(b) $A = 4\pi r^2 = 4\pi(0.02\text{ m})^2 = 5.03\times10^{-3}\text{ m}^2$

$$\mathscr{P} = \sigma A e T^4 = 5.67\times10^{-8}\text{ W/m}^2\text{K}^4\ 5.03\times10^{-3}\text{ m}^2\ 0.86(293\text{ K})^4 = \boxed{1.81\text{ W}}$$

(c) It emits but does not absorb radiation, so its temperature must drop according to

$$Q = mc\Delta T = mc(T_f - T_i) \qquad \frac{dQ}{dt} = mc\frac{dT_f}{dt}$$

$$\frac{dT_f}{dt} = \frac{\frac{dQ}{dt}}{mc} = \frac{-\mathscr{P}}{mc} = \frac{-1.81\text{ J/s}}{0.263\text{ kg } 448\text{ J/kg}\cdot\text{C}^\circ} = \boxed{-0.015\ 3\ ^\circ\text{C/s}} = -0.919\ ^\circ\text{C/min}$$

(d) $\lambda_{max}T = 2.898\times10^{-3}\text{ m}\cdot\text{K}$

$$\lambda_{max} = \frac{2.898\times10^{-3}\text{ m}\cdot\text{K}}{293\text{ K}} = \boxed{9.89\times10^{-6}\text{ m}}\text{ infrared}$$

(e) $E = hf = \frac{hc}{\lambda} = \frac{6.63\times10^{-34}\text{ Js } 3\times10^8\text{ m/s}}{9.89\times10^{-6}\text{ m}} = \boxed{2.01\times10^{-20}\text{ J}}$

(f) The energy output each second is carried by photons according to

$$\mathscr{P} = \left(\frac{N}{\Delta t}\right)E$$

$$\frac{N}{\Delta t} = \frac{\mathscr{P}}{E} = \frac{1.81\text{ J/s}}{2.01\times10^{-20}\text{ J/photon}} = \boxed{8.98\times10^{19}\text{ photon/s}}$$

Matter is coupled to radiation, quite strongly, in terms of photon numbers.

:ion 28.2 The Photoelectric Effect

28.9

(a) $\lambda_c = \frac{hc}{\phi} = \frac{(6.626\times10^{-34}\text{ J}\cdot\text{s})(3.00\times10^8\text{ m/s})}{(4.20\text{ eV})(1.60\times10^{-19}\text{ J/eV})} = \boxed{296\text{ nm}}$

$$f_c = \frac{c}{\lambda_c} = \frac{3.00\times10^8\text{ m/s}}{296\times10^{-9}\text{ m}} = \boxed{1.01\times10^{15}\text{ Hz}}$$

(b) $\frac{hc}{\lambda} = \phi + e\Delta V_S$: $\frac{(6.626\times10^{-34})(3.00\times10^8)}{180\times10^{-9}} = (4.20\text{ eV})(1.60\times10^{-19}\text{ J/eV}) + (1.60\times10^{-19})\Delta V_S$

Therefore, $\boxed{\Delta V_S = 2.71\text{ V}}$

P28.10 $K_{\max}=\frac{1}{2}mv_{\max}^2=\frac{1}{2}\left(9.11\times10^{-31}\right)\left(4.60\times10^5\right)^2=9.64\times10^{-20}\text{ J}=0.602\text{ eV}$

(a) $\phi=E-K_{\max}=\frac{1\,240\text{ eV}\cdot\text{nm}}{625\text{ nm}}-0.602\text{ nm}=\boxed{1.38\text{ eV}}$

(b) $f_c=\frac{\phi}{h}=\frac{1.38\text{ eV}}{6.626\times10^{-34}\text{ J}\cdot\text{s}}\left(\frac{1.60\times10^{-19}\text{ J}}{1\text{ eV}}\right)=\boxed{3.34\times10^{14}\text{ Hz}}$

P28.11 (a) $e\Delta V_S=\frac{hc}{\lambda}-\phi\rightarrow\phi=\frac{1\,240\text{ nm}\cdot\text{eV}}{546.1\text{ nm}}-0.376\text{ eV}=\boxed{1.90\text{ eV}}$

(b) $e\Delta V_S=\frac{hc}{\lambda}-\phi=\frac{1\,240\text{ nm}\cdot\text{eV}}{587.5\text{ nm}}-1.90\text{ eV}\rightarrow\boxed{\Delta V_S=0.216\text{ V}}$

P28.12 The energy needed is $E=1.00\text{ eV}=1.60\times10^{-19}\text{ J}.$

The energy absorbed in time interval Δt is $E=\mathcal{P}\Delta t=IA\,\Delta t$

so $\Delta t=\frac{E}{IA}=\frac{1.60\times10^{-19}\text{ J}}{\left(500\text{ J/s}\cdot\text{m}^2\right)\left[\pi\left(2.82\times10^{-15}\text{ m}\right)^2\right]}=1.28\times10^7\text{ s}=\boxed{148\text{ days}}.$

The gross failure of the classical theory of the photoelectric effect contrasts with the success of quantum mechanics.

P28.13 Ultraviolet photons will be absorbed to knock electrons out of the sphere with maximum kinetic energy $K_{\max}=hf-\phi$,

or $K_{\max}=\frac{\left(6.626\times10^{-34}\text{ J}\cdot\text{s}\right)\left(3.00\times10^8\text{ m/s}\right)}{200\times10^{-9}\text{ m}}\left(\frac{1.00\text{ eV}}{1.60\times10^{-19}\text{ J}}\right)-4.70\text{ eV}=1.51\text{ eV}.$

The sphere is left with positive charge and so with positive potential relative to $V=0$ at $r=\infty$. As its potential approaches 1.51 V, no further electrons will be able to escape, but will fall back onto the sphere. Its charge is then given by

$V=\frac{k_eQ}{r}$ or $Q=\frac{rV}{k_e}=\frac{\left(5.00\times10^{-2}\text{ m}\right)\left(1.51\text{ N}\cdot\text{m/C}\right)}{8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2}=\boxed{8.41\times10^{-12}\text{ C}}.$

Section 28.3 The Compton Effect

P28.14 $E=\frac{hc}{\lambda}=\frac{\left(6.626\times10^{-34}\text{ J}\cdot\text{s}\right)\left(3.00\times10^8\text{ m/s}\right)}{700\times10^{-9}\text{ m}}=2.84\times10^{-19}\text{ J}=\boxed{1.78\text{ eV}}$

$p=\frac{h}{\lambda}=\frac{6.626\times10^{-34}\text{ J}\cdot\text{s}}{700\times10^{-9}\text{ m}}=\boxed{9.47\times10^{-28}\text{ kg}\cdot\text{m/s}}$

P28.37 (a) $\psi(x) = Ae^{i(5.00\times10^{10}x)} = A\cos(5\times10^{10}x) + Ai\sin(5\times10^{10}x) = A\cos(kx) + Ai\sin(kx)$ goes through a full cycle when x changes by λ and when kx changes by 2π. Then $k\lambda = 2\pi$ where $k = 5.00\times10^{10}\ \text{m}^{-1} = \frac{2\pi}{\lambda}$. Then $\lambda = \frac{2\pi\ \text{m}}{(5.00\times10^{10})} = \boxed{1.26\times10^{-10}\ \text{m}}$.

(b) $$p = \frac{h}{\lambda} = \frac{6.626\times10^{-34}\ \text{J}\cdot\text{s}}{1.26\times10^{-10}\ \text{m}} = \boxed{5.27\times10^{-24}\ \text{kg}\cdot\text{m/s}}$$

(c) $m_e = 9.11\times10^{-31}\ \text{kg}$

$$K = \frac{m_e^2v^2}{2m_e} = \frac{p^2}{2m} = \frac{(5.27\times10^{-24}\ \text{kg}\cdot\text{m/s})^2}{(2\times9.11\times10^{-31}\ \text{kg})} = 1.52\times10^{-17}\ \text{J} = \frac{1.52\times10^{-17}\ \text{J}}{1.60\times10^{-19}\ \text{J/eV}} = \boxed{95.5\ \text{eV}}$$

Section 28.10 A Particle in a Box

P28.38 For an electron wave to "fit" into an infinitely deep potential well, an integral number of half-wavelengths must equal the width of the well.

$$\frac{n\lambda}{2} = 1.00\times10^{-9}\ \text{m} \qquad \text{so} \qquad \lambda = \frac{2.00\times10^{-9}}{n} = \frac{h}{p}$$

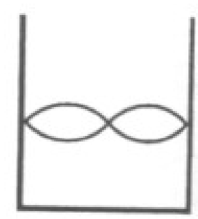

FIG. P28.38

(a) Since $$K = \frac{p^2}{2m_e} = \frac{(h^2/\lambda^2)}{2m_e} = \frac{h^2}{2m_e}\frac{n^2}{(2\times10^{-9})^2} = (0.377n^2)\ \text{eV}$$

For $K \approx 6$ eV $\boxed{n = 4}$

(b) With $n = 4$, $\boxed{K = 6.03\ \text{eV}}$

28.39 (a) We can draw a diagram that parallels our treatment of standing mechanical waves. In each state, we measure the distance d from one node to another (N to N), and base our solution upon that:

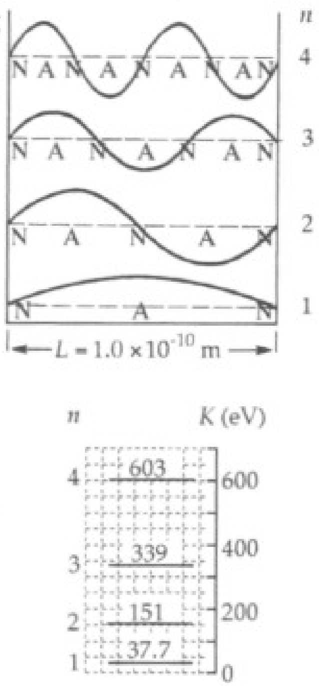

FIG. P28.39

Since $d_{\text{N to N}} = \dfrac{\lambda}{2}$ and $\lambda = \dfrac{h}{p}$

$$p = \frac{h}{\lambda} = \frac{h}{2d}.$$

Next, $K = \dfrac{p^2}{2m_e} = \dfrac{h^2}{8m_e d^2} = \dfrac{1}{d^2}\left[\dfrac{\left(6.626\times10^{-34}\ \text{J}\cdot\text{s}\right)^2}{8\left(9.11\times10^{-31}\ \text{kg}\right)}\right].$

Evaluating, $K = \dfrac{6.02\times10^{-38}\ \text{J}\cdot\text{m}^2}{d^2}$ $\quad K = \dfrac{3.77\times10^{-19}\ \text{eV}\cdot\text{m}^2}{d^2}.$

In state 1, $d = 1.00\times10^{-10}$ m $\quad K_1 = 37.7$ eV.

In state 2, $d = 5.00\times10^{-11}$ m $\quad K_2 = 151$ eV.

In state 3, $d = 3.33\times10^{-11}$ m $\quad K_3 = 339$ eV.

In state 4, $d = 2.50\times10^{-11}$ m $\quad K_4 = 603$ eV.

(b) When the electron falls from state 2 to state 1, it puts out energy

$$E = 151\ \text{eV} - 37.7\ \text{eV} = 113\ \text{eV} = hf = \frac{hc}{\lambda}$$

into emitting a photon of wavelength

$$\lambda = \frac{hc}{E} = \frac{\left(6.626\times10^{-34}\ \text{J}\cdot\text{s}\right)\left(3.00\times10^{8}\ \text{m/s}\right)}{(113\ \text{eV})\left(1.60\times10^{-19}\ \text{J/eV}\right)} = 11.0\ \text{nm}.$$

The wavelengths of the other spectral lines we find similarly:

Transition	$4\to3$	$4\to2$	$4\to1$	$3\to2$	$3\to1$	$2\to1$
E(eV)	264	452	565	188	302	113
λ(nm)	4.71	2.75	2.20	6.60	4.12	11.0

P28.40 The confined proton can be described in the same way as a standing wave on a string. At level 1, the node-to-node distance of the standing wave is 1.00×10^{-14} m, so the wavelength is twice this distance: $\frac{h}{p}=2.00\times10^{-14}$ m.

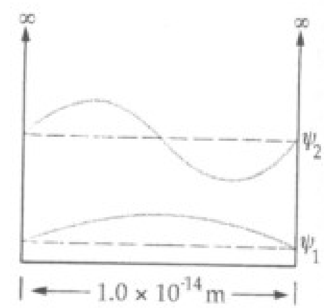

FIG. P28.40

The proton's kinetic energy is

$$K=\frac{1}{2}mv^2=\frac{p^2}{2m}=\frac{h^2}{2m\lambda^2}=\frac{\left(6.626\times10^{-34}\text{ J}\cdot\text{s}\right)^2}{2\left(1.67\times10^{-27}\text{ kg}\right)\left(2.00\times10^{-14}\text{ m}\right)^2}$$

$$=\frac{3.29\times10^{-13}\text{ J}}{1.60\times10^{-19}\text{ J/eV}}=2.05\text{ MeV}$$

In the first excited state, level 2, the node-to-node distance is half as long as in state 1. The momentum is two times larger and the energy is four times larger: $K=8.22$ MeV.

The proton has mass, has charge, moves slowly compared to light in a standing wave state, and stays inside the nucleus. When it falls from level 2 to level 1, its energy change is

$2.05\text{ MeV}-8.22\text{ MeV}=-6.16\text{ MeV}$.

Therefore, we know that a photon (a traveling wave with no mass and no charge) is emitted at the speed of light, and that it has an energy of $\boxed{+6.16\text{ MeV}}$.

Its frequency is $$f=\frac{E}{h}=\frac{\left(6.16\times10^6\text{ eV}\right)\left(1.60\times10^{-19}\text{ J/eV}\right)}{6.626\times10^{-34}\text{ J}\cdot\text{s}}=1.49\times10^{21}\text{ Hz}.$$

And its wavelength is $$\lambda=\frac{c}{f}=\frac{3.00\times10^8\text{ m/s}}{1.49\times10^{21}\text{ s}^{-1}}=\boxed{2.02\times10^{-13}\text{ m}}.$$

This is a gamma ray, according to the electromagnetic spectrum chart in Chapter 24.

***P28.41** (a) The energies of the confined electron are $E_n=\frac{h^2}{8m_eL^2}n^2$. Its energy gain in the quantum jump from state 1 to state 4 is $\frac{h^2}{8m_eL^2}\left(4^2-1^2\right)$ and this is the photon energy: $\frac{h^2 15}{8m_eL^2}=hf=\frac{hc}{\lambda}$. Then $8m_ecL^2=15h\lambda$ and $\boxed{L=\left(\frac{15h\lambda}{8m_ec}\right)^{1/2}}$.

(b) Let λ' represent the wavelength of the photon emitted: $\frac{hc}{\lambda'}=\frac{h^2}{8m_eL^2}4^2-\frac{h^2}{8m_eL^2}2^2=\frac{12h^2}{8m_eL^2}$.

Then $\frac{hc}{\lambda}\frac{\lambda'}{hc}=\frac{h^2 15\left(8m_eL^2\right)}{8m_eL^2 12h^2}=\frac{5}{4}$ and $\boxed{\lambda'=1.25\lambda}$.