{V}}$$

A free positive charge q shown, turning with the disk, feels a magnetic force $q\vec{\mathbf{v}}\times\vec{\mathbf{B}}$ radially outward. Thus the $\boxed{\text{outer contact is positive}}$.

*P23.18 The speed of waves on the wire is

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{267\ \mathrm{N\cdot m}}{3\times10^{-3}\ \mathrm{kg}}} = 298\ \mathrm{m/s}.$$

In the simplest standing-wave vibration state,

$d_{NN} = 0.64\ \mathrm{m} = \frac{\lambda}{2} \qquad \lambda = 1.28\ \mathrm{m}$

and $f = \frac{v}{\lambda} = \frac{298\ \mathrm{m/s}}{1.28\ \mathrm{m}} = 233\ \mathrm{Hz}$.

(a) The changing flux of magnetic field through the circuit containing the wire will drive current to the left in the wire as it moves up and to the right as it moves down. The emf will have this same frequency of $\boxed{233\ \mathrm{Hz}}$.

(b) The vertical coordinate of the center of the wire is described by

$x = A\cos\omega t = (1.5\ \mathrm{cm})\cos(2\pi 233t/\mathrm{s})$.

Its velocity is $v = \frac{dx}{dt} = -(1.5\ \mathrm{cm})(2\pi 233/\mathrm{s})\sin(2\pi 233\ t/\mathrm{s})$.

Its maximum speed is $1.5\ \mathrm{cm}(2\pi)233/\mathrm{s} = 22.0\ \mathrm{m/s}$.

The induced emf is $\mathcal{E} = -B\ell v$, with amplitude

$\mathcal{E}_{\max} = B\ell v_{\max} = 4.50\times10^{-3}\ \mathrm{T}(0.02\ \mathrm{m})22\ \mathrm{m/s} = \boxed{1.98\times10^{-3}\ \mathrm{V}}$

P23.19 $\omega = (2.00\ \text{rev/s})(2\pi\ \text{rad/rev}) = 4.00\pi\ \text{rad/s}$

$$\mathcal{E} = \frac{1}{2}B\omega\ell^2 = \boxed{2.83\ \text{mV}}$$

P23.20 (a) $\vec{B}_{ext} = B_{ext}\hat{i}$ and B_{ext} decreases; therefore, the induced field is $\vec{B}_0 = B_0\hat{i}$ (to the right) and the current in the resistor is directed $\boxed{\text{to the right}}$.

(b) $\vec{B}_{ext} = B_{ext}(-\hat{i})$ increases; therefore, the induced field $\vec{B}_0 = B_0(+\hat{i})$ is to the right, and the current in the resistor is directed $\boxed{\text{to the right}}$.

(c) $\vec{B}_{ext} = B_{ext}(-\hat{k})$ into the paper and B_{ext} decreases; therefore, the induced field is $\vec{B}_0 = B_0(-\hat{k})$ into the paper, and the current in the resistor is directed $\boxed{\text{to the right}}$

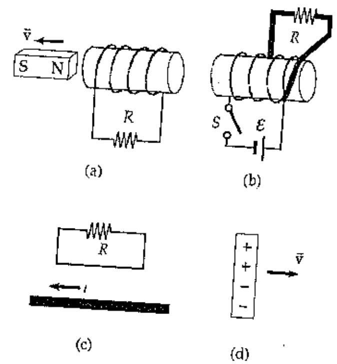

FIG. P23.20

(d) By the magnetic force law, $\vec{F}_B = q(\vec{v}\times\vec{B})$. Therefore, a positive charge will move to the top of the bar if $\vec{B}$ is $\boxed{\text{into the paper}}$.

P23.21 (a) At terminal speed,

$$Mg = F_B = IwB = \left(\frac{\mathcal{E}}{R}\right)wB = \left(\frac{Bwv_T}{R}\right)wB = \frac{B^2w^2v_T}{R}$$

or $$\boxed{v_T = \frac{MgR}{B^2w^2}}.$$

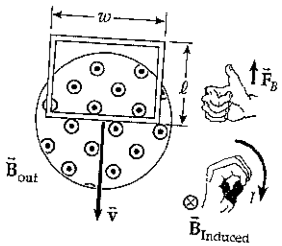

FIG. P23.21

(b) The emf is directly proportional to v_T, but the current is inversely proportional to R. A large R means a small current at a given speed, so the loop must travel faster to get $F_B = mg$.

(c) At a given speed, the current is directly proportional to the magnetic field. But the force is proportional to the product of the current and the field. For a small B, the speed must increase to compensate for both the small B and also the current, so $v_T \propto \frac{1}{B^2}$.

P23.22 (a) The force on the side of the coil entering the field (consisting of N wires) is

$$F = N(ILB) = N(IwB).$$

The induced emf in the coil is

$$|\mathcal{E}| = N\frac{d\Phi_B}{dt} = N\frac{d(Bwx)}{dt} = NBwv.$$

so the current is $I = \frac{|\mathcal{E}|}{R} = \frac{NBwv}{R}$ counterclockwise.

The force on the leading side of the coil is then:

$$F = N\left(\frac{NBwv}{R}\right)wB = \boxed{\frac{N^2B^2w^2v}{R}\ \text{to the left}}$$

(b) Once the coil is entirely inside the field, $\Phi_B = NBA = \text{constant}$,

so $\mathcal{E} = 0$, $I = 0$, and $F = \boxed{0}$

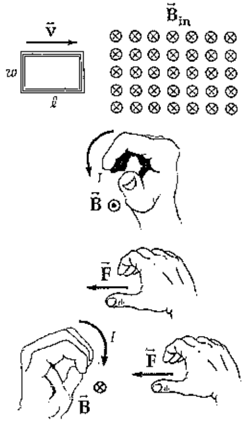

FIG. P23.22

(c) As the coil starts to leave the field, the flux *decreases* at the rate Bwv, so the magnitude of the current is the same as in part (a), but now the current is clockwise. Thus, the force exerted on the trailing side of the coil is:

$$F = \boxed{\frac{N^2B^2w^2v}{R}\ \text{to the left again}}.$$

P23.23 (a) $\mathcal{E}_{max} = NAB\omega = (1\,000)(0.100)(0.200)(120\pi) = \boxed{7.54\ \text{kV}}$

(b) $\mathcal{E}(t) = NBA\omega\sin\omega t = NBA\omega\sin\theta$

$|\mathcal{E}|$ is maximal when $|\sin\theta| = 1$

or $\theta = \pm\frac{\pi}{2}$

so the $\boxed{\text{plane of coil is parallel to } \vec{B}}$.

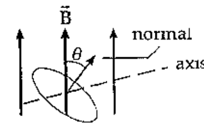

FIG. P23.23

***P23.24** $B = \mu_0 nI = (4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(200\ \text{m}^{-1})(15.0\ \text{A}) = 3.77\times10^{-3}\ \text{T}$

For the small coil, $\Phi_B = N\vec{B}\cdot\vec{A} = NBA\cos\omega t = NB(\pi r^2)\cos\omega t$.

Thus, $$\mathcal{E} = -\frac{d\Phi_B}{dt} = NB\pi r^2\omega\sin\omega t$$

$$\mathcal{E} = (30.0)(3.77\times10^{-3}\ \text{T})\pi(0.080\,0\ \text{m})^2(4.00\pi\,\text{s}^{-1})\sin(4.00\pi t) = \boxed{(28.6\ \text{mV})\sin(4.00\pi t)}$$

Section 23.4 Induced emfs and Electric Fields

P23.25 $\frac{dB}{dt} = 0.060\,0t$ $\quad |\mathcal{E}| = \frac{d\Phi_B}{dt} = \pi r_1^2 \frac{dB}{dt} = 2\pi r_1 E$

At $t = 3.00$ s,

$$E = \left(\frac{\pi r_1^2}{2\pi r_1}\right)\frac{dB}{dt} = \frac{0.020\,0\text{ m}}{2}(0.060\,0\text{ T/s}^2)(3.00\text{ s})\left(\frac{1\text{ N}\cdot\text{s}}{1\text{ T}\cdot\text{C}\cdot\text{m}}\right)$$

$\vec{E} = \boxed{1.80\times10^{-3}\text{ N/C perpendicular to } r_1 \text{ and counterclockwise}}$

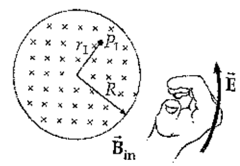

FIG. P23.25

P23.26 (a) $\frac{dB}{dt} = 6.00t^2 - 8.00t$ $\quad |\mathcal{E}| = \frac{d\Phi_B}{dt}$

At $t = 2.00$ s, $\quad E = \frac{\pi R^2(dB/dt)}{2\pi r_2} = \frac{8.00\pi(0.025\,0)^2}{2\pi(0.050\,0)}$

$F = qE = \boxed{8.00\times10^{-21}\text{ N}}$

(b) When $6.00t^2 - 8.00t = 0$, at $t = \boxed{1.33\text{ s}}$ and at $t = \boxed{0}$.

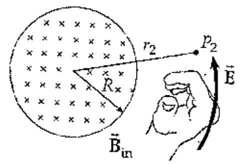

FIG. P23.26

Section 23.5 Self-Inductance

P23.27 $|\overline{\mathcal{E}}| = L\frac{\Delta I}{\Delta t} = (3.00\times10^{-3}\text{ H})\left(\frac{1.50\text{ A} - 0.200\text{ A}}{0.200\text{ s}}\right) = 1.95\times10^{-2}\text{ V} = \boxed{19.5\text{ mV}}$

P23.28 Treating the telephone cord as a solenoid, we have:

$$L = \frac{\mu_0 N^2 A}{\ell} = \frac{(4\pi\times10^{-7}\text{ T}\cdot\text{m/A})(70.0)^2\pi(6.50\times10^{-3}\text{ m})^2}{0.600\text{ m}} = \boxed{1.36\ \mu\text{H}}.$$

P23.29 $\mathcal{E} = -L\frac{dI}{dt} = -L\frac{d}{dt}(I_{\max}\sin\omega t) = -L\omega I_{\max}\cos\omega t = -(10.0\times10^{-3})(120\pi)(5.00)\cos\omega t$

$\mathcal{E} = -(6.00\pi)\cos(120\pi t) = \boxed{-(18.8\text{ V})\cos(377t)}$

P23.30 From $|\mathcal{E}| = L\left(\frac{\Delta I}{\Delta t}\right)$, we have $\quad L = \frac{\mathcal{E}}{(\Delta I/\Delta t)} = \frac{24.0\times10^{-3}\text{ V}}{10.0\text{ A/s}} = 2.40\times10^{-3}\text{ H}.$

From $L = \frac{N\Phi_B}{I}$, we have $\quad \Phi_B = \frac{LI}{N} = \frac{(2.40\times10^{-3}\text{ H})(4.00\text{ A})}{500} = \boxed{19.2\ \mu\text{T}\cdot\text{m}^2}.$

P23.31 $|\mathcal{E}| = L\frac{dI}{dt} = (90.0\times10^{-3})\frac{d}{dt}(t^2 - 6t)\text{ V}$

(a) At $t = 1.00$ s, $\quad \mathcal{E} = \boxed{360\text{ mV}}$

(b) At $t = 4.00$ s, $\quad \mathcal{E} = \boxed{180\text{ mV}}$

(c) $\mathcal{E} = (90.0\times10^{-3})(2t - 6) = 0$

when $\quad \boxed{t = 3.00\text{ s}}$.

P23.32 $L = \frac{N\Phi_B}{I} = \frac{NBA}{I} \approx \frac{NA}{I}\cdot\frac{\mu_0 NI}{2\pi R} = \boxed{\frac{\mu_0 N^2 A}{2\pi R}}$

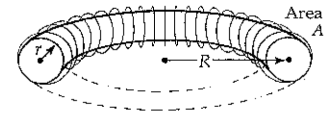

FIG. P23.32

Section 23.6 *RL* Circuits

P23.33 (a) At time t, $\quad I(t) = \frac{\mathcal{E}(1 - e^{-t/\tau})}{R}$

where $\quad \tau = \frac{L}{R} = 0.200$ s.

After a long time, $\quad I_{\max} = \frac{\mathcal{E}(1 - e^{-\infty})}{R} = \frac{\mathcal{E}}{R}$

At $I(t) = 0.500 I_{\max}$ $\quad (0.500)\frac{\mathcal{E}}{R} = \frac{\mathcal{E}(1 - e^{-t/0.200\text{ s}})}{R}$

so $\quad 0.500 = 1 - e^{-t/0.200\text{ s}}$

Isolating the constants on the right, $\quad \ln(e^{-t/0.200\text{ s}}) = \ln(0.500)$

and solving for t, $\quad -\frac{t}{0.200\text{ s}} = -0.693$

or $\quad t = \boxed{0.139\text{ s}}$.

(b) Similarly, to reach 90% of $I_{\max}$, $\quad 0.900 = 1 - e^{-t/\tau}$

and $\quad t = -\tau\ln(1 - 0.900)$.

Thus, $\quad t = -(0.200\text{ s})\ln(0.100) = \boxed{0.461\text{ s}}$.

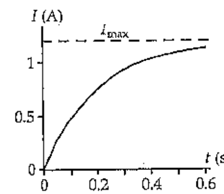

FIG. P23.33

P23.34 Taking $\tau = \frac{L}{R}$: $\quad I = I_0 e^{-t/\tau} \qquad \frac{dI}{dt} = I_0 e^{-t/\tau}\left(-\frac{1}{\tau}\right)$

$IR + L\frac{dI}{dt} = 0$ will be true if $\quad I_0 R e^{-t/\tau} + L\left(I_0 e^{-t/\tau}\right)\left(-\frac{1}{\tau}\right) = 0$.

Because $\tau = \frac{L}{R}$, we have agreement with $0 = 0$.

P23.35 (a) $\tau = \frac{L}{R} = 2.00 \times 10^{-3}\ \text{s} = \boxed{2.00\ \text{ms}}$

(b) $I = I_{\max}\left(1 - e^{-t/\tau}\right) = \left(\frac{6.00\ \text{V}}{4.00\ \Omega}\right)\left(1 - e^{-0.250/2.00}\right) = \boxed{0.176\ \text{A}}$

(c) $I_{\max} = \frac{\mathcal{E}}{R} = \frac{6.00\ \text{V}}{4.00\ \Omega} = \boxed{1.50\ \text{A}}$

(d) $0.800 = 1 - e^{-t/2.00\ \text{ms}} \rightarrow t = -(2.00\ \text{ms})\ln(0.200) = \boxed{3.22\ \text{ms}}$

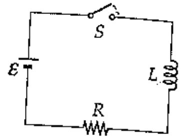

FIG. P23.35

P23.36 (a) $\Delta V_R = IR = (8.00\ \Omega)(2.00\ \text{A}) = 16.0\ \text{V}$

and $\quad \Delta V_L = \mathcal{E} - \Delta V_R = 36.0\ \text{V} - 16.0\ \text{V} = 20.0\ \text{V}$.

Therefore, $\quad \frac{\Delta V_R}{\Delta V_L} = \frac{16.0\ \text{V}}{20.0\ \text{V}} = \boxed{0.800}$.

(b) $\Delta V_R = IR = (4.50\ \text{A})(8.00\ \Omega) = 36.0\ \text{V}$

$\Delta V_L = \mathcal{E} - \Delta V_R = \boxed{0}$

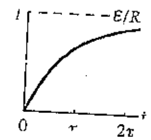

FIG. P23.36

***P23.37** For the increasing current $I = \frac{\mathcal{E}}{R}\left(1 - e^{-Lt/R}\right)$. The final value is $\frac{\mathcal{E}}{R}$, so the condition on Δt is

$$0.8\frac{\mathcal{E}}{R} = \frac{\mathcal{E}}{R}\left(1 - e^{-L\Delta t/R}\right)$$
$$e^{-L\Delta t/R} = 0.2$$
$$e^{+L\Delta t/R} = 5$$
$$\frac{L\Delta t}{R} = \ln 5$$
$$\Delta t = \frac{R\ln 5}{L}$$

At the moment when the battery is removed, the current in the coil is quite precisely $\frac{\mathcal{E}}{R}$. During the decrease, $I = I_i e^{-Lt/R} = \frac{\mathcal{E}}{R}e^{-Lt/R}$

(a) at $t = \Delta t = \frac{R\ln 5}{L}$, $\qquad \frac{I}{I_i} = e^{-L\Delta t/R} = 0.200 = \boxed{20.0\%}$

(b) at $t = 2\Delta t$, $\qquad \frac{I}{I_i} = e^{-L2\Delta t/R} = \left(e^{-L\Delta t/R}\right)^2 = (0.2)^2 = 0.04 = \boxed{4.00\%}$

P23.38 $I = I_{\max}\left(1 - e^{-t/\tau}\right)$: $\quad 0.980 = 1 - e^{-3.00\times10^{-3}/\tau}$

$$0.020\,0 = e^{-3.00\times10^{-3}/\tau}$$
$$\tau = -\frac{3.00\times10^{-3}}{\ln(0.020\,0)} = 7.67\times10^{-4}\ \text{s}$$

$\tau = \frac{L}{R}$, so $\qquad L = \tau R = \left(7.67\times10^{-4}\right)(10.0) = \boxed{7.67\ \text{mH}}$

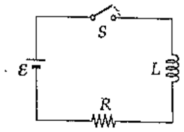

FIG. P23.38

P23.39 Name the currents as shown. By Kirchhoff's laws:

$+I_1 - I_2 - I_3 = 0 \qquad (1)$

$+10.0\ \text{V} - 4.00I_1 - 4.00I_2 = 0 \qquad (2)$

$+10.0\ \text{V} - 4.00I_1 - 8.00I_3 - (1.00)\frac{dI_3}{dt} = 0 \qquad (3)$

From (1) and (2), $\quad +10.0 - 4.00I_1 - 4.00I_1 + 4.00I_3 = 0$

and $\quad I_1 = 0.500I_3 + 1.25\ \text{A}$.

Then (3) becomes $\quad 10.0\ \text{V} - 4.00(0.500I_3 + 1.25\ \text{A}) - 8.00I_3 - (1.00)\frac{dI_3}{dt} = 0$

$$(1.00\ \text{H})\left(\frac{dI_3}{dt}\right) + (10.0\ \Omega)I_3 = 5.00\ \text{V}.$$

We solve the differential equation using Equations 23.13 and 23.14:

$$I_3(t) = \left(\frac{5.00\ \text{V}}{10.0\ \Omega}\right)\left[1 - e^{-(10.0\ \Omega)t/1.00\ \text{H}}\right] = \boxed{(0.500\ \text{A})\left[1 - e^{-10t/\text{s}}\right]}$$
$$I_1 = 1.25 + 0.500I_3 = \boxed{1.50\ \text{A} - (0.250\ \text{A})e^{-10t/\text{s}}}$$

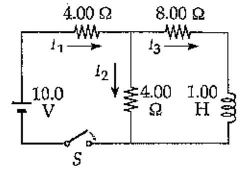

FIG. P23.39

P23.40 (a) $I = \frac{\mathcal{E}}{R} = \frac{12.0\ \text{V}}{12.0\ \Omega} = \boxed{1.00\ \text{A}}$

(b) Initial current is 1.00 A: $\Delta V_{12} = (1.00\ \text{A})(12.00\ \Omega) = \boxed{12.0\ \text{V}}$

$$\Delta V_{1\,200} = (1.00\ \text{A})(1\,200\ \Omega) = \boxed{1.20\ \text{kV}}$$
$$\Delta V_L = \boxed{1.21\ \text{kV}}.$$

(c) $I = I_{\max}e^{-Rt/L}$. $\qquad \frac{dI}{dt} = -I_{\max}\frac{R}{L}e^{-Rt/L}$

and $\qquad -L\frac{dI}{dt} = \Delta V_L = I_{\max}Re^{-Rt/L}$

Solving $\qquad 12.0\ \text{V} = (1\,212\ \text{V})e^{-1\,212t/2.00}$

so $\qquad 9.90\times10^{-3} = e^{-606t}$

Thus, $\qquad \boxed{t = 7.62\ \text{ms}}$.

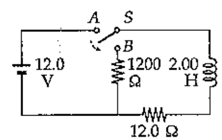

FIG. P23.40

P23.41 $\tau = \frac{L}{R} = \frac{0.140}{4.90} = 28.6 \text{ ms}$

$I_{max} = \frac{\mathcal{E}}{R} = \frac{6.00 \text{ V}}{4.90\ \Omega} = 1.22 \text{ A}$

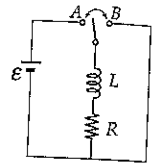

FIG. P23.41

(a) $I = I_{max}\left(1 - e^{-t/\tau}\right)$ so $0.220 = 1.22\left(1 - e^{-t/\tau}\right)$

$e^{-t/\tau} = 0.820$: $t = -\tau \ln(0.820) = \boxed{5.66 \text{ ms}}$

(b) $I = I_{max}\left(1 - e^{-10.0/0.028\,6}\right) = (1.22 \text{ A})\left(1 - e^{-350}\right) = \boxed{1.22 \text{ A}}$

(c) $I = I_{max} e^{-t/\tau}$ and $0.160 = 1.22 e^{-t/\tau}$

so $t = -\tau \ln(0.131) = \boxed{58.1 \text{ ms}}$.

Section 23.7 Energy Stored in a Magnetic Field

P23.42 (a) The magnetic energy density is given by

$$u = \frac{B^2}{2\mu_0} = \frac{(4.50 \text{ T})^2}{2\left(1.26\times10^{-6} \text{ T}\cdot\text{m/A}\right)} = \boxed{8.06\times10^6 \text{ J/m}^3}$$

(b) The magnetic energy stored in the field equals u times the volume of the solenoid (the volume in which B is non-zero).

$$U = uV = \left(8.06\times10^6 \text{ J/m}^3\right)\left[(0.260 \text{ m})\pi(0.031\,0 \text{ m})^2\right] = \boxed{6.32 \text{ kJ}}$$

P23.43 $L = \mu_0 \frac{N^2 A}{\ell} = \mu_0 \frac{(68.0)^2\left[\pi\left(0.600\times10^{-2}\right)^2\right]}{0.080\,0} = 8.21\ \mu\text{H}$

$U = \frac{1}{2}LI^2 = \frac{1}{2}\left(8.21\times10^{-6} \text{ H}\right)(0.770 \text{ A})^2 = \boxed{2.44\ \mu\text{J}}$

P23.44 (a) $U = \frac{1}{2}LI^2 = \frac{1}{2}(4.00 \text{ H})(0.500 \text{ A})^2$ $U = \boxed{0.500 \text{ J}}$

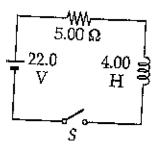

FIG. P23.44

(b) When the current is 1.00 A,

Kirchhoff's loop rule reads $+22.0 \text{ V} - (1.00 \text{ A})(5.00\ \Omega) - \Delta V_L = 0$.

Then $\Delta V_L = 17.0 \text{ V}$.

The power being stored in the inductor is

$$I\Delta V_L = (1.00 \text{ A})(17.0 \text{ V}) = \boxed{17.0 \text{ W}}.$$

(c) $\mathcal{P} = I\Delta V = (0.500 \text{ A})(22.0 \text{ V})$ $\mathcal{P} = \boxed{11.0 \text{ W}}$

P23.45 $u = \epsilon_0 \frac{E^2}{2} = \boxed{44.2 \text{ nJ/m}^3}$ $u = \frac{B^2}{2\mu_0} = \boxed{995\ \mu\text{J/m}^3}$

Section 23.8 Context Connection—The Repulsive Model for Magnetic Levitation

P23.46 The induced emf in the leading edge of the loop is

$$\mathcal{E} = B\ell v = B(0.2 \text{ m})(400 \text{ km/h})\left(\frac{1\,000 \text{ m}}{\text{km}}\right)\left(\frac{\text{h}}{3\,600 \text{ s}}\right) = \left(22.2 \text{ m}^2/\text{s}\right)B$$

The induced current is $I = \frac{\mathcal{E}}{R} = \frac{\left(22.2 \text{ m}^2\right)B}{25\ \Omega\cdot\text{s}}$

The magnetic force on the lower section of one loop is $\vec{\mathbf{F}}_B = I\vec{\ell}\times\vec{\mathbf{B}}$:

$$\vec{\mathbf{F}}_B = \left(\frac{22.2 \text{ m}^2\text{A}}{25 \text{ V}\cdot\text{s}}\right)B(0.1 \text{ m})B\sin 90° = \left(0.0889 \text{ m}^3\cdot\text{C}^2/\text{J}\cdot\text{s}^2\right)B^2 \text{ up}$$

We require $\Sigma F_y = 0$: $100F_B - mg = 0$

$B^2\left(8.89 \text{ m}^2\cdot\text{C}^2/\text{N}\cdot\text{s}^2\right) = 5\times10^4(9.80 \text{ N})$

$B = 235 \text{ N}\cdot\text{s/C}\cdot\text{m} = \boxed{2\times10^2 \text{ T}}$

P23.47 (a) $B = \frac{\mu_0 NI}{\ell} = \frac{\left(4\pi\times10^{-7} \text{ T}\cdot\text{m/A}\right)(1\,400)(2.00 \text{ A})}{1.20 \text{ m}} = \boxed{2.93\times10^{-3} \text{ T (upward)}}$

(b) $u = \frac{B^2}{2\mu_0} = \frac{\left(2.93\times10^{-3} \text{ T}\right)^2}{2\left(4\pi\times10^{-7} \text{ T}\cdot\text{m/A}\right)} = \left(3.42 \text{ J/m}^3\right)\left(\frac{1 \text{ N}\cdot\text{m}}{1 \text{ J}}\right) = 3.42 \text{ N/m}^2 = \boxed{3.42 \text{ Pa}}$

(c) To produce a downward magnetic field, the surface of the superconductor must carry a $\boxed{\text{clockwise}}$ current.

(d) The vertical component of the field of the solenoid exerts an inward force on the superconductor. The total horizontal force is zero. Over the top end of the solenoid, its field diverges and has a radially outward horizontal component. This component exerts upward force on the clockwise superconductor current. The total force on the core is $\boxed{\text{upward}}$. You can think of it as a force of repulsion between the solenoid with its north end pointing up, and the core, with its north end pointing down.

(e) $F = PA = (3.42 \text{ Pa})\left[\pi\left(1.10\times10^{-2} \text{ m}\right)^2\right] = \boxed{1.30\times10^{-3} \text{ N}}$

Note that we have not proven that energy density is pressure. In fact, it is not in some cases; Equation 16.13 shows that the pressure is two-thirds of the translational energy density in an ideal gas.

Additional Problems

P23.48 (a) Doubling the number of turns.

$\boxed{\text{Amplitude doubles: period unchanged}}$

(b) Doubling the angular velocity.

$\boxed{\text{doubles the amplitude: cuts the period in half}}$

(c) Doubling the angular velocity while reducing the number of turns to one half the original value.

$\boxed{\text{Amplitude unchanged: cuts the period in half}}$

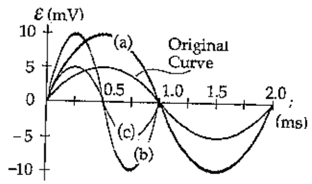

FIG. P23.48

P23.49 $\mathcal{E} = -N\frac{d}{dt}(BA\cos\theta) = -N(\pi r^2)\cos 0°\left(\frac{dB}{dt}\right)$

$$\mathcal{E} = -(30.0)\left[\pi(2.70\times10^{-3}\text{ m})^2\right](1)\frac{d}{dt}\left[50.0\text{ mT}+(3.20\text{ mT})\sin(2\pi[523t\text{ s}^{-1}])\right]$$

$$\mathcal{E} = -(30.0)\left[\pi(2.70\times10^{-3}\text{ m})^2\right](3.20\times10^{-3}\text{ T})\left[2\pi(523\text{ s}^{-1})\cos(2\pi[523t\text{ s}^{-1}])\right]$$

$$\mathcal{E} = \boxed{-(7.22\times10^{-3}\text{ V})\cos\left[2\pi(523t\text{ s}^{-1})\right]}$$

***P23.50** $\mathcal{E} = -N\frac{\Delta}{\Delta t}(BA\cos\theta) = -N(\pi r^2)\cos 0°\frac{\Delta B}{\Delta t} = -1(0.005\,00\text{ m}^2)(1)\left(\frac{1.50\text{ T}-5.00\text{ T}}{20.0\times10^{-3}\text{ s}}\right) = 0.875\text{ V}$

(a) $I = \frac{\mathcal{E}}{R} = \frac{0.875\text{ V}}{0.020\,0\ \Omega} = \boxed{43.8\text{ A}}$

(b) $\mathcal{P} = \mathcal{E}I = (0.875\text{ V})(43.8\text{ A}) = \boxed{38.3\text{ W}}$

***P23.51** Suppose we wrap twenty turns of wire into a flat compact circular coil of diameter 3 cm. Suppose we use a bar magnet to produce field 10^{-3} T through the coil in one direction along its axis. Suppose we then flip the magnet to reverse the flux in 10^{-1} s. The average induced emf is then

$$\bar{\mathcal{E}} = -N\frac{\Delta\Phi_B}{\Delta t} = -N\frac{\Delta[BA\cos\theta]}{\Delta t} = -NB(\pi r^2)\left(\frac{\cos180° - \cos0°}{\Delta t}\right)$$

$$\bar{\mathcal{E}} = -(20)(10^{-3}\text{ T})\pi(0.015\,0\text{ m})^2\left(\frac{-2}{10^{-1}\text{ s}}\right)\boxed{\sim10^{-4}\text{ V}}$$

P23.52 $I = \frac{\mathcal{E}+\mathcal{E}_{\text{induced}}}{R}$ and $\mathcal{E}_{\text{induced}} = -\frac{d}{dt}(BA)$

$$F = m\frac{dv}{dt} = IBd$$

$$\frac{dv}{dt} = \frac{IBd}{m} = \frac{Bd}{mR}(\mathcal{E}+\mathcal{E}_{\text{induced}})$$

$$\frac{dv}{dt} = \frac{Bd}{mR}(\mathcal{E}-Bvd)$$

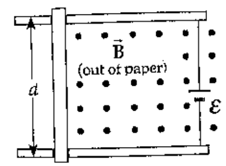

FIG. P23.52

To solve the differential equation, let $u = \mathcal{E} - Bvd$

$$\frac{du}{dt} = -Bd\frac{dv}{dt}$$

$$-\frac{1}{Bd}\frac{du}{dt} = \frac{Bd}{mR}u$$

so $\int_{u_0}^{u}\frac{du}{u} = -\int_0^t\frac{(Bd)^2}{mR}dt.$

Integrating from $t=0$ to $t=t$, $\ln\frac{u}{u_0} = -\frac{(Bd)^2}{mR}t$

or $\frac{u}{u_0} = e^{-B^2d^2t/mR}$

Since $v=0$ when $t=0$, $u_0 = \mathcal{E}$

and $u = \mathcal{E} - Bvd$

$$\mathcal{E} - Bvd = \mathcal{E}e^{-B^2d^2t/mR}$$

Therefore, $\boxed{v = \frac{\mathcal{E}}{Bd}\left(1-e^{-B^2d^2t/mR}\right)}.$

P23.53 The enclosed flux is $\Phi_B = BA = B\pi r^2$

The particle moves according to $\sum\vec{F} = m\vec{a}$: $qvB\sin90° = \frac{mv^2}{r}$

$$r = \frac{mv}{qB}$$

Then $\Phi_B = \frac{B\pi m^2v^2}{q^2B^2}$

(a) $v = \sqrt{\frac{\Phi_B q^2 B}{\pi m^2}} = \sqrt{\frac{(15\times10^{-6}\text{ T}\cdot\text{m}^2)(30\times10^{-9}\text{ C})^2(0.6\text{ T})}{\pi(2\times10^{-16}\text{ kg})^2}} = \boxed{2.54\times10^5\text{ m/s}}$

continued on next page

(b) Energy for the particle-electric field system is conserved in the firing process:

$U_i = K_f: \quad q\Delta V = \frac{1}{2}mv^2$

$$\Delta V = \frac{mv^2}{2q} = \frac{(2\times10^{-16}\text{ kg})(2.54\times10^5\text{ m/s})^2}{2(30\times10^{-9}\text{ C})} = \boxed{215\text{ V}}$$

P23.54 (a) Consider an annulus of radius r, width dr, height b, and resistivity ρ. Around its circumference, a voltage is induced according to

$$\mathcal{E} = -N\frac{d}{dt}\vec{\mathbf{B}}\cdot\vec{\mathbf{A}} = -1\frac{d}{dt}B_{max}(\cos\omega t)\pi r^2 = +B_{max}\pi r^2\omega\sin\omega t.$$

The resistance around the loop is $\frac{\rho\ell}{A_x} = \frac{\rho(2\pi r)}{bdr}$

The eddy current in the ring is $dI = \frac{\mathcal{E}}{\text{resistance}} = \frac{B_{max}\pi r^2\omega(\sin\omega t)bdr}{\rho(2\pi r)} = \frac{B_{max}rb\omega dr\sin\omega t}{2\rho}$

The instantaneous power is $d\mathscr{P}_i = \mathcal{E}dI = \frac{B_{max}^2\pi r^3 b\omega^2 dr\sin^2\omega t}{2\rho}$.

The time average of the function $\sin^2\omega t = \frac{1}{2} - \frac{1}{2}\cos 2\omega t$ is $\frac{1}{2} - 0 = \frac{1}{2}$

so the time-averaged power delivered to the annulus is

$$d\mathscr{P} = \frac{B_{max}^2\pi r^3 b\omega^2 dr}{4\rho}$$

The power delivered to the disk is $\mathscr{P} = \int d\mathscr{P} = \int_0^R \frac{B_{max}^2\pi b\omega^2}{4\rho}r^3 dr$

$$\mathscr{P} = \frac{B_{max}^2\pi b\omega^2}{4\rho}\left(\frac{R^4}{4} - 0\right) = \boxed{\frac{\pi B_{max}^2 R^4 b\omega^2}{16\rho}}.$$

(b) When B_{max} gets two times larger, B_{max}^2 and $\mathscr{P}$ get $\boxed{4}$ times larger.

(c) When f and $\omega = 2\pi f$ double. ω^2 and $\mathscr{P}$ get $\boxed{4}$ times larger.

(d) When R doubles, R^4 and $\mathscr{P}$ become $2^4 = \boxed{16}$ times larger.

P23.55 We are given $\Phi_B = (6.00t^3 - 18.0t^2)\text{ T}\cdot\text{m}^2$

and $\mathcal{E} = -\frac{d\Phi_B}{dt} = -18.0t^2 + 36.0t$.

Maximum $\mathcal{E}$ occurs when $\frac{d\mathcal{E}}{dt} = -36.0t + 36.0 = 0$

which gives $t = 1.00$ s.

Therefore, the maximum current (at $t = 1.00$ s) is $I = \frac{\mathcal{E}}{R} = \frac{(-18.0+36.0)\text{ V}}{3.00\ \Omega} = \boxed{6.00\text{ A}}$.

*P23.56 (a) $\mathcal{E} = -N\frac{d}{dt}BA\cos\theta = -1\frac{d}{dt}B\frac{\theta a^2}{2}\cos 0° = -\frac{Ba^2}{2}\frac{d\theta}{dt} = -\frac{1}{2}Ba^2\omega = -\frac{1}{2}(0.5\text{ T})(0.5\text{ m})^2 2\text{ rad/s}$

$= -0.125\text{ V} = \boxed{0.125\text{ V clockwise}}$

The – sign indicates that the induced emf produces clockwise current, to make its own magnetic field into the page.

(b) At this instant $\theta = \omega t = 2\text{ rad/s}(0.25\text{ s}) = 0.5$ rad. The arc PQ has length $r\theta = (0.5\text{ rad})(0.5\text{ m}) = 0.25$ m. The length of the circuit is $0.5\text{ m} + 0.5\text{ m} + 0.25\text{ m} = 1.25$ m its resistance is $1.25\text{ m}(5\ \Omega/\text{m}) = 6.25\ \Omega$. The current is $\frac{0.125\text{ V}}{6.25\ \Omega} = \boxed{0.020\,0\text{ A clockwise}}$

P23.57 Suppose the field is vertically down. When an electron is moving away from you, the force on it is in the direction given by

$q\vec{\mathbf{v}}\times\vec{\mathbf{B}}_r$ as $-(\text{away})\times\text{down} = -$ (↻) $= -\text{left} = \text{right}$.

Therefore, the electrons circulate clockwise.

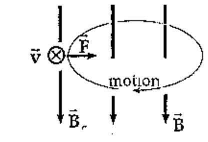

FIG. P23.57

(a) As the downward field increases, an emf is induced to produce some current that in turn produces an upward field. This current is directed counterclockwise, carried by negative electrons moving clockwise. Therefore the original electron motion speeds up.

(b) At the circumference, we have $\sum F_c = ma_c$: $\quad |q|vB_r\sin 90° = \frac{mv^2}{r}$

$mv = |q|rB_r$.

The increasing magnetic field $\vec{\mathbf{B}}_{av}$ in the area enclosed by the orbit produces a tangential electric field according to

$\left|\oint\vec{\mathbf{E}}\cdot d\vec{\mathbf{s}}\right| = \left|-\frac{d}{dt}\vec{\mathbf{B}}_{av}\cdot\vec{\mathbf{A}}\right| \qquad E(2\pi r) = \pi r^2\frac{dB_{av}}{dt} \qquad E = \frac{r}{2}\frac{dB_{av}}{dt}$

An electron feels a tangential force according to $\sum F_t = ma_t$: $\quad |q|E = m\frac{dv}{dt}$

Then $\quad |q|\frac{r}{2}\frac{dB_{av}}{dt} = m\frac{dv}{dt} \qquad |q|\frac{r}{2}B_{av} = mv = |q|rB_r$

and $\quad B_{av} = 2B_r$.

P23.58 $\mathcal{E} = -N\frac{d\Phi_B}{dt} = -N\frac{d}{dt}(BA\cos\theta)$

$$\mathcal{E} = -NB\cos\theta\left(\frac{\Delta A}{\Delta t}\right) = -200(50.0\times10^{-6}\text{ T})(\cos 62.0°)\left(\frac{39.0\times10^{-4}\text{ m}^2}{1.80\text{ s}}\right) = \boxed{-10.2\ \mu\text{V}}$$

P23.59 The magnetic field produced by the current in the straight wire is perpendicular to the plane of the coil at all points within the coil. The magnitude of the field is $B = \frac{\mu_0 I}{2\pi r}$. Thus, the flux linkage is

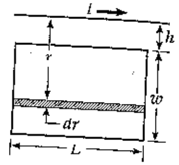

FIG. P23.59

$$N\Phi_B = \frac{\mu_0 NIL}{2\pi}\int_h^{h+w}\frac{dr}{r} = \frac{\mu_0 NI_{max}L}{2\pi}\ln\left(\frac{h+w}{h}\right)\sin(\omega t+\phi).$$

Finally, the induced emf is

$$\mathcal{E} = -\frac{\mu_0 NI_{max}L\omega}{2\pi}\ln\left(1+\frac{w}{h}\right)\cos(\omega t+\phi)$$

$$\mathcal{E} = -\frac{(4\pi\times10^{-7})(100)(50.0)(0.200\text{ m})(200\pi\text{s}^{-1})}{2\pi}\ln\left(1+\frac{5.00\text{ cm}}{5.00\text{ cm}}\right)\cos(\omega t+\phi)$$

$$\mathcal{E} = \boxed{-(87.1\text{ mV})\cos(200\pi t+\phi)}.$$

The term $\sin(\omega t+\phi)$ in the expression for the current in the straight wire does not change appreciably when ωt changes by 0.10 rad or less. Thus, the current does not change appreciably during a time interval

$$\Delta t < \frac{0.10}{(200\pi\text{s}^{-1})} = 1.6\times10^{-4}\text{ s}.$$

We define a critical length, $c\Delta t = (3.00\times10^8\text{ m/s})(1.6\times10^{-4}\text{ s}) = 4.8\times10^4\text{ m}$ equal to the distance to which field changes could be propagated during an interval of 1.6×10^{-4} s. This length is so much larger than any dimension of the coil or its distance from the wire that, although we consider the straight wire to be infinitely long, we can also safely ignore the field propagation effects in the vicinity of the coil. Moreover, the phase angle can be considered to be constant along the wire in the vicinity of the coil.

If the angular frequency ω were much larger, say, $200\pi\times10^5\text{ s}^{-1}$, the corresponding critical length would be only 48 cm. In this situation propagation effects would be important and the above expression for $\mathcal{E}$ would require modification. As a "rule of thumb" we can consider field propagation effects for circuits of laboratory size to be negligible for frequencies, $f = \frac{\omega}{2\pi}$, that are less than about 10^6 Hz.

P23.60 $B = \frac{\mu_0 NI}{2\pi r}$

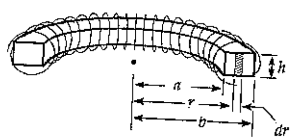

FIG. P23.60

(a) $\Phi_B = \int B\,dA = \int_a^b \frac{\mu_0 NI}{2\pi r}h\,dr = \frac{\mu_0 NIh}{2\pi}\int_a^b\frac{dr}{r} = \frac{\mu_0 NIh}{2\pi}\ln\left(\frac{b}{a}\right)$

$L = \frac{N\Phi_B}{I} = \boxed{\frac{\mu_0 N^2 h}{2\pi}\ln\left(\frac{b}{a}\right)}$

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(b) $L = \frac{\mu_0(500)^2(0.010\,0)}{2\pi}\ln\left(\frac{12.0}{10.0}\right) = \boxed{91.2\ \mu\text{H}}$

(c) $L_{appx} = \frac{\mu_0 N^2}{2\pi}\left(\frac{A}{R}\right) = \frac{\mu_0(500)^2}{2\pi}\left(\frac{2.00\times10^{-4}\text{ m}^2}{0.110}\right) = \boxed{90.9\ \mu\text{H}}$, only 0.3% different.

P23.61 (a) At the center, $B = \frac{N\mu_0 IR^2}{2(R^2+0^2)^{3/2}} = \frac{N\mu_0 I}{2R}$

So the coil creates flux through itself $\Phi_B = BA\cos\theta = \frac{N\mu_0 I}{2R}\pi R^2\cos0° = \frac{\pi}{2}N\mu_0 IR$.

When the current it carries changes, $\mathcal{E}_L = -N\frac{d\Phi_B}{dt} \approx -N\left(\frac{\pi}{2}\right)N\mu_0 R\frac{dI}{dt} = -L\frac{dI}{dt}$

so $\boxed{L \approx \frac{\pi}{2}N^2\mu_0 R}$

(b) $2\pi r = 3(0.3\text{ m})$ so $r \approx 0.14$ m

$L \approx \frac{\pi}{2}(1^2)(4\pi\times10^{-7}\text{ T}\cdot\text{m/A})(0.14\text{ m}) = 2.8\times10^{-7}$ H

$\boxed{L \sim 100\text{ nH}}$

(c) $\frac{L}{R} = \frac{2.8\times10^{-7}\text{ V}\cdot\text{s/A}}{270\text{ V/A}} = 1.0\times10^{-9}$ s $\boxed{\frac{L}{R} \sim 1\text{ ns}}$

P23.62 When the switch is closed, as shown in figure (a), the current in the inductor is I:

$12.0 - 7.50I - 10.0 = 0 \to I = 0.267$ A.

When the switch is opened, the initial current in the inductor remains at 0.267 A.

$IR = \Delta V$: $(0.267\text{ A})R \le 80.0\text{ V}$

$\boxed{R \le 300\ \Omega}$

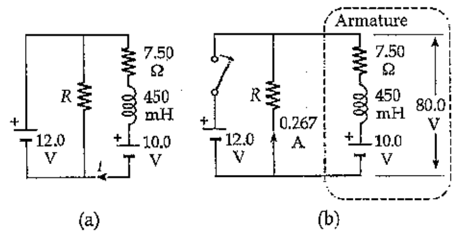

FIG. P23.62

P23.63 For an RL circuit,

$$I(t) = I_{\max} e^{-(R/L)t} \qquad \frac{I(t)}{I_{\max}} = 1 - 10^{-9} = e^{-(R/L)t} \approx 1 - \frac{R}{L}t$$

$$\frac{R}{L}t = 10^{-9} \quad \text{so} \quad R_{\max} = \frac{(3.14\times10^{-8})(10^{-9})}{(2.50\text{ yr})(3.16\times10^{7}\text{ s/yr})} = \boxed{3.97\times10^{-25}\ \Omega}.$$

(If the ring were of purest copper, of diameter 1 cm, and cross-sectional area 1 mm^2, its resistance would be at least $10^{-6}\ \Omega$.)

P23.64 (a) $U_B = \frac{1}{2}LI^2 = \frac{1}{2}(50.0\text{ H})(50.0\times10^3\text{ A})^2 = \boxed{6.25\times10^{10}\text{ J}}$

(b) Two adjacent turns are parallel wires carrying current in the same direction. Since the loops have such large radius, a one-meter section can be regarded as straight.

Then one wire creates a field of $B = \frac{\mu_0 I}{2\pi r}$

This causes a force on the next wire of $F = I\ell B\sin\theta$

giving $F = I\ell\frac{\mu_0 I}{2\pi r}\sin 90° = \frac{\mu_0 \ell I^2}{2\pi r}$

Evaluating the force, $F = (4\pi\times10^{-7}\text{ N/A}^2)\frac{(1.00\text{ m})(50.0\times10^3\text{ A})^2}{2\pi(0.250\text{ m})} = \boxed{2\,000\text{ N}}$.

P23.65 $\mathcal{P} = I\Delta V \qquad I = \frac{\mathcal{P}}{\Delta V} = \frac{1.00\times10^9\text{ W}}{200\times10^3\text{ V}} = 5.00\times10^3\text{ A}$

From Ampère's law, $B(2\pi r) = \mu_0 I_{\text{enclosed}}$ or $B = \frac{\mu_0 I_{\text{enclosed}}}{2\pi r}$

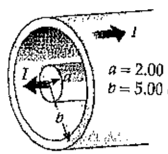

FIG. P23.65

(a) At $r = a = 0.020\,0\text{ m}$, $I_{\text{enclosed}} = 5.00\times10^3\text{ A}$

and $B = \frac{(4\pi\times10^{-7}\text{ T}\cdot\text{m/A})(5.00\times10^3\text{ A})}{2\pi(0.020\,0\text{ m})} = 0.050\,0\text{ T} = \boxed{50.0\text{ mT}}$

(b) At $r = b = 0.050\,0\text{ m}$, $I_{\text{enclosed}} = I = 5.00\times10^3\text{ A}$

and $B = \frac{(4\pi\times10^{-7}\text{ T}\cdot\text{m/A})(5.00\times10^3\text{ A})}{2\pi(0.050\,0\text{ m})} = 0.020\,0\text{ T} = \boxed{20.0\text{ mT}}$.

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(c) $U = \int u\,dV = \int_a^b \frac{[B(r)]^2(2\pi r\ell dr)}{2\mu_0} = \frac{\mu_0 I^2 \ell}{4\pi}\int_a^b \frac{dr}{r} = \frac{\mu_0 I^2 \ell}{4\pi}\ln\left(\frac{b}{a}\right)$

$$U = \frac{(4\pi\times10^{-7}\text{ T}\cdot\text{m/A})(5.00\times10^3\text{ A})^2(1\,000\times10^3\text{ m})}{4\pi}\ln\left(\frac{5.00\text{ cm}}{2.00\text{ cm}}\right) = 2.29\times10^6\text{ J} = \boxed{2.29\text{ MJ}}$$

(d) The magnetic field created by the inner conductor exerts a force of repulsion on the current in the outer sheath. The strength of this field, from part (b), is 20.0 mT. Consider a small rectangular section of the outer cylinder of length ℓ and width w.

It carries a current of $(5.00\times10^3\text{ A})\left(\frac{w}{2\pi(0.050\,0\text{ m})}\right)$

and experiences an outward force $F = I\ell B\sin\theta = \frac{(5.00\times10^3\text{ A})w}{2\pi(0.050\,0\text{ m})}\ell(20.0\times10^{-3}\text{ T})\sin 90.0°$.

The pressure on it is $P = \frac{F}{A} = \frac{F}{w\ell} = \frac{(5.00\times10^3\text{ A})(20.0\times10^{-3}\text{ T})}{2\pi(0.050\,0\text{ m})} = \boxed{318\text{ Pa}}$.

ANSWERS TO EVEN PROBLEMS

P23.2	9.82 mV	**P23.22**	(a) $F = \frac{N^2B^2w^2v}{R}$ to the left; (b) 0; (c) $F = \frac{N^2B^2w^2v}{R}$ to the left
P23.4	(a) $\frac{\mu_0 n\pi r_2^2}{2R}\left(\frac{\Delta I}{\Delta t}\right)$; (b) $\frac{\mu_0^2 n\pi r_2^2}{4r_1 R}\left(\frac{\Delta I}{\Delta t}\right)$; (c) upward	**P23.24**	$(28.6\text{ mV})\sin(4\pi t)$
P23.6	$-(14.2\text{ mV})\cos(120t)$	**P23.26**	(a) 8.00×10^{-21} N, clockwise; (b) 1.33 s and 0
P23.8	(a) see the solution; (b) 625 m/s	**P23.28**	1.36 μH
P23.10	13.3 mA counterclockwise in the lower loop and clockwise in the upper loop	**P23.30**	19.2 $\mu\text{T}\cdot\text{m}^2$
P23.12	1.00 m/s	**P23.32**	see the solution
P23.14	(a) 0.500 A; (b) 2.00 W; (c) 2.00 W	**P23.34**	see the solution
P23.16	360 T	**P23.36**	(a) 0.800; (b) 0
P23.18	(a) 233 Hz; (b) 1.98 mV	**P23.38**	7.67 mH
P23.20	(a) to the right; (b) to the right; (c) to the right; (d) into the paper		