P1.60 $0.579t\ \text{ft}^3/\text{s} + 1.19\times10^{-9}t^2\ \text{ft}^3/\text{s}^2$

P1.62 0.541 rad

P1.64 (a) 1 000 kg; (b) 5.24×10^{-16} kg, 0.268 kg, 1.26×10^{-5} kg

P1.66 $2\tan^{-1}\left(\frac{1}{n}\right)$

P1.68 2.29 km

P1.70 (a) zero; (b) zero

P1.72 106°

Chapter 2

Motion in One Dimension

CHAPTER OUTLINE

ANSWERS TO QUESTIONS

Q2.1 If I count 5.0 s between lightning and thunder, the sound has traveled $(331\ \text{m/s})(5.0\ \text{s}) = 1.7\ \text{km}$. The transit time for the light is smaller by

$$\frac{3.00\times10^8\ \text{m/s}}{331\ \text{m/s}} = 9.06\times10^5\ \text{times},$$

so it is negligible in comparison.

Q2.2 Yes. Yes, if the particle winds up in the $+x$ region at the end.

Q2.3 Zero.

Q2.4 Yes. Yes.

Q2.5 No. Consider a sprinter running a straight-line race. His average velocity would simply be the length of the race divided by the time it took for him to complete the race. If he stops along the way to tie his shoe, then his instantaneous velocity at that point would be zero.

Q2.6 We assume the object moves along a straight line. If its average velocity is zero, then the displacement must be zero over the time interval, according to Equation 2.2. The object might be stationary throughout the interval. If it is moving to the right at first, it must later move to the left to return to its starting point. Its velocity must be zero as it turns around. The graph of the motion shown to the right represents such motion, as the initial and final positions are the same. In an x vs. t graph, the instantaneous velocity at any time t is the slope of the curve at that point. At t_0 in the graph, the slope of the curve is zero, and thus the instantaneous velocity at that time is also zero.

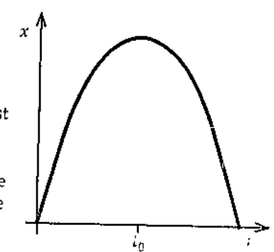


FIG. Q2.6

Q2.7 Yes. If the velocity of the particle is nonzero, the particle is in motion. If the acceleration is zero, the velocity of the particle is unchanging, or is a constant.

Q2.8 Yes. If you drop a doughnut from rest ($v = 0$), then its acceleration is not zero. A common misconception is that immediately after the doughnut is released, both the velocity and acceleration are zero. If the acceleration were zero, then the velocity would not change, leaving the doughnut floating at rest in mid-air.

Q2.9 No: Car A might have greater acceleration than B, but they might both have zero acceleration, or otherwise equal accelerations; or the driver of B might have tramped hard on the gas pedal in the recent past.

Q2.10 Yes. Consider throwing a ball straight up. As the ball goes up, its velocity is upward ($v > 0$), and its acceleration is directed down ($a < 0$). A graph of v vs. t for this situation would look like the figure to the right. The acceleration is the slope of a v vs. t graph, and is always negative in this case, even when the velocity is positive.

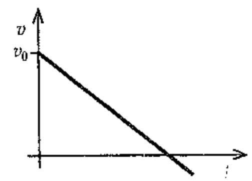


FIG. Q2.10

Q2.11 (a) Speeding up as it moves east (b) Braking East

(c) Cruising East (d) Braking West

(e) Speeding up as it moves west (f) Cruising West

(g) Stopped but starting to move East (h) Stopped but starting to move West

Q2.12 No. Constant acceleration only. Yes. Zero is a constant.

Q2.13 Once the objects leave the hand, both are in free fall, and both experience the same downward acceleration equal to the free-fall acceleration, $-g$.

Q2.14 With $h = \frac{1}{2}gt^2$,

(a) $0.5h = \frac{1}{2}g(0.707t)^2$. The time is later than $0.5t$.

(b) The distance fallen is $0.25h = \frac{1}{2}g(0.5t)^2$. The elevation is $0.75h$, greater than $0.5h$.

Q2.15 They are the same. After the first ball reaches its apex and falls back downward past the student, it will have a downward velocity equal to v_i. This velocity is the same as the velocity of the second ball, so after they fall through equal heights their impact speeds will also be the same.

Q2.16 Above. Your ball has zero initial speed and smaller average speed during the time of flight to the passing point.

SOLUTIONS TO PROBLEMS

Section 2.1 Average Velocity

P2.1 (a) $\bar{v} = \boxed{2.30\ \text{m/s}}$

(b) $\bar{v} = \frac{\Delta x}{\Delta t} = \frac{57.5\ \text{m} - 9.20\ \text{m}}{3.00\ \text{s}} = \boxed{16.1\ \text{m/s}}$

(c) $\bar{v} = \frac{\Delta x}{\Delta t} = \frac{57.5\ \text{m} - 0\ \text{m}}{5.00\ \text{s}} = \boxed{11.5\ \text{m/s}}$

P2.2 $x = 10t^2$: For $\begin{cases} t(\text{s}) & = & 2.0 & 2.1 & 3.0 \\ x(\text{m}) & = & 40 & 44.1 & 90 \end{cases}$

(a) $\bar{v} = \frac{\Delta x}{\Delta t} = \frac{50\ \text{m}}{1.0\ \text{s}} = \boxed{50.0\ \text{m/s}}$

(b) $\bar{v} = \frac{\Delta x}{\Delta t} = \frac{4.1\ \text{m}}{0.1\ \text{s}} = \boxed{41.0\ \text{m/s}}$

P2.3 (a) $\bar{v} = \frac{\Delta x}{\Delta t} = \frac{10\ \text{m}}{2\ \text{s}} = \boxed{5\ \text{m/s}}$

(b) $\bar{v} = \frac{5\ \text{m}}{4\ \text{s}} = \boxed{1.2\ \text{m/s}}$

(c) $\bar{v} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{5\ \text{m} - 10\ \text{m}}{4\ \text{s} - 2\ \text{s}} = \boxed{-2.5\ \text{m/s}}$

(d) $\bar{v} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{-5\ \text{m} - 5\ \text{m}}{7\ \text{s} - 4\ \text{s}} = \boxed{-3.3\ \text{m/s}}$

(e) $\bar{v} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{0 - 0}{8 - 0} = \boxed{0\ \text{m/s}}$

*P2.4 (a) Let d represent the distance between A and B. Let t_1 be the time for which the walker has the higher speed in $5.00\ \text{m/s} = \frac{d}{t_1}$. Let t_2 represent the longer time for the return trip in $-3.00\ \text{m/s} = -\frac{d}{t_2}$. Then the times are $t_1 = \frac{d}{(5.00\ \text{m/s})}$ and $t_2 = \frac{d}{(3.00\ \text{m/s})}$. The average speed is:

continued on next page

$$\bar{v} = \frac{\text{Total distance}}{\text{Total time}} = \frac{d+d}{\frac{d}{(5.00 \text{ m/s})} + \frac{d}{(3.00 \text{ m/s})}} = \frac{2d}{\frac{(8.00 \text{ m/s})d}{(15.0 \text{ m}^2/\text{s}^2)}}$$

$$\bar{v} = \frac{2(15.0 \text{ m}^2/\text{s}^2)}{8.00 \text{ m/s}} = \boxed{3.75 \text{ m/s}}$$

(b) She starts and finishes at the same point A. With total displacement = 0, average velocity $= \boxed{0}$.

Section 2.2 Instantaneous Velocity

P2.5 (a) at $t_i = 1.5$ s, $x_i = 8.0$ m (Point A)

at $t_f = 4.0$ s, $x_f = 2.0$ m (Point B)

$$\bar{\mathbf{v}} = \frac{x_f - x_i}{t_f - t_i} = \frac{(2.0 - 8.0) \text{ m}}{(4 - 1.5) \text{ s}} = -\frac{6.0 \text{ m}}{2.5 \text{ s}} = \boxed{-2.4 \text{ m/s}}$$

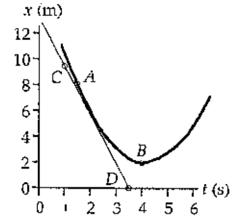


FIG. P2.5

(b) The slope of the tangent line can be found from Points C and D. ($t_C = 1.0$ s, $x_C = 9.5$ m) and ($t_D = 3.5$ s, $x_D = 0$),

$$v \approx \boxed{-3.8 \text{ m/s}}.$$

(c) The velocity is zero when x is a minimum. This is at $t \approx \boxed{4 \text{ s}}$.

P2.6 (a) At any time, t, the position is given by $x = (3.00 \text{ m/s}^2)t^2$

Thus, at $t_i = 3.00$ s: $x_i = (3.00 \text{ m/s}^2)(3.00 \text{ s})^2 = \boxed{27.0 \text{ m}}$

(b) At $t_f = 3.00 \text{ s} + \Delta t$: $x_f = (3.00 \text{ m/s}^2)(3.00 \text{ s} + \Delta t)^2$, or

$$x_f = \boxed{27.0 \text{ m} + (18.0 \text{ m/s})\Delta t + (3.00 \text{ m/s}^2)(\Delta t)^2}.$$

(c) The instantaneous velocity at $t = 3.00$ s is:

$$v = \lim_{\Delta t \to 0}\left(\frac{x_f - x_i}{\Delta t}\right) = \lim_{\Delta t \to 0}\left(18.0 \text{ m/s} + (3.00 \text{ m/s}^2)\Delta t\right) = \boxed{18.0 \text{ m/s}}.$$

P2.7 (a)

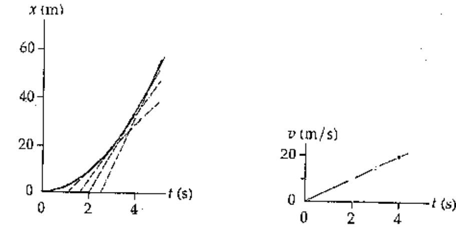


FIG. P2.7(a)

(b) At $t = 5.0$ s, the slope is $v \approx \frac{58 \text{ m}}{2.5 \text{ s}} \approx \boxed{23 \text{ m/s}}$.

At $t = 4.0$ s, the slope is $v \approx \frac{54 \text{ m}}{3 \text{ s}} \approx \boxed{18 \text{ m/s}}$.

At $t = 3.0$ s, the slope is $v \approx \frac{49 \text{ m}}{3.4 \text{ s}} \approx \boxed{14 \text{ m/s}}$.

At $t = 2.0$ s, the slope is $v \approx \frac{36 \text{ m}}{4.0 \text{ s}} \approx \boxed{9.0 \text{ m/s}}$

(c) $\bar{a} = \frac{\Delta v}{\Delta t} \approx \frac{23 \text{ m/s}}{5.0 \text{ s}} \approx \boxed{4.6 \text{ m/s}^2}$

(d) Initial velocity of the car was $\boxed{\text{zero}}$.

P2.8 (a) $v = \frac{(5-0) \text{ m}}{(1-0) \text{ s}} = \boxed{5 \text{ m/s}}$

(b) $v = \frac{(5-10) \text{ m}}{(4-2) \text{ s}} = \boxed{-2.5 \text{ m/s}}$

(c) $v = \frac{(5 \text{ m} - 5 \text{ m})}{(5 \text{ s} - 4 \text{ s})} = \boxed{0}$

(d) $v = \frac{0 - (-5 \text{ m})}{(8 \text{ s} - 7 \text{ s})} = \boxed{+5 \text{ m/s}}$

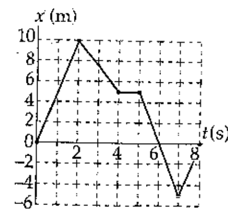


FIG. P2.8

Section 2.3 Analysis Models—The Particle Under Constant Velocity

P2.9 Once it resumes the race, the hare will run for a time of

$$t=\frac{x_f-x_i}{v_x}=\frac{1\,000\text{ m}-800\text{ m}}{8\text{ m/s}}=25\text{ s}.$$

In this time, the tortoise can crawl a distance

$$x_f-x_i=(0.2\text{ m/s})(25\text{ s})=\boxed{5.00\text{ m}}.$$

Section 2.4 Acceleration

P2.10 Choose the positive direction to be the outward direction, perpendicular to the wall.

$$v_f=v_i+at:\ a=\frac{\Delta v}{\Delta t}=\frac{22.0\text{ m/s}-(-25.0\text{ m/s})}{3.50\times10^{-3}\text{ s}}=\boxed{1.34\times10^4\text{ m/s}^2}.$$

P2.11 (a) Acceleration is constant over the first ten seconds, so at the end.

$$v_f=v_i+at=0+(2.00\text{ m/s}^2)(10.0\text{ s})=\boxed{20.0\text{ m/s}}.$$

Then $a=0$ so v is constant from $t=10.0$ s to $t=15.0$ s. And over the last five seconds the velocity changes to

$$v_f=v_i+at=20.0\text{ m/s}+(-3.00\text{ m/s}^2)(5.00\text{ s})=\boxed{5.00\text{ m/s}}$$

(b) In the first ten seconds,

$$x_f=x_i+v_it+\frac{1}{2}at^2=0+0+\frac{1}{2}(2.00\text{ m/s}^2)(10.0\text{ s})^2=100\text{ m}.$$

Over the next five seconds the position changes to

$$x_f=x_i+v_it+\frac{1}{2}at^2=100\text{ m}+(20.0\text{ m/s})(5.00\text{ s})+0=200\text{ m}.$$

And at $t=20.0$ s,

$$x_f=x_i+v_it+\frac{1}{2}at^2=200\text{ m}+(20.0\text{ m/s})(5.00\text{ s})+\frac{1}{2}(-3.00\text{ m/s}^2)(5.00\text{ s})^2=\boxed{262\text{ m}}.$$

P2.12 (a) At $t=2.00$ s, $x=\left[3.00(2.00)^2-2.00(2.00)+3.00\right]\text{ m}=11.0\text{ m}.$

At $t=3.00$ s, $x=\left[3.00(9.00)^2-2.00(3.00)+3.00\right]\text{ m}=24.0\text{ m}$

so

$$\overline{v}=\frac{\Delta x}{\Delta t}=\frac{24.0\text{ m}-11.0\text{ m}}{3.00\text{ s}-2.00\text{ s}}=\boxed{13.0\text{ m/s}}.$$

(b) At all times the instantaneous velocity is

$$v=\frac{d}{dt}(3.00t^2-2.00t+3.00)=(6.00t-2.00)\text{ m/s}$$

At $t=2.00$ s, $v=\left[6.00(2.00)-2.00\right]\text{ m/s}=\boxed{10.0\text{ m/s}}$

At $t=3.00$ s, $v=\left[6.00(3.00)-2.00\right]\text{ m/s}=\boxed{16.0\text{ m/s}}$.

(c) $$\overline{a}=\frac{\Delta v}{\Delta t}=\frac{16.0\text{ m/s}-10.0\text{ m/s}}{3.00\text{ s}-2.00\text{ s}}=\boxed{6.00\text{ m/s}^2}$$

(d) At all times $a=\frac{d}{dt}(6.00t-2.00)=\boxed{6.00\text{ m/s}^2}$. (This includes both $t=2.00$ s and $t=3.00$ s).

P2.13 $x=2.00+3.00t-t^2$, $\quad v=\frac{dx}{dt}=3.00-2.00t$, $\quad a=\frac{dv}{dt}=-2.00$

At $t=3.00$ s:

(a) $x=(2.00+9.00-9.00)\text{ m}=\boxed{2.00\text{ m}}$

(b) $v=(3.00-6.00)\text{ m/s}=\boxed{-3.00\text{ m/s}}$

(c) $a=\boxed{-2.00\text{ m/s}^2}$

P2.14 (a) See the graphs at the right.

Choose $x=0$ at $t=0$.

At $t=3$ s, $x=\frac{1}{2}(8\text{ m/s})(3\text{ s})=12$ m.

At $t=5$ s, $x=12\text{ m}+(8\text{ m/s})(2\text{ s})=28$ m.

At $t=7$ s, $x=28\text{ m}+\frac{1}{2}(8\text{ m/s})(2\text{ s})=36$ m.

(b) For $0<t<3$ s, $a=\frac{8\text{ m/s}}{3\text{ s}}=2.67\text{ m/s}^2$

For $3<t<5$ s, $a=0$.

(c) For $5\text{ s}<t<9$ s, $a=-\frac{16\text{ m/s}}{4\text{ s}}=\boxed{-4\text{ m/s}^2}$

(d) At $t=6$ s, $x=28\text{ m}+(6\text{ m/s})(1\text{ s})=\boxed{34\text{ m}}$.

(e) At $t=9$ s, $x=36\text{ m}+\frac{1}{2}(-8\text{ m/s})(2\text{ s})=\boxed{28\text{ m}}$.

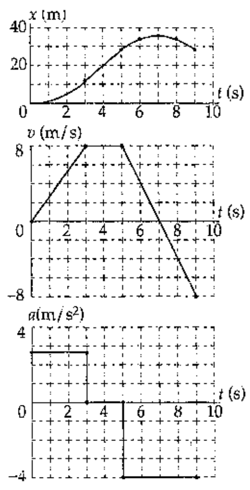


FIG. P2.14

P2.15 (a) $a=\frac{\Delta v}{\Delta t}=\frac{8.00\text{ m/s}}{6.00\text{ s}}=\boxed{1.3\text{ m/s}^2}$

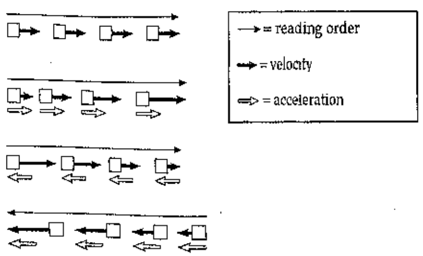
(b) Maximum positive acceleration is at $t=3$ s, and is approximately $\boxed{2\text{ m/s}^2}$.

(c) $a=0$, at $\boxed{t=6\text{ s}}$, and also for $\boxed{t>10\text{ s}}$.

(d) Maximum negative acceleration is at $t=8$ s, and is approximately $\boxed{-1.5\text{ m/s}^2}$.

Section 2.5 Motion Diagrams

P2.16 (a) (b) (c) (d)



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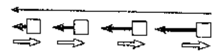
(e)



(f) One way of phrasing the answer: The spacing of the successive positions would change with less regularity.

Another way: The object would move with some combination of the kinds of motion shown in (a) through (e). Within one drawing, the accelerations vectors would vary in magnitude and direction.

Section 2.6 The Particle Under Constant Acceleration

***P2.17** (a) $x_f-x_i=\frac{1}{2}(v_i+v_f)t$ becomes $40\text{ m}=\frac{1}{2}(v_i+2.80\text{ m/s})(8.50\text{ s})$ which yields $v_i=\boxed{6.61\text{ m/s}}$.

(b) $a=\frac{v_f-v_i}{t}=\frac{2.80\text{ m/s}-6.61\text{ m/s}}{8.50\text{ s}}=\boxed{-0.448\text{ m/s}^2}$

P2.18 Method One

Suppose the unknown acceleration is constant as a car moving at $v_{i1}=35.0$ mi/h comes to a $v_f=0$ stop in $x_{f1}-x_i=40.0$ ft. We find its acceleration from $v_{f1}{}^2=v_{i1}{}^2+2a(x_{f1}-x_i)$:

$$a=\frac{v_{f1}{}^2-v_i{}^2}{2(x_{f1}-x_i)}=\frac{0-(35.0\text{ mi/h})^2}{2(40.0\text{ ft})}\left(\frac{5280\text{ ft}}{\text{mi}}\right)^2\left(\frac{1\text{ h}}{3600\text{ s}}\right)^2=-32.9\text{ ft/s}^2$$

Now consider a car moving at $v_{i2}=70.0$ mi/h and stopping to $v_f=0$ with $a=-32.9\text{ ft/s}^2$. From the same equation its stopping distance is

$$x_{f2}-x_i=\frac{v_{f2}{}^2-v_i{}^2}{2a}=\frac{0-(70.0\text{ mi/h})^2}{2(-32.9\text{ ft/s}^2)}\left(\frac{5280\text{ ft}}{1\text{ mi}}\right)^2\left(\frac{1\text{ h}}{3600\text{ s}}\right)^2=\boxed{160\text{ ft}}$$

Method Two

For the process of stopping from the lower speed v_{i1} we have $v_f^2=v_{i1}^2+2a(x_{f1}-x_i)$; $0=v_{i1}^2+2ax_{f1}$, $v_{i1}^2=-2ax_{f1}$. For stopping from $v_{i2}=2v_{i1}$, similarly $0=v_{i2}^2+2a(x_{f2}-0)$; $v_{i2}^2=-2ax_{f2}$. Dividing gives $\frac{v_{i2}^2}{v_{i1}^2}=\frac{x_{f2}}{x_{i1}}$; $x_{f2}=40\text{ ft}\times 2^2=\boxed{160\text{ ft}}$.

P2.19 Given $v_i=12.0$ cm/s when $x_i=3.00$ cm$(t=0)$, and at $t=2.00$ s, $x_f=-5.00$ cm,

$$x_f-x_i=v_it+\frac{1}{2}at^2\quad -5.00-3.00=12.0(2.00)+\frac{1}{2}a(2.00)^2$$

$$-8.00=24.0+2a\qquad a=-\frac{32.0}{2}=\boxed{-16.0\text{ cm/s}^2}.$$

P2.20 (a) Choose the initial point where the pilot reduces the throttle and the final point where the boat passes the buoy:

$$x_i = 0,\ x_f = 100\text{ m},\ v_{xi} = 30\text{ m/s},\ v_{xf} = ?,\ a_x = -3.5\text{ m/s}^2,\ t = ?$$

$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$:

$$100\text{ m} = 0 + (30\text{ m/s})t + \frac{1}{2}(-3.5\text{ m/s}^2)t^2$$

$$(1.75\text{ m/s}^2)t^2 - (30\text{ m/s})t + 100\text{ m} = 0.$$

We use the quadratic formula:

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$t = \frac{30\text{ m/s} \pm \sqrt{900\text{ m}^2/\text{s}^2 - 4(1.75\text{ m/s}^2)(100\text{ m})}}{2(1.75\text{ m/s}^2)} = \frac{30\text{ m/s} \pm 14.1\text{ m/s}}{3.5\text{ m/s}^2} = 12.6\text{ s or } \boxed{4.53\text{ s}}.$$

The smaller value is the physical answer. If the boat kept moving with the same acceleration, it would stop and move backward, then gain speed, and pass the buoy again at 12.6 s.

(b) $v_{xf} = v_{xi} + a_x t = 30\text{ m/s} - (3.5\text{ m/s}^2)4.53\text{ s} = \boxed{14.1\text{ m/s}}$

P2.21 (a) $v_i = 100\text{ m/s}$, $a = -5.00\text{ m/s}^2$, $v_f = v_i + at$ so $0 = 100 - 5t$, $v_f^2 = v_i^2 + 2a(x_f - x_i)$ so $0 = (100)^2 - 2(5.00)(x_f - 0)$. Thus $x_f = 1\,000\text{ m}$ and $t = \boxed{20.0\text{ s}}$.

(b) At this acceleration the plane would overshoot the runway: $\boxed{\text{No}}$.

P2.22 (a) Compare the position equation $x = 2.00 + 3.00t - 4.00t^2$ to the general form

$$x_f = x_i + v_i t + \frac{1}{2}at^2$$

to recognize that $x_i = 2.00\text{ m}$, $v_i = 3.00\text{ m/s}$, and $a = -8.00\text{ m/s}^2$. The velocity equation, $v_f = v_i + at$, is then

$$v_f = 3.00\text{ m/s} - (8.00\text{ m/s}^2)t.$$

The particle changes direction when $v_f = 0$, which occurs at $t = \frac{3}{8}$ s. The position at this time is:

$$x = 2.00\text{ m} + (3.00\text{ m/s})\left(\frac{3}{8}\text{ s}\right) - (4.00\text{ m/s}^2)\left(\frac{3}{8}\text{ s}\right)^2 = \boxed{2.56\text{ m}}$$

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(b) From $x_f = x_i + v_i t + \frac{1}{2}at^2$, observe that when $x_f = x_i$, the time is given by $t = -\frac{2v_i}{a}$. Thus, when the particle returns to its initial position, the time is

$$t = \frac{-2(3.00\text{ m/s})}{-8.00\text{ m/s}^2} = \frac{3}{4}\text{ s}$$

and the velocity is $v_f = 3.00\text{ m/s} - (8.00\text{ m/s}^2)\left(\frac{3}{4}\text{ s}\right) = \boxed{-3.00\text{ m/s}}$.

P2.23 In the simultaneous equations:

$$\begin{Bmatrix} v_{xf} = v_{xi} + a_x t \\ x_f - x_i = \frac{1}{2}(v_{xi} + v_{xf})t \end{Bmatrix} \text{ we have } \begin{Bmatrix} v_{xf} = v_{xi} - (5.60\text{ m/s}^2)(4.20\text{ s}) \\ 62.4\text{ m} = \frac{1}{2}(v_{xi} + v_{xf})(4.20\text{ s}) \end{Bmatrix}.$$

So substituting for v_{xi} gives $62.4\text{ m} = \frac{1}{2}\left[v_{xf} + (5.60\text{ m/s}^2)(4.20\text{ s}) + v_{xf}\right](4.20\text{ s})$

$$14.9\text{ m/s} = v_{xf} + \frac{1}{2}(5.60\text{ m/s}^2)(4.20\text{ s}).$$

Thus

$$v_{xf} = \boxed{3.10\text{ m/s}}.$$

P2.24 Take any two of the standard four equations, such as $\begin{Bmatrix} v_{xf} = v_{xi} + a_x t \\ x_f - x_i = \frac{1}{2}(v_{xi} + v_{xf})t \end{Bmatrix}$. Solve one for v_{xi}, and substitute into the other: $v_{xi} = v_{xf} - a_x t$

$$x_f - x_i = \frac{1}{2}(v_{xf} - a_x t + v_{xf})t.$$

Thus

$$\boxed{x_f - x_i = v_{xf}t - \frac{1}{2}a_x t^2}.$$

We note that the equation is dimensionally correct. The units are units of length in each term. Like the standard equation $x_f - x_i = v_{xi}t + \frac{1}{2}a_x t^2$, this equation represents that displacement is a quadratic function of time. Back in problem 23, $62.4\text{ m} = v_{xf}(4.20\text{ s}) - \frac{1}{2}(-5.60\text{ m/s}^2)(4.20\text{ s})^2$

$$v_{xf} = \frac{62.4\text{ m} - 49.4\text{ m}}{4.20\text{ s}} = \boxed{3.10\text{ m/s}}$$

P2.25 (a) The time it takes the truck to reach 20.0 m/s is found from $v_f = v_i + at$. Solving for t yields

$$t = \frac{v_f - v_i}{a} = \frac{20.0\ \text{m/s} - 0\ \text{m/s}}{2.00\ \text{m/s}^2} = 10.0\ \text{s}.$$

The total time is thus

$$10.0\ \text{s} + 20.0\ \text{s} + 5.00\ \text{s} = \boxed{35.0\ \text{s}}$$

(b) The average velocity is the total distance traveled divided by the total time taken. The distance traveled during the first 10.0 s is

$$x_1 = \bar{v}t = \left(\frac{0 + 20.0}{2}\right)(10.0) = 100\ \text{m}.$$

With a being 0 for this interval, the distance traveled during the next 20.0 s is

$$x_2 = v_i t + \frac{1}{2}at^2 = (20.0)(20.0) + 0 = 400\ \text{m}.$$

The distance traveled in the last 5.00 s is

$$x_3 = \bar{v}t = \left(\frac{20.0 + 0}{2}\right)(5.00) = 50.0\ \text{m}.$$

The total distance $x = x_1 + x_2 + x_3 = 100 + 400 + 50 = 550\ \text{m}$, and the average velocity is given by $\bar{v} = \frac{x}{t} = \frac{550}{35.0} = \boxed{15.7\ \text{m/s}}$.

***P2.26** We have $v_i = 2.00 \times 10^4\ \text{m/s}$, $v_f = 6.00 \times 10^6\ \text{m/s}$, $x_f - x_i = 1.50 \times 10^{-4}\ \text{m}$.

(a) $x_f - x_i = \frac{1}{2}(v_i + v_f)t$: $t = \frac{2(x_f - x_i)}{v_i + v_f} = \frac{2(1.50 \times 10^{-4}\ \text{m})}{2.00 \times 10^4\ \text{m/s} + 6.00 \times 10^6\ \text{m/s}} = \boxed{4.98 \times 10^{-9}\ \text{s}}$

(b) $v_f^2 = v_i^2 + 2a_x(x_f - x_i)$:

$$a_x = \frac{v_f^2 - v_i^2}{2(x_f - x_i)} = \frac{(6.00 \times 10^6\ \text{m/s})^2 - (2.00 \times 10^4\ \text{m/s})^2}{2(1.50 \times 10^{-2}\ \text{m})} = \boxed{1.20 \times 10^{15}\ \text{m/s}^2}$$

P2.27 Take the original point to be when Sue notices the van. Choose the origin of the x-axis at Sue's car. For her we have $x_{is} = 0$, $v_{is} = 30.0\ \text{m/s}$, $a_s = -2.00\ \text{m/s}^2$ so her position is given by

$$x_s(t) = x_{is} + v_{is}t + \frac{1}{2}a_s t^2 = (30.0\ \text{m/s})t + \frac{1}{2}(-2.00\ \text{m/s}^2)t^2$$

For the van, $x_{iv} = 155\ \text{m}$, $v_{iv} = 5.00\ \text{m/s}$, $a_v = 0$ and

$$x_v(t) = x_{iv} + v_{iv}t + \frac{1}{2}a_v t^2 = 155 + (5.00\ \text{m/s})t + 0.$$

To test for a collision, we look for an instant t_c when both are at the same place:

$$30.0t_c - t_c^2 = 155 + 5.00t_c$$
$$0 = t_c^2 - 25.0t_c + 155.$$

From the quadratic formula

$$t_c = \frac{25.0 \pm \sqrt{(25.0)^2 - 4(155)}}{2} = 13.6\ \text{s or } \boxed{11.4\ \text{s}}.$$

The smaller value is the collision time. (The larger value tells when the van would pull ahead again if the vehicles could move through each other). The wreck happens at position

$$155\ \text{m} + (5.00\ \text{m/s})(11.4\ \text{s}) = \boxed{212\ \text{m}}.$$

Section 2.7 Freely Falling Objects

***P2.28** Choose the origin ($y = 0$, $t = 0$) at the starting point of the cat and take upward as positive. Then $y_i = 0$, $v_i = 0$, and $a = -g = -9.80\ \text{m/s}^2$. The position and the velocity at time t become:

$$y_f - y_i = v_i t + \frac{1}{2}at^2: \ y_f = -\frac{1}{2}gt^2 = -\frac{1}{2}(9.80\ \text{m/s}^2)t^2$$

and

$$v_f = v_i + at: \ v_f = -gt = -(9.80\ \text{m/s}^2)t.$$

(a) at $t = 0.1$ s: $y_f = -\frac{1}{2}(9.80\ \text{m/s}^2)(0.1\ \text{s})^2 = \boxed{-0.0490\ \text{m}}$

(b) at $t = 0.2$ s: $y_f = -\frac{1}{2}(9.80\ \text{m/s}^2)(0.2\ \text{s})^2 = \boxed{-0.196\ \text{m}}$

(c) at $t = 0.3$ s: $y_f = -\frac{1}{2}(9.80\ \text{m/s}^2)(0.3\ \text{s})^2 = \boxed{-0.441\ \text{m}}$

continued on next page

(a) at $t = 0.1$ s: $v_f = -(9.80\ \text{m/s}^2)(0.1\ \text{s}) = \boxed{-0.980\ \text{m/s}}$

(b) at $t = 0.2$ s: $v_f = -(9.80\ \text{m/s}^2)(0.2\ \text{s}) = \boxed{-1.96\ \text{m/s}}$

(c) at $t = 0.3$ s: $v_f = -(9.80\ \text{m/s}^2)(0.3\ \text{s}) = \boxed{-2.94\ \text{m/s}}$

P2.29 (a) $v_f = v_i - gt$: $v_f = 0$ when $t = 3.00$ s, $g = 9.80\ \text{m/s}^2$. Therefore,

$$v_i = gt = (9.80\ \text{m/s}^2)(3.00\ \text{s}) = \boxed{29.4\ \text{m/s}}.$$

(b) $y_f - y_i = \frac{1}{2}(v_f + v_i)t$

$$y_f - y_i = \frac{1}{2}(29.4\ \text{m/s})(3.00\ \text{s}) = \boxed{44.1\ \text{m}}$$

P2.30 Assume that air resistance may be neglected. Then, the acceleration at all times during the flight is that due to gravity, $a = -g = -9.80\ \text{m/s}^2$. During the flight, Goff went 1 mile (1 609 m) up and then 1 mile back down. Determine his speed just after launch by considering his upward flight:

$$v_f^2 = v_i^2 + 2a(y_f - y_i):\quad 0 = v_i^2 - 2(9.80\ \text{m/s}^2)(1\,609\ \text{m})$$
$$v_i = 178\ \text{m/s}.$$

His time in the air may be found by considering his motion from just after launch to just before impact:

$$y_f - y_i = v_i t + \frac{1}{2}at^2:\ 0 = (178\ \text{m/s})t - \frac{1}{2}(-9.80\ \text{m/s}^2)t^2$$

The root $t = 0$ describes launch; the other root, $t = 36.2$ s, describes his flight time. His rate of pay may then be found from

$$\text{pay rate} = \frac{\$1.00}{36.2\ \text{s}} = (0.027\,6\ \$/\text{s})(3\,600\ \text{s/h}) = \boxed{\$99.3/\text{h}}.$$

We have assumed that the workman's flight time, "a mile", and "a dollar", were measured to three-digit precision. We have interpreted "up in the sky" as referring to the free fall time, not to the launch and landing times. Both the takeoff and landing times must be several seconds away from the job, in order for Goff to survive to resume work.

P2.31 (a) $y_f - y_i = v_i t + \frac{1}{2}at^2$: $4.00 = (1.50)v_i - (4.90)(1.50)^2$ and $v_i = \boxed{10.0\ \text{m/s upward}}$.

(b) $v_f = v_i + at = 10.0 - (9.80)(1.50) = -4.68\ \text{m/s}$

$$v_f = \boxed{4.68\ \text{m/s downward}}$$

P2.32 We have $y_f = -\frac{1}{2}gt^2 + v_i t + y_i$

$$0 = -(4.90\ \text{m/s}^2)t^2 - (8.00\ \text{m/s})t + 30.0\ \text{m}.$$

Solving for t,

$$t = \frac{8.00 \pm \sqrt{64.0 + 588}}{-9.80}$$

Using only the positive value for t, we find that $t = \boxed{1.79\ \text{s}}$.

P2.33 Time to fall 3.00 m is found from Eq. 2.12 with $v_i = 0$, $3.00\ \text{m} = \frac{1}{2}(9.80\ \text{m/s}^2)t^2$, $t = 0.782$ s.

(a) With the horse galloping at 10.0 m/s, the horizontal distance is $vt = \boxed{7.82\ \text{m}}$.

(b) $t = \boxed{0.782\ \text{s}}$

***P2.34** (a) Consider the upward flight of the arrow.

$$v_{yf}^2 = v_{yi}^2 + 2a_y(y_f - y_i)$$
$$0 = (100\ \text{m/s})^2 + 2(-9.8\ \text{m/s}^2)\Delta y$$
$$\Delta y = \frac{10\,000\ \text{m}^2/\text{s}^2}{19.6\ \text{m/s}^2} = \boxed{510\ \text{m}}$$

(b) Consider the whole flight of the arrow.

$$y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2$$
$$0 = 0 + (100\ \text{m/s})t + \frac{1}{2}(-9.8\ \text{m/s}^2)t^2$$

The root $t = 0$ refers to the starting point. The time of flight is given by

$$t = \frac{100\ \text{m/s}}{4.9\ \text{m/s}^2} = \boxed{20.4\ \text{s}}$$

Section 2.8 Context Connection—Acceleration Required by Consumers

*P2.35 (a) The Lamborghini requires the shortest time to speed up from 0 to 60 mi/h, so it has the greatest average acceleration:

$$v_{xf} = v_{xi} + a_x t$$

$$a_x = \frac{v_{xf} - v_{xi}}{t} = \frac{60\ \text{mi/h} - 0}{3.6\ \text{s}} = 16.7\ \text{mi/h}\cdot\text{s},\ \text{in agreement}$$

with the value listed. For the Toyota Prius,

$$a_x = \frac{v_{xf} - v_{xi}}{t} = \frac{60\ \text{mi/h} - 0}{12.7\ \text{s}} = 4.72\ \text{mi/h}\cdot\text{s},\ \text{in agreement}$$

with 4.7 mi/h·s listed.

(b) Lamborghini: $\dfrac{60\ \text{mi/h}}{3.6\ \text{s}}\left(\dfrac{1\,609\ \text{m}}{1\ \text{mi}}\right)\left(\dfrac{1\ \text{h}}{3\,600\ \text{s}}\right) = \boxed{7.4\ \text{m/s}^2}$ to two significant digits.

Prius: $\dfrac{60\ \text{mi/h}}{12.7\ \text{s}}\left(\dfrac{1\,609\ \text{m}}{1\ \text{mi}}\right)\left(\dfrac{1\ \text{h}}{3\,600\ \text{s}}\right) = \boxed{2.1\ \text{m/s}^2}$

(c) $x_f - x_i = \frac{1}{2}(v_{xi} + v_{xf})t$

Lamborghini: $x_f - x_i = \frac{1}{2}(0 + 60\ \text{mi/h})3.6\ \text{s}\left(\dfrac{1\,609\ \text{m}}{1\ \text{mi}}\right)\left(\dfrac{1\ \text{h}}{3\,600\ \text{s}}\right) = \boxed{48\ \text{m}}$

Prius: $x_f - x_i = \frac{1}{2}(60\ \text{mi/h})12.7\ \text{s}\left(\dfrac{1\,609\ \text{m}}{1\ \text{mi}}\right)\left(\dfrac{1\ \text{h}}{3\,600\ \text{s}}\right) = \boxed{170\ \text{m}}$

(d) $v_{xf} = v_{xi} + a_x t$

$$t = \frac{v_{xf} - v_{xi}}{a_x} = \frac{60\ \text{mi/h} - 0}{9.80\ \text{m/s}^2}\left(\frac{1\,609\ \text{mi}}{1\ \text{mi}}\right)\left(\frac{1\ \text{h}}{3\,600\ \text{s}}\right) = \boxed{2.74\ \text{s}}$$

Building a car capable of such a large acceleration would be difficult. So would steering it as it is speeding up.

P2.36 (a) Assuming a constant acceleration: $a = \dfrac{v_f - v_i}{t} = \dfrac{42.0\ \text{m/s}}{8.00\ \text{s}} = \boxed{5.25\ \text{m/s}^2}$

(b) Taking the origin at the original position of the car.

$$x_f = \frac{1}{2}(v_i + v_f)t = \frac{1}{2}(42.0\ \text{m/s})(8.00\ \text{s}) = \boxed{168\ \text{m}}$$

(c) From $v_f = v_i + at$, the velocity 10.0 s after the car starts from rest is:

$$v_f = 0 + (5.25\ \text{m/s}^2)(10.0\ \text{s}) = \boxed{52.5\ \text{m/s}}$$

*P2.37 (a) $v_{xf} = v_{xi} + a_x t$

$$a_x = \frac{v_{xf} - v_{xi}}{t} = \frac{175\ \text{mi/h} - 0}{2.50\ \text{s}} = 70.0\ \text{mi/h}\cdot\text{s} = 70.0\ \text{mi/h}\cdot\text{s}\left(\frac{1\,609\ \text{mi}}{1\ \text{m}}\right)\left(\frac{1\ \text{h}}{3\,600\ \text{s}}\right)$$

$$= \boxed{31.3\ \text{m/s}^2} = 31.3\ \text{m/s}^2\left(\frac{g}{9.8\ \text{m/s}^2}\right) = 3.19g$$

(b) $x_f - x_i = \frac{1}{2}(v_{xi} + v_{xf})t = \frac{1}{2}(0 + 175\ \text{mi/h})2.5\ \text{s}\left(\dfrac{1\,609\ \text{mi}}{1\ \text{mi}}\right)\left(\dfrac{1\ \text{h}}{3\,600\ \text{s}}\right) = \boxed{97.8\ \text{m}}$

*P2.38 (a) Starting from rest and accelerating at $a_b = 13.0\ \text{mi/h}\cdot\text{s}$, the bicycle reaches its maximum speed of $v_{b,\max} = 20.0\ \text{mi/h}$ in a time

$$t_{b,1} = \frac{v_{b,\max} - 0}{a_b} = \frac{20.0\ \text{mi/h}}{13.0\ \text{mi/h}\cdot\text{s}} = 1.54\ \text{s}$$

Since the acceleration a_c of the car is less than that of the bicycle, the car cannot catch the bicycle until some time $t > t_{b,1}$ (that is, until the bicycle is at its maximum speed and coasting). The total displacement of the bicycle at time t is

$$\Delta x_b = \frac{1}{2}a_b t_{b,1}^2 + v_{b,\max}(t - t_{b,1})$$

$$= \left(\frac{1.47\ \text{ft/s}}{1\ \text{mi/h}}\right)\left[\frac{1}{2}\left(13.0\ \frac{\text{mi/h}}{\text{s}}\right)(1.54\ \text{s})^2 + (20.0\ \text{mi/h})(t - 1.54\ \text{s})\right]$$

$$= (29.4\ \text{ft/s})t - 22.6\ \text{ft}$$

The total displacement of the car at this time is

$$\Delta x_c = \frac{1}{2}a_c t^2 = \left(\frac{1.47\ \text{ft/s}}{1\ \text{mi/h}}\right)\left[\frac{1}{2}\left(9.00\ \frac{\text{mi/h}}{\text{s}}\right)t^2\right] = (6.62\ \text{ft/s})t^2$$

At the time the car catches the bicycle $\Delta x_c = \Delta x_b$. This gives

$$(6.62\ \text{ft/s}^2)t^2 = (29.4\ \text{ft/s})t - 22.6\ \text{ft} \qquad \text{or} \qquad t^2 - (4.44\ \text{s})t + 3.42\ \text{s}^2 = 0$$

that has only one acceptable solution $t > t_{b,1}$. This solution gives the total time the bicycle leads the car and is $t = \boxed{3.45\ \text{s}}$.

(b) The lead the bicycle has over the car continues to increase as long as the bicycle is moving faster than the car. This means until the car attains a speed of $v_c = v_{b,\max} = 20.0\ \text{mi/h}$. Thus, the elapsed time when the bicycle's lead ceases to increase is

$$t = \frac{v_{b,\max}}{a_c} = \frac{20.0\ \text{mi/h}}{9.00\ \text{mi/h}\cdot\text{s}} = 2.22\ \text{s}$$

At this time, the lead is

$$(\Delta x_b - \Delta x_c)_{\max} = (\Delta x_b - \Delta x_c)\big|_{t=2.22\ \text{s}} = \left|(29.4\ \text{ft/s})(2.22\ \text{s}) - 22.6\ \text{ft}\right| - \left|(6.62\ \text{ft/s}^2)(2.22\ \text{s})^2\right|$$

or $(\Delta x_b - \Delta x_c)_{\max} = \boxed{10.0\ \text{ft}}$.

Additional Problems

P2.39 (a) $a = \dfrac{v_f - v_i}{t} = \dfrac{632\left(\frac{5\,280}{3\,600}\right)}{1.40} = \boxed{-662\ \text{ft/s}^2} = -202\ \text{m/s}^2 = -20.6g$

(b) $x_f = v_i t + \dfrac{1}{2}at^2 = (632)\left(\dfrac{5\,280}{3\,600}\right)(1.40) - \dfrac{1}{2}(662)(1.40)^2 = \boxed{649\ \text{ft}} = \boxed{198\ \text{m}}$

***P2.40** Take downward as the positive y direction.

(a) While the woman was in free fall,

$$\Delta y = 144\ \text{ft},\ v_i = 0,\ \text{and we take } a = g = 32.0\ \text{ft/s}^2.$$

Thus, $\Delta y = v_i t + \dfrac{1}{2}at^2 \rightarrow 144\ \text{ft} = 0 + (16.0\ \text{ft/s}^2)t^2$ giving $t_{\text{fall}} = 3.00$ s. Her velocity just before impact is:

$$v_f = v_i + gt = 0 + (32.0\ \text{ft/s}^2)(3.00\ \text{s}) = \boxed{96.0\ \text{ft/s}}.$$

(b) While crushing the box, $v_i = 96.0$ ft/s, $v_f = 0$, and $\Delta y = 18.0$ in. $= 1.50$ ft. Therefore,

$a = \dfrac{v_f^2 - v_i^2}{2(\Delta y)} = \dfrac{0-(96.0\ \text{ft/s})^2}{2(1.50\ \text{ft})} = -3.07\times10^3\ \text{ft/s}^2$, or $\boxed{a = 3.07\times10^3\ \text{ft/s}^2\ \text{upward}} = 96.0g$.

(c) Time to crush box: $\Delta t = \dfrac{\Delta y}{\bar{v}} = \dfrac{\Delta y}{\frac{v_f+v_i}{2}} = \dfrac{2(1.50\ \text{ft})}{0+96.0\ \text{ft/s}}$ or $\boxed{\Delta t = 3.13\times10^{-2}\ \text{s}}$

P2.41 From $v_f^2 = v_i^2 + 2ax$, we have $(10.97\times10^3\ \text{m/s})^2 = 0 + 2a(220\ \text{m})$, so that $\boxed{a = 2.74\times10^5\ \text{m/s}^2}$ which is $\boxed{a = 2.79\times10^4\ \text{times } g}$.

***P2.42** Take the x-axis along the tail section of the snake. The displacement from tail to head is

$$240\ \text{m}\hat{i} + (420-240)\ \text{m}\cos(180°-105°)\hat{i} - 180\ \text{m}\sin 75°\,\hat{j} = 287\ \text{m}\hat{i} - 174\ \text{m}\hat{j}.$$

Its magnitude is $\sqrt{(287)^2+(174)^2}$ m $= 335$ m. From $v = \dfrac{\text{distance}}{\Delta t}$, the time for each child's run is

$$\text{Inge: } \Delta t = \frac{\text{distance}}{v} = \frac{335\ \text{m(h)}(1\ \text{km})(3\,600\ \text{s})}{(12\ \text{km})(1\,000\ \text{m})(1\ \text{h})} = 101\ \text{s}$$

$$\text{Olaf: } \Delta t = \frac{420\ \text{m}\cdot\text{s}}{3.33\ \text{m}} = 126\ \text{s}.$$

Inge wins by $126 - 101 = \boxed{25.4\ \text{s}}$.

P2.43 (a) Take initial and final points at top and bottom of the incline. If the ball starts from rest,

$$v_i = 0,\ a = 0.500\ \text{m/s}^2,\ x_f - x_i = 9.00\ \text{m}.$$

Then

$$v_f^2 = v_i^2 + 2a(x_f - x_i) = 0^2 + 2(0.500\ \text{m/s}^2)(9.00\ \text{m})$$
$$v_f = \boxed{3.00\ \text{m/s}}.$$

(b) $x_f - x_i = v_i t + \dfrac{1}{2}at^2$

$$9.00 = 0 + \frac{1}{2}(0.500\ \text{m/s}^2)t^2$$
$$t = \boxed{6.00\ \text{s}}$$

(c) Take initial and final points at the bottom of the planes and the top of the second plane, respectively:

$$v_i = 3.00\ \text{m/s},\ v_f = 0,\ x_f - x_i = 15.00\ \text{m}.$$

$v_f^2 = v_i^2 + 2a(x_f - x_i)$ gives

$$a = \frac{v_f^2 - v_i^2}{2(x_f - x_i)} = \frac{\left[0-(3.00\ \text{m/s})^2\right]}{2(15.0\ \text{m})} = \boxed{-0.300\ \text{m/s}^2}.$$

(d) Take the initial point at the bottom of the planes and the final point 8.00 m along the second: $v_i = 3.00$ m/s, $x_f - x_i = 8.00$ m, $a = -0.300\ \text{m/s}^2$

$$v_f^2 = v_i^2 + 2a(x_f - x_i) = (3.00\ \text{m/s})^2 + 2(-0.300\ \text{m/s}^2)(8.00\ \text{m}) = 4.20\ \text{m}^2/\text{s}^2$$
$$v_f = \boxed{2.05\ \text{m/s}}.$$

***P2.44** Let the glider enter the photogate with velocity v_i and move with constant acceleration a. For its motion from entry to exit,

$$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$$
$$\ell = 0 + v_i\Delta t_d + \frac{1}{2}a\Delta t_d^2 = v_d\Delta t_d$$
$$v_d = v_i + \frac{1}{2}a\Delta t_d$$

(a) The speed halfway through the photogate in space is given by

$$v_{hs}^2 = v_i^2 + 2a\left(\frac{\ell}{2}\right) = v_i^2 + av_d\Delta t_d.$$

$v_{hs} = \sqrt{v_i^2 + av_d\Delta t_d}$ and this is not equal to v_d unless $a = 0$.

(b) The speed halfway through the photogate in time is given by $v_{ht} = v_i + a\left(\dfrac{\Delta t_d}{2}\right)$ and this is equal to v_d as determined above.

P2.45 Average speed of every point on the train as the first car passes Liz:

$$\frac{\Delta x}{\Delta t}=\frac{8.60\ \text{m}}{1.50\ \text{s}}=5.73\ \text{m/s}.$$

The train has this as its instantaneous speed halfway through the 1.50 s time. Similarly, halfway through the next 1.10 s, the speed of the train is $\frac{8.60\ \text{m}}{1.10\ \text{s}}=7.82\ \text{m/s}$. The time required for the speed to change from 5.73 m/s to 7.82 m/s is

$$\frac{1}{2}(1.50\ \text{s})+\frac{1}{2}(1.10\ \text{s})=1.30\ \text{s}$$

so the acceleration is: $a_x=\frac{\Delta v_x}{\Delta t}=\frac{7.82\ \text{m/s}-5.73\ \text{m/s}}{1.30\ \text{s}}=\boxed{1.60\ \text{m/s}^2}$.

***P2.46** (a) As we see from the graph, from about −50 s to 50 s Acela is cruising at a constant positive velocity in the $+x$ direction. From 50 s to 200 s, Acela accelerates in the $+x$ direction reaching a top speed of about 170 mi/h. Around 200 s, the engineer applies the brakes, and the train, still traveling in the $+x$ direction, slows down and then stops at 350 s. Just after 350 s, Acela reverses direction (v becomes negative) and steadily gains speed in the $-x$ direction.

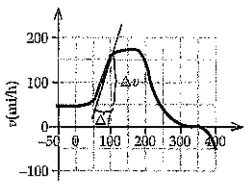

FIG. P2.46(a)

(b) The peak acceleration between 45 and 170 mi/h is given by the slope of the steepest tangent to the v versus t curve in this interval. From the tangent line shown, we find

$$a=\text{slope}=\frac{\Delta v}{\Delta t}=\frac{(155-45)\ \text{mi/h}}{(100-50)\ \text{s}}=\boxed{2.2\ (\text{mi/h})/\text{s}}=0.98\ \text{m/s}^2.$$

(c) Let us use the fact that the area under the v versus t curve equals the displacement. The train's displacement between 0 and 200 s is equal to the area of the gray shaded region, which we have approximated with a series of triangles and rectangles.

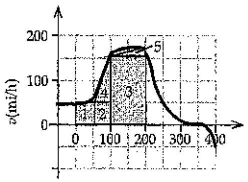

FIG. P2.46(c)

$$\begin{aligned}\Delta x_{0\to 200\ \text{s}}&=\text{area}_1+\text{area}_2+\text{area}_3+\text{area}_4+\text{area}_5\\&\approx(50\ \text{mi/h})(50\ \text{s})+(50\ \text{mi/h})(50\ \text{s})\\&\quad+(160\ \text{mi/h})(100\ \text{s})+\frac{1}{2}(50\ \text{s})(100\ \text{mi/h})\\&\quad+\frac{1}{2}(100\ \text{s})(170\ \text{mi/h}-160\ \text{mi/h})\\&=24\,000(\text{mi/h})(\text{s})\end{aligned}$$

Now, at the end of our calculation, we can find the displacement in miles by converting hours to seconds. As $1\ \text{h}=3\,600\ \text{s}$,

$$\Delta x_{0\to 200\ \text{s}}\approx\left(\frac{24\,000\ \text{mi}}{3\,600\ \text{s}}\right)(\text{s})=\boxed{6.7\ \text{mi}}.$$

P2.47 Let point 0 be at ground level and point 1 be at the end of the engine burn. Let point 2 be the highest point the rocket reaches and point 3 be just before impact. The data in the table are found for each phase of the rocket's motion.

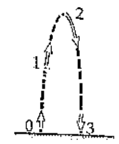

FIG. P2.47

(0 to 1) $v_f^2-(80.0)^2=2(4.00)(1\,000)$ so $v_f=120\ \text{m/s}$
$120=80.0+(4.00)t$ giving $t=10.0\ \text{s}$

(1 to 2) $0-(120)^2=2(-9.80)(x_f-x_i)$ giving $x_f-x_i=735\ \text{m}$
$0-120=-9.80t$ giving $t=12.2\ \text{s}$
This is the time of maximum height of the rocket.

(2 to 3) $v_f^2-0=2(-9.80)(-1\,735)$
$v_f=-184=(-9.80)t$ giving $t=18.8\ \text{s}$

(a) $t_{\text{total}}=10+12.2+18.8=\boxed{41.0\ \text{s}}$

(b) $(x_f-x_i)_{\text{total}}=\boxed{1.73\ \text{km}}$

(c) $v_{\text{final}}=\boxed{-184\ \text{m/s}}$

		t	x	v	a
0		0.0	0	80	+4.00
#1	End Thrust	10.0	1 000	120	+4.00
#2	Rise Upwards	22.2	1 735	0	−9.80
#3	Fall to Earth	41.0	0	−184	−9.80

P2.48 Distance traveled by motorist $=(15.0\ \text{m/s})t$

Distance traveled by policeman $=\frac{1}{2}(2.00\ \text{m/s}^2)t^2$

(a) intercept occurs when $15.0t=t^2$, or $t=\boxed{15.0\ \text{s}}$

(b) $v(\text{officer})=(2.00\ \text{m/s}^2)t=\boxed{30.0\ \text{m/s}}$

(c) $x(\text{officer})=\frac{1}{2}(2.00\ \text{m/s}^2)t^2=\boxed{225\ \text{m}}$

P2.49 (a) Let x be the distance traveled at acceleration a until maximum speed v is reached. If this is achieved in time t_1 we can use the following three equations:

$$x=\frac{1}{2}(v+v_i)t_1, \qquad 100-x=v(10.2-t_1) \qquad \text{and } v=v_i+at_1.$$

The first two give

$$100=\left(10.2-\frac{1}{2}t_1\right)v=\left(10.2-\frac{1}{2}t_1\right)at_1$$

$$a=\frac{200}{(20.4-t_1)t_1}$$

$$\text{For Maggie: } a=\frac{200}{(18.4)(2.00)}=\boxed{5.43\ \text{m/s}^2}$$

$$\text{For Judy: } a=\frac{200}{(17.4)(3.00)}=\boxed{3.83\ \text{m/s}^2}$$

(b) $v=at_1$

$$\text{Maggie: } v=(5.43)(2.00)=\boxed{10.9\ \text{m/s}}$$

$$\text{Judy: } v=(3.83)(3.00)=\boxed{11.5\ \text{m/s}}$$

(c) At the six-second mark

$$x=\frac{1}{2}at_1^2+v(6.00-t_1)$$

$$\text{Maggie: } x=\frac{1}{2}(5.43)(2.00)^2+(10.9)(4.00)=54.3\ \text{m}$$

$$\text{Judy: } x=\frac{1}{2}(3.83)(3.00)^2+(11.5)(3.00)=51.7\ \text{m}$$

Maggie is ahead by $\boxed{2.62\ \text{m}}$.

P2.50 $a_1=0.100\ \text{m/s}^2$ $\qquad a_2=-0.500\ \text{m/s}^2$

$$x=1\,000\ \text{m}=\frac{1}{2}a_1t_1^2+v_1t_2+\frac{1}{2}a_2t_2^2 \qquad t=t_1+t_2 \text{ and } v_1=a_1t_1=-a_2t_2$$

$$1\,000=\frac{1}{2}a_1t_1^2+a_1t_1\left(-\frac{a_1t_1}{a_2}\right)+\frac{1}{2}a_2\left(\frac{a_1t_1}{a_2}\right)^2 \qquad 1\,000=\frac{1}{2}a_1\left(1-\frac{a_1}{a_2}\right)t_1^2$$

$$t_1=\sqrt{\frac{20\,000}{1.20}}=\boxed{129\ \text{s}}$$

$$t_2=\frac{a_1t_1}{-a_2}=\frac{12.9}{0.500}\approx 26\ \text{s} \qquad \text{Total time}=t=\boxed{155\ \text{s}}$$

P2.51 (a) $y_f=v_{i1}t+\frac{1}{2}at^2=50.0=2.00t+\frac{1}{2}(9.80)t^2$,

$4.90t^2+2.00t-50.0=0$

$$t=\frac{-2.00+\sqrt{2.00^2-4(4.90)(-50.0)}}{2(4.90)}$$

Only the positive root is physically meaningful:

$$t=\boxed{3.00\ \text{s}} \text{ after the first stone is thrown.}$$

(b) $y_f=v_{i2}t+\frac{1}{2}at^2$ and $t=3.00-1.00=2.00$ s.

substitute $50.0=v_{i2}(2.00)+\frac{1}{2}(9.80)(2.00)^2$.

$$v_{i2}=\boxed{15.3\ \text{m/s}} \text{ downward}$$

(c) $v_{1f}=v_{i1}+at=2.00+(9.80)(3.00)=\boxed{31.4\ \text{m/s}}$ downward

$v_{2f}=v_{i2}+at=15.3+(9.80)(2.00)=\boxed{34.8\ \text{m/s}}$ downward

P2.52 Let the ball fall 1.50 m. It strikes at speed given by

$$v_{xf}^2 = v_{xi}^2 + 2a(x_f - x_i):$$

$$v_{xf}^2 = 0 + 2(-9.80\ \text{m/s}^2)(-1.50\ \text{m})$$

$$v_{xf} = -5.42\ \text{m/s}$$

and its stopping is described by

$$v_{xf}^2 = v_{xi}^2 + 2a_x(x_f - x_i)$$

$$0 = (-5.42\ \text{m/s})^2 + 2a_x(-10^{-2}\ \text{m})$$

$$a_x = \frac{-29.4\ \text{m}^2/\text{s}^2}{-2.00\times 10^{-2}\ \text{m}} = +1.47\times 10^3\ \text{m/s}^2$$

Its maximum acceleration will be larger than the average acceleration we estimate by imagining constant acceleration, but will still be of order of magnitude $\boxed{\sim 10^3\ \text{m/s}^2}$.

P2.53 (a) In walking a distance Δx, in a time Δt, the length of rope ℓ is only increased by $\Delta x \sin\theta$.

$\therefore$ The pack lifts at a rate $\frac{\Delta x}{\Delta t}\sin\theta$.

$$v = \frac{\Delta x}{\Delta t}\sin\theta = v_{\text{boy}}\frac{x}{\ell} = \boxed{v_{\text{boy}}\frac{x}{\sqrt{x^2+h^2}}}$$

(b) $$a = \frac{dv}{dt} = \frac{v_{\text{boy}}}{\ell}\frac{dx}{dt} + v_{\text{boy}}x\frac{d}{dt}\left(\frac{1}{\ell}\right)$$

$$a = v_{\text{boy}}\frac{v_{\text{boy}}}{\ell} - \frac{v_{\text{boy}}x}{\ell^2}\frac{d\ell}{dt},\ \text{but}\ \frac{d\ell}{dt} = v = v_{\text{boy}}\frac{x}{\ell}$$

$$\therefore a = \frac{v_{\text{boy}}^2}{\ell}\left(1 - \frac{x^2}{\ell^2}\right) = \frac{v_{\text{boy}}^2}{\ell}\frac{h^2}{\ell^2} = \boxed{\frac{h^2 v_{\text{boy}}^2}{(x^2+h^2)^{3/2}}}$$

(c) $\frac{v_{\text{boy}}^2}{h}$, 0

(d) v_{boy}, 0

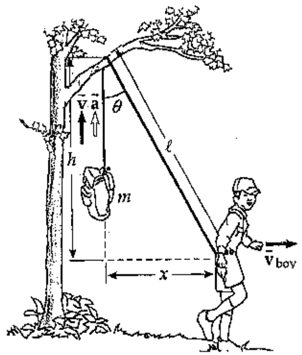

FIG. P2.53

P2.54 $h = 6.00$ m, $v_{\text{boy}} = 2.00$ m/s $v = \frac{\Delta x}{\Delta t}\sin\theta = v_{\text{boy}}\frac{x}{\ell} = \frac{v_{\text{boy}}x}{(x^2+h^2)^{1/2}}$.

However, $x = v_{\text{boy}}t$ $\therefore v = \frac{v_{\text{boy}}^2 t}{(v_{\text{boy}}^2 t^2 + h^2)^{1/2}} = \frac{4t}{(4t^2+36)^{1/2}}$.

(a)

t(s)	v(m/s)
0	0
0.5	0.33
1	0.63
1.5	0.89
2	1.11
2.5	1.28
3	1.41
3.5	1.52
4	1.60
4.5	1.66
5	1.71

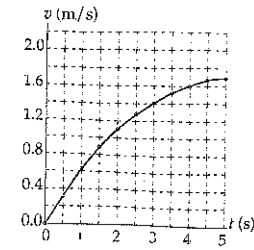

FIG. P2.54(a)

(b) From problem 2.53 above, $a = \frac{h^2 v_{\text{boy}}^2}{(x^2+h^2)^{3/2}} = \frac{h^2 v_{\text{boy}}^2}{(v_{\text{boy}}^2 t^2 + h^2)^{3/2}} = \frac{144}{(4t^2+36)^{3/2}}$.

t(s)	a(m/s²)
0	0.67
0.5	0.64
1	0.57
1.5	0.48
2	0.38
2.5	0.30
3.	0.24
3.5	0.18
4.	0.14
4.5	0.11
5	0.09

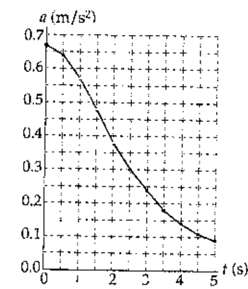

FIG. P2.54(b)

P2.55 (a) $d = \frac{1}{2}(9.80)t_1^2$ $\qquad d = 336t_2$

$t_1 + t_2 = 2.40$ $\qquad 336t_2 = 4.90(2.40 - t_2)^2$

$4.90t_2^2 - 359.5t_2 + 28.22 = 0$ $\qquad t_2 = \frac{359.5 \pm \sqrt{359.5^2 - 4(4.90)(28.22)}}{9.80}$

$t_2 = \frac{359.5 \pm 358.75}{9.80} = 0.0765$ s so $d = 336t_2 = \boxed{26.4\ \text{m}}$

(b) Ignoring the sound travel time, $d = \frac{1}{2}(9.80)(2.40)^2 = 28.2$ m, an error of $\boxed{6.82\%}$.

P2.56 (a)

Time t (s)	Height h (m)	Δh (m)	Δt (s)	$\bar{v}$ (m/s)	midpoint time t (s)
0.00	5.00				
		0.75	0.25	3.00	0.13
0.25	5.75				
		0.65	0.25	2.60	0.38
0.50	6.40				
		0.54	0.25	2.16	0.63
0.75	6.94				
		0.44	0.25	1.76	0.88
1.00	7.38				
		0.34	0.25	1.36	1.13
1.25	7.72				
		0.24	0.25	0.96	1.38
1.50	7.96				
		0.14	0.25	0.56	1.63
1.75	8.10				
		0.03	0.25	0.12	1.88
2.00	8.13				
		–0.06	0.25	–0.24	2.13
2.25	8.07				
		–0.17	0.25	–0.68	2.38
2.50	7.90				
		–0.28	0.25	–1.12	2.63
2.75	7.62				
		–0.37	0.25	–1.48	2.88
3.00	7.25				
		–0.48	0.25	–1.92	3.13
3.25	6.77				
		–0.57	0.25	–2.28	3.38
3.50	6.20				
		–0.68	0.25	–2.72	3.63
3.75	5.52				
		–0.79	0.25	–3.16	3.88
4.00	4.73				
		–0.88	0.25	–3.52	4.13
4.25	3.85				
		–0.99	0.25	–3.96	4.38
4.50	2.86				
		–1.09	0.25	–4.36	4.63
4.75	1.77				
		–1.19	0.25	–4.76	4.88
5.00	0.58				

TABLE P2.56

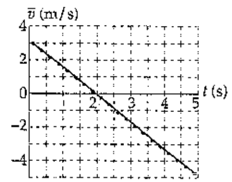

FIG. P2.56(b)

acceleration = slope of line is constant.

$\bar{a} = -1.63\ \text{m/s}^2 = \boxed{1.63\ \text{m/s}^2\ \text{downward}}$

P2.57 The distance x and y are always related by $x^2 + y^2 = L^2$. Differentiating this equation with respect to time, we have

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$

Now $\frac{dy}{dt}$ is v_B, the unknown velocity of B; and $\frac{dx}{dt} = -v$.
From the equation resulting from differentiation, we have

$$\frac{dy}{dt} = -\frac{x}{y}\left(\frac{dx}{dt}\right) = -\frac{x}{y}(-v).$$

But $\frac{y}{x} = \tan\alpha$ so $v_B = \left(\frac{1}{\tan\alpha}\right)v$. When $\alpha = 60.0°$, $v_B = \frac{v}{\tan 60.0°} = \frac{v\sqrt{3}}{3} = \boxed{0.577v}$.

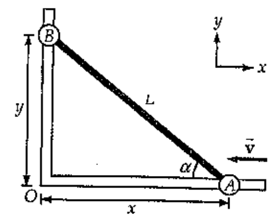

FIG. P2.57

ANSWERS TO EVEN PROBLEMS

P2.2 (a) 50.0 m/s; (b) 41.0 m/s

P2.4 (a) 3.75 m/s; (b) 0

P2.6 (a) 27.0 m;
(b) $x_f = 27.0\ \text{m} + (18.0\ \text{m/s})\Delta t + (3.00\ \text{m/s}^2)(\Delta t)^2$;
(c) 18.0 m/s

P2.8 (a) 5.0 m/s; (b) –2.5 m/s; (c) 0; (d) 5.0 m/s

P2.10 $1.34\times10^4\ \text{m/s}^2$

P2.12 (a) 13.0 m/s; (b) 10.0 m/s, 16.0 m/s;
(c) 6.00 m/s^2; (d) 6.00 m/s^2

P2.14 (a) see the solution; (b) see the solution;
(c) –4 m/s^2; (d) 34 m; (e) 28 m

P2.16 (a) see the solution; (b) see the solution;
(c) see the solution; (d) see the solution;
(e) see the solution;
(f) The spacing of the successive positions would change with less regularity.

P2.18 160 ft

P2.20 (a) 4.53 s; (b) 14.1 m/s

P2.22 (a) 2.56 m; (b) –3.00 m/s

P2.24 $x_f - x_i = v_{xi}t - \frac{1}{2}a_x t^2$; 3.10 m/s

P2.26 (a) 4.98×10^{-9} s; (b) $1.20\times10^{15}\ \text{m/s}^2$

P2.28 (a) 4.90 cm below the top of the TV, 0.980 m/s downward;
(b) 19.6 cm down, 1.96 m/s downward;
(c) 44.1 cm down, 2.94 m/s downward

P2.30 \$99.3/h

P2.32 1.79 s

P2.34 (a) 510 m; (b) 20.4 s

P2.36 (a) 5.25 m/s^2; (b) 168 m; (c) 52.5 m/s

P2.38 (a) 3.45 s; (b) 10.0 ft

P2.40 (a) 96.0 ft/s;
(b) $a = 3.07\times10^3\ \text{ft/s}^2$ upward
(c) $\Delta t = 3.13\times10^{-4}$ s

P2.42 25.4 s

P2.44 (a) false unless the acceleration is zero; see the solution;
(b) true

P2.46 (a) see the solution; (b) 2.2 (mi/h)/s; (c) 6.7 mi

P2.48 (a) 15.0 s; (b) 30.0 m/s; (c) 225 m

P2.50 155 s, 129 s

P2.52 $\sim 10^3$ m/s^2

P2.54 see the solution

P2.56 see the solution, $a = 1.63$ m/s^2 downward

Chapter 3

Motion in Two Dimensions

ANSWERS TO QUESTIONS

Q3.1 No, you cannot determine the instantaneous velocity. Yes, you can determine the average velocity. The points could be widely separated. In this case, you can only determine the average velocity, which is

$$\vec{v} = \frac{\Delta \vec{x}}{\Delta t}$$

Q3.2 (a)

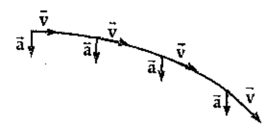

FIG. Q3.2(a)

(b)

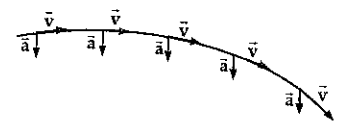

FIG. Q3.2(b)

Q3.3 The easiest way to approach this problem is to determine acceleration first, velocity second and finally position. Determine

(c) acceleration first,

(b) velocity second, and

(a) finally position.

Vertical: In free flight, $a_y = -g$. At the top of a projectile's trajectory, $v_y = 0$. Using this, the maximum height can be found using $v_{fy}^2 = v_{iy}^2 + 2a_y(y_f - y_i)$.

Horizontal: $a_x = 0$, so v_x is always the same. To find the horizontal position at maximum height, one needs the flight time, t. Using the vertical information found previously, the flight time can be found using $v_{fy} = v_{iy} + a_y t$. The horizontal position is $x_f = v_{ix} t$.

If air resistance were taken into account, then the acceleration in both the x and y-directions would have an additional term due to the drag.

Q3.4 The balls will be closest together as the second ball is thrown. Yes, the first ball will always be moving faster, since its flight time is larger, and thus the vertical component of the velocity is larger. The time interval will be one second. No, since the vertical component of the motion determines the flight time.

Q3.5 A parabola.

Q3.6 (a) no (b) yes (c) yes (d) no

Q3.7 The projectile is in free fall. Its vertical component of acceleration is the downward acceleration of gravity. Its horizontal component of acceleration is zero.

Q3.8 The optimal angle would be less than 45°. The longer the projectile is in the air, the more that air resistance will change the components of the velocity. Since the vertical component of the motion determines the flight time, an angle less than 45° would increase range.

Q3.9 The projectile on the moon would have both the larger range and the greater altitude. *Apollo* astronauts performed the experiment with golf balls.

Q3.10 The racing car rounds the turn at a constant *speed* of 90 miles per hour.

Q3.11 (a) no (b) yes

In the second case, the particle is continuously changing the direction of its velocity vector.

Q3.12 (a) The velocity is not constant because the object is constantly changing the direction of its motion.

(b) The acceleration is not constant because the acceleration always points towards the center of the circle. The magnitude of the acceleration is constant, but not the direction.

Q3.13 (a) straight ahead (b) in a circle or straight ahead

Q3.14

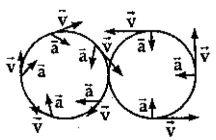

FIG. Q3.14

Q3.15 The wrench will hit at the base of the mast. If air resistance is a factor, it will hit slightly leeward of the base of the mast, displaced in the direction in which air is moving relative to the deck. If the boat is scudding before the wind, for example, the wrench's impact point can be in front of the mast.

Q3.16 (a) The ball would move straight up and down as observed by the passenger. The ball would move in a parabolic trajectory as seen by the ground observer.

(b) Both the passenger and the ground observer would see the ball move in a parabolic trajectory, although the two observed paths would not be the same.

SOLUTIONS TO PROBLEMS

Section 3.1 The Position, Velocity, and Acceleration Vectors

P3.1

$x(m)$	$y(m)$
0	−3 600
−3 000	0
−1 270	1 270
−4 270 m	−2 330 m

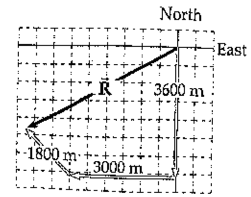

FIG. P3.1

(a) Net displacement $= \sqrt{x^2+y^2}$

$= \boxed{4.87 \text{ km at } 28.6° \text{ S of W}}$

(b) $$\text{Average speed} = \frac{(20.0 \text{ m/s})(180 \text{ s}) + (25.0 \text{ m/s})(120 \text{ s}) + (30.0 \text{ m/s})(60.0 \text{ s})}{180 \text{ s} + 120 \text{ s} + 60.0 \text{ s}} = \boxed{23.3 \text{ m/s}}$$

(c) $$\text{Average velocity} = \frac{4.87 \times 10^3 \text{ m}}{360 \text{ s}} = \boxed{13.5 \text{ m/s along } \vec{\mathbf{R}}}$$

P3.2 (a) For the average velocity, we have

$$\vec{\mathbf{v}}_{av} = \left(\frac{x(4.00) - x(2.00)}{4.00 \text{ s} - 2.00 \text{ s}}\right)\hat{\mathbf{i}} + \left(\frac{y(4.00) - y(2.00)}{4.00 \text{ s} - 2.00 \text{ s}}\right)\hat{\mathbf{j}} = \left(\frac{5.00 \text{ m} - 3.00 \text{ m}}{2.00 \text{ s}}\right)\hat{\mathbf{i}} + \left(\frac{3.00 \text{ m} - 1.50 \text{ m}}{2.00 \text{ s}}\right)\hat{\mathbf{j}}$$

$$\vec{\mathbf{v}}_{av} = \boxed{\left(1.00\hat{\mathbf{i}} + 0.750\hat{\mathbf{j}}\right) \text{ m/s}}$$

(b) For the velocity components,

we have $\quad v_x = \frac{dx}{dt} = a = 1.00 \text{ m/s}$

and $\quad v_y = \frac{dy}{dt} = 2ct = \left(0.250 \text{ m/s}^2\right)t$

Therefore, $\quad \vec{\mathbf{v}} = v_x\hat{\mathbf{i}} + v_y\hat{\mathbf{j}} = (1.00 \text{ m/s})\hat{\mathbf{i}} + \left(0.250 \text{ m/s}^2\right)t\,\hat{\mathbf{j}}$

$$\boxed{\vec{\mathbf{v}}(t = 2.00 \text{ s}) = (1.00 \text{ m/s})\hat{\mathbf{i}} + (0.500 \text{ m/s})\hat{\mathbf{j}}}$$

and the speed is $\quad \left|\vec{\mathbf{v}}(t = 2.00 \text{ s})\right| = \sqrt{(1.00 \text{ m/s})^2 + (0.500 \text{ m/s})^2} = \boxed{1.12 \text{ m/s}}$

Section 3.2 Two-Dimensional Motion with Constant Acceleration

P3.3 $\vec{v}_i = \left(4.00\hat{i}+1.00\hat{j}\right)$ m/s and $\vec{v}(20.0) = \left(20.0\hat{i}-5.00\hat{j}\right)$ m/s

(a) $$a_x = \frac{\Delta v_x}{\Delta t} = \frac{20.0-4.00}{20.0}\ \text{m/s}^2 = \boxed{0.800\ \text{m/s}^2}$$

$$a_y = \frac{\Delta v_y}{\Delta t} = \frac{-5.00-1.00}{20.0}\ \text{m/s}^2 = \boxed{-0.300\ \text{m/s}^2}$$

(b) $$\theta = \tan^{-1}\left(\frac{-0.300}{0.800}\right) = -20.6° = \boxed{339° \text{ from } +x \text{ axis}}$$

(c) At $t = 25.0$ s

$$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2 = 10.0 + 4.00(25.0) + \frac{1}{2}(0.800)(25.0)^2 = \boxed{360\ \text{m}}$$

$$y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2 = -4.00 + 1.00(25.0) + \frac{1}{2}(-0.300)(25.0)^2 = \boxed{-72.7\ \text{m}}$$

$$v_{xf} = v_{xi} + a_x t = 4 + 0.8(25) = 24\ \text{m/s}$$

$$v_{yf} = v_{yi} + a_y t = 1 - 0.3(25) = -6.5\ \text{m/s}$$

$$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{-6.50}{24.0}\right) = \boxed{-15.2°}$$

P3.4 (a) $\vec{v}_f = \vec{v}_i + \vec{a}t$

$$\vec{a} = \frac{\vec{v}_f - \vec{v}_i}{t} = \frac{\left(9.00\hat{i}+7.00\hat{j}\right)-\left(3.00\hat{i}-2.00\hat{j}\right)}{3.00} = \boxed{\left(2.00\hat{i}+3.00\hat{j}\right)\ \text{m/s}^2}$$

(b) $$\vec{r}_f = \vec{r}_i + \vec{v}_i t + \frac{1}{2}\vec{a}t^2 = \left(3.00\hat{i}-2.00\hat{j}\right)t + \frac{1}{2}\left(2.00\hat{i}+3.00\hat{j}\right)t^2$$

$$\boxed{x = \left(3.00t + t^2\right)\ \text{m}} \text{ and } \boxed{y = \left(1.50t^2 - 2.00t\right)\ \text{m}}$$

P3.5 $\vec{a} = 3.00\hat{j}$ m/s^2 ; $\vec{v}_i = 5.00\hat{i}$ m/s ; $\vec{r}_i = 0\hat{i} + 0\hat{j}$

(a) $$\vec{r}_f = \vec{r}_i + \vec{v}_i t + \frac{1}{2}\vec{a}t^2 = \boxed{\left[5.00t\hat{i} + \frac{1}{2}3.00t^2\hat{j}\right]\ \text{m}}$$

$$\vec{v}_f = \vec{v}_i + \vec{a}t = \boxed{\left(5.00\hat{i} + 3.00t\hat{j}\right)\ \text{m/s}}$$

continued on next page

(b) $t = 2.00$ s, $\vec{r}_f = 5.00(2.00)\hat{i} + \frac{1}{2}(3.00)(2.00)^2\hat{j} = \left(10.0\hat{i} + 6.00\hat{j}\right)$ m

so $x_f = \boxed{10.0\ \text{m}}$, $y_f = \boxed{6.00\ \text{m}}$

$$\vec{v}_f = 5.00\hat{i} + 3.00(2.00)\hat{j} = \left(5.00\hat{i} + 6.00\hat{j}\right)\ \text{m/s}$$

$$v_f = \left|\vec{v}_f\right| = \sqrt{v_{xf}^2 + v_{yf}^2} = \sqrt{(5.00)^2 + (6.00)^2} = \boxed{7.81\ \text{m/s}}$$

P3.6 (a) For the x-component of the motion we have $x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$.

$$0.01\ \text{m} = 0 + \left(1.80\times10^7\ \text{m/s}\right)t + \frac{1}{2}\left(8\times10^{14}\ \text{m/s}^2\right)t^2$$

$$\left(4\times10^{14}\ \text{m/s}^2\right)t^2 + \left(1.80\times10^7\ \text{m/s}\right)t - 10^{-2}\ \text{m} = 0$$

$$t = \frac{-1.80\times10^7\ \text{m/s} \pm \sqrt{\left(1.8\times10^7\ \text{m/s}\right)^2 - 4\left(4\times10^{14}\ \text{m/s}^2\right)\left(-10^{-2}\ \text{m}\right)}}{2\left(4\times10^{14}\ \text{m/s}^2\right)} = \frac{-1.8\times10^7 \pm 1.84\times10^7\ \text{m/s}}{8\times10^{14}\ \text{m/s}^2}$$

We choose the + sign to represent the physical situation

$$t = \frac{4.39\times10^5\ \text{m/s}}{8\times10^{14}\ \text{m/s}^2} = 5.49\times10^{-10}\ \text{s}.$$

Here

$$y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2 = 0 + 0 + \frac{1}{2}\left(1.6\times10^{15}\ \text{m/s}^2\right)\left(5.49\times10^{-10}\ \text{s}\right)^2 = 2.41\times10^{-4}\ \text{m}.$$

So, $\boxed{\vec{r}_f = \left(10.0\ \hat{i} + 0.241\ \hat{j}\right)\ \text{mm}}$.

(b) $$\vec{v}_f = \vec{v}_i + \vec{a}t = 1.80\times10^7\ \text{m/s}\,\hat{i} + \left(8\times10^{14}\ \text{m/s}^2\ \hat{i} + 1.6\times10^{15}\ \text{m/s}^2\ \hat{j}\right)\left(5.49\times10^{-10}\ \text{s}\right)$$

$$= \left(1.80\times10^7\ \text{m/s}\right)\hat{i} + \left(4.39\times10^5\ \text{m/s}\right)\hat{i} + \left(8.78\times10^5\ \text{m/s}\right)\hat{j}$$

$$= \boxed{\left(1.84\times10^7\ \text{m/s}\right)\hat{i} + \left(8.78\times10^5\ \text{m/s}\right)\hat{j}}$$

(c) $$\left|\vec{v}_f\right| = \sqrt{\left(1.84\times10^7\ \text{m/s}\right)^2 + \left(8.78\times10^5\ \text{m/s}\right)^2} = \boxed{1.85\times10^7\ \text{m/s}}$$

(d) $$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{8.78\times10^5}{1.84\times10^7}\right) = \boxed{2.73°}$$

Section 3.3 Projectile Motion

P3.7 (a) The mug leaves the counter horizontally with a velocity v_{xi} (say). If time t elapses before it hits the ground, then since there is no horizontal acceleration, $x_f = v_{xi}t$, i.e.,

$$t = \frac{x_f}{v_{xi}} = \frac{(1.40 \text{ m})}{v_{xi}}$$

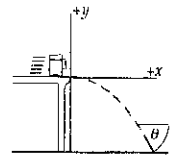

FIG. P3.7

In the same time it falls a distance of 0.860 m with acceleration downward of 9.80 m/s^2. Then

$$y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2:\ 0 = 0.860 \text{ m} + \frac{1}{2}\left(-9.80 \text{ m/s}^2\right)\left(\frac{1.40 \text{ m}}{v_{xi}}\right)^2.$$

Thus,

$$v_{xi} = \sqrt{\frac{\left(4.90 \text{ m/s}^2\right)\left(1.96 \text{ m}^2\right)}{0.860 \text{ m}}} = \boxed{3.34 \text{ m/s}}.$$

(b) The vertical velocity component with which it hits the floor is

$$v_{yf} = v_{yi} + a_y t = 0 + \left(-9.80 \text{ m/s}^2\right)\left(\frac{1.40 \text{ m}}{3.34 \text{ m/s}}\right) = -4.11 \text{ m/s}.$$

Hence, the angle θ at which the mug strikes the floor is given by

$$\theta = \tan^{-1}\left(\frac{v_{yf}}{v_{xf}}\right) = \tan^{-1}\left(\frac{-4.11}{3.34}\right) = \boxed{-50.9°}$$

P3.8 The mug is a projectile from just after leaving the counter until just before it reaches the floor. Taking the origin at the point where the mug leaves the bar, the coordinates of the mug at any time are

$$x_f = v_{xi}t + \frac{1}{2}a_x t^2 = v_{xi}t + 0 \text{ and } y_f = v_{yi}t + \frac{1}{2}a_y t^2 = 0 - \frac{1}{2}gt^2$$

When the mug reaches the floor, $y_f = -h$ so

$$-h = -\frac{1}{2}gt^2$$

which gives the time of impact as $t = \sqrt{\frac{2h}{g}}$

continued on next page

(a) Since $x_f = d$ when the mug reaches the floor, $x_f = v_{xi}t$ becomes $d = v_{xi}\sqrt{\frac{2h}{g}}$ giving the initial velocity as $\boxed{v_{xi} = d\sqrt{\frac{g}{2h}}}$.

(b) Just before impact, the x-component of velocity is still

$$v_{xf} = v_{xi}$$

while the y-component is $v_{yf} = v_{yi} + a_y t = 0 - g\sqrt{\frac{2h}{g}}$.

Then the direction of motion just before impact is below the horizontal at an angle of

$$\theta = \tan^{-1}\left(\frac{|v_{yf}|}{v_{xf}}\right) = \tan^{-1}\left(\frac{g\sqrt{\frac{2h}{g}}}{d\sqrt{\frac{g}{2h}}}\right) = \boxed{\tan^{-1}\left(\frac{2h}{d}\right)}$$

The answer for v_{xi} indicates that a larger measured value for d would imply larger takeoff speed in direct proportion. A tape measure lying on the floor could be calibrated as a speedometer. A larger value for h would imply a smaller value for speed by an inverse proportionality to the square root of h. That is, if h were 9 times larger, v_{xi} would be 3 times smaller. The answer for θ shows that the impact velocity makes an angle with the horizontal whose tangent is just twice as large as that of the elevation angle α of the edge of the table as seen from the impact point.

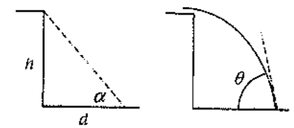

FIG. P3.8

***P3.9** At the maximum height $v_y = 0$, and the time to reach this height is found from

$$v_{yf} = v_{iy} + a_y t \text{ as } t = \frac{v_{yf} - v_{iy}}{a_y} = \frac{0 - v_{iy}}{-g} = \frac{v_{iy}}{g}.$$

The vertical displacement that has occurred during this time is

$$(\Delta y)_{max} = v_{y,avg}t = \left(\frac{v_{yf} + v_{iy}}{2}\right)t = \left(\frac{0 + v_{iy}}{2}\right)\left(\frac{v_{iy}}{g}\right) = \frac{v_{iy}^2}{2g}$$

Thus, if $(\Delta y)_{max} = 12 \text{ ft}\left(\frac{1 \text{ m}}{3.281 \text{ ft}}\right) = 3.66 \text{ m}$, then

$$v_{iy} = \sqrt{2g(\Delta y)_{max}} = \sqrt{2\left(9.80 \text{ m/s}^2\right)(3.66 \text{ m})} = 8.47 \text{ m/s},$$

continued on next page

and if the angle of projection is $\theta = 45°$, the launch speed is

$$v_i = \frac{v_{iy}}{\sin\theta} = \frac{8.47\ \text{m/s}}{\sin 45°} = \boxed{12.0\ \text{m/s}}.$$

P3.10 From Equation 3.16 with $R = 15.0$ m, $v_i = 3.00$ m/s, $\theta_{max} = 45.0°$

$$\therefore g = \frac{v_i^2}{R} = \frac{9.00}{15.0} = \boxed{0.600\ \text{m/s}^2}$$

P3.11 Take the origin at the mouth of the cannon.

$x_f = v_{xi}t$ $\qquad 2\,000\ \text{m} = (1\,000\ \text{m/s})\cos\theta_i t$

Therefore, $\qquad t = \dfrac{2.00\ \text{s}}{\cos\theta_i}$

$y_f = v_{yi}t + \frac{1}{2}a_y t^2$: $\qquad 800\ \text{m} = (1\,000\ \text{m/s})\sin\theta_i t + \frac{1}{2}(-9.80\ \text{m/s}^2)t^2$

$$800\ \text{m} = (1\,000\ \text{m/s})\sin\theta_i\left(\frac{2.00\ \text{s}}{\cos\theta_i}\right) - \frac{1}{2}(9.80\ \text{m/s}^2)\left(\frac{2.00\ \text{s}}{\cos\theta_i}\right)^2$$

$$800\ \text{m}(\cos^2\theta_i) = 2\,000\ \text{m}(\sin\theta_i\cos\theta_i) - 19.6\ \text{m}$$

$$19.6\ \text{m} + 800\ \text{m}(\cos^2\theta_i) = 2\,000\ \text{m}\sqrt{1-\cos^2\theta_i}(\cos\theta_i)$$

$$384 + (31\,360)\cos^2\theta_i + (640\,000)\cos^4\theta_i = (4\,000\,000)\cos^2\theta_i - (4\,000\,000)\cos^4\theta_i$$

$$4\,640\,000\cos^4\theta_i - 3\,968\,640\cos^2\theta_i + 384 = 0$$

$$\cos^2\theta_i = \frac{3\,968\,640 \pm \sqrt{(3\,968\,640)^2 - 4(4\,640\,000)(384)}}{9\,280\,000}$$

$\cos\theta_i = 0.925$ or 0.00984

$\theta_i = \boxed{22.4° \text{ or } 89.4°}$ (Both solutions are valid.)

P3.12 (a) $x_f = v_{xi}t = 8.00\cos 20.0°(3.00) = \boxed{22.6\ \text{m}}$

(b) Taking y positive downwards, and the final point just before ground impact,

$$y_f = v_{yi}t + \frac{1}{2}gt^2$$
$$y_f = 8.00\sin 20.0°(3.00) + \frac{1}{2}(9.80)(3.00)^2 = \boxed{52.3\ \text{m}}.$$

continued on next page

(c) Now take the final point 10 m below the window.

$$10.0 = 8.00(\sin 20.0°)t + \frac{1}{2}(9.80)t^2$$

$$4.90t^2 + 2.74t - 10.0 = 0$$

$$t = \frac{-2.74 \pm \sqrt{(2.74)^2 + 196}}{9.80} = \boxed{1.18\ \text{s}}$$

P3.13 Consider the motion from original zero height to maximum height h:

$v_{yf}^2 = v_{yi}^2 + 2a_y(y_f - y_i)$ gives $0 = v_{yi}^2 - 2g(h-0)$ or $v_{yi} = \sqrt{2gh}$

Now consider the motion from the original point to half the maximum height:

$v_{yf}^2 = v_{yi}^2 + 2a_y(y_f - y_i)$ gives $v_{yh}^2 = 2gh + 2(-g)\left(\frac{1}{2}h - 0\right)$ so $v_{yh} = \sqrt{gh}$

At maximum height, the speed is $\qquad v_x = \frac{1}{2}\sqrt{v_x^2 + v_{yh}^2} = \frac{1}{2}\sqrt{v_x^2 + gh}$

Solving, $\qquad v_x = \sqrt{\dfrac{gh}{3}}$

Now the projection angle is $\qquad \theta_i = \tan^{-1}\dfrac{v_{yi}}{v_x} = \tan^{-1}\dfrac{\sqrt{2gh}}{\sqrt{gh/3}} = \tan^{-1}\sqrt{6} = \boxed{67.8°}.$

P3.14 We interpret the problem to mean that the displacement from fish to bug is

$$2.00\ \text{m at } 30° = (2.00\ \text{m})\cos 30°\,\hat{i} + (2.00\ \text{m})\sin 30°\,\hat{j} = (1.73\ \text{m})\hat{i} + (1.00\ \text{m})\hat{j}.$$

If the water should drop 0.03 m during its flight, then the fish must aim at a point 0.03 m above the bug. The initial velocity of the water then is directed through the point with displacement

$$(1.73\ \text{m})\hat{i} + (1.03\ \text{m})\hat{j} = 2.015\ \text{m at } 30.7°$$

For the time of flight of a water drop we have $x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$

$$1.73\ \text{m} = 0 + (v_i\cos 30.7°)t + 0$$

so $t = \dfrac{1.73\ \text{m}}{v_i\cos 30.7°}$. The vertical motion is described by $y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2$. The "drop on its path" is

$-3.00\ \text{cm} = \dfrac{1}{2}(-9.80\ \text{m/s}^2)\left(\dfrac{1.73\ \text{m}}{v_i\cos 30.7°}\right)^2$. Thus,

$$v_i = \frac{1.73\ \text{m}}{\cos 30.7°}\sqrt{\frac{9.80\ \text{m/s}^2}{2\times 0.03\ \text{m}}} = 2.015\ \text{m}(12.8\ \text{s}^{-1}) = \boxed{25.8\ \text{m/s}}.$$

P3.15 (a) We use the trajectory equation: $y_f = x_f \tan\theta_i - \dfrac{g x_f^2}{2v_i^2 \cos^2\theta_i}$. With $x_f = 36.0$ m, $v_i = 20.0$ m/s, and $\theta = 53.0°$ we find $y_f = (36.0 \text{ m})\tan 53.0° - \dfrac{(9.80 \text{ m/s}^2)(36.0 \text{ m})^2}{2(20.0 \text{ m/s})^2 \cos^2(53.0°)} = 3.94$ m. The ball clears the bar by $(3.94 - 3.05)$ m $= \boxed{0.889 \text{ m}}$.

(b) The time the ball takes to reach the maximum height is

$$t_1 = \frac{v_i \sin\theta_i}{g} = \frac{(20.0 \text{ m/s})(\sin 53.0°)}{9.80 \text{ m/s}^2} = 1.63 \text{ s}.$$

The time to travel 36.0 m horizontally is $t_2 = \dfrac{x_f}{v_{ix}}$

$$t_2 = \frac{36.0 \text{ m}}{(20.0 \text{ m/s})(\cos 53.0°)} = 2.99 \text{ s}.$$

Since $t_2 > t_1$ $\boxed{\text{the ball clears the goal on its way down}}$.

P3.16 The horizontal component of displacement is $x_f = v_{xi}t = (v_i \cos\theta_i)t$. Therefore, the time required to reach the building a distance d away is $t = \dfrac{d}{v_i \cos\theta_i}$. At this time, the altitude of the water is

$$y_f = v_{yi}t + \frac{1}{2}a_y t^2 = v_i \sin\theta_i \left(\frac{d}{v_i \cos\theta_i}\right) - \frac{g}{2}\left(\frac{d}{v_i \cos\theta_i}\right)^2.$$

Therefore the water strikes the building at a height h above ground level of

$$h = y_f = \boxed{d \tan\theta_i - \frac{g d^2}{2v_i^2 \cos^2\theta_i}}.$$

P3.17 (a) For the horizontal motion, we have

$$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$$
$$24 \text{ m} = 0 + v_i(\cos 53°)(2.2 \text{ s}) + 0$$
$$v_i = \boxed{18.1 \text{ m/s}}.$$

(b) As it passes over the wall, the ball is above the street by $y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2$

$$y_f = 0 + (18.1 \text{ m/s})(\sin 53°)(2.2 \text{ s}) + \frac{1}{2}(-9.8 \text{ m/s}^2)(2.2 \text{ s})^2 = 8.13 \text{ m}.$$

So it clears the parapet by $8.13 \text{ m} - 7 \text{ m} = \boxed{1.13 \text{ m}}$.

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(c) Note that the highest point of the ball's trajectory is not directly above the wall. For the whole flight, we have from the trajectory equation $y_f = (\tan\theta_i)x_f - \left(\dfrac{g}{2v_i^2 \cos^2\theta_i}\right)x_f^2$ or

$6 \text{ m} = (\tan 53°)x_f - \left(\dfrac{9.8 \text{ m/s}^2}{2(18.1 \text{ m/s})^2 \cos^2 53°}\right)x_f^2$. Solving, $(0.0412 \text{ m}^{-1})x_f^2 - 1.33x_f + 6 \text{ m} = 0$ and

$x_f = \dfrac{1.33 \pm \sqrt{1.33^2 - 4(0.0412)(6)}}{2(0.0412 \text{ m}^{-1})}$. This yields two results: $x_f = 26.8$ m or 5.44 m. The ball passes twice through the level of the roof. It hits the roof at distance from the wall $26.8 \text{ m} - 24 \text{ m} = \boxed{2.79 \text{ m}}$.

***P3.18** We match the given equations

$$x_f = 0 + (11.2 \text{ m/s})\cos 18.5° t$$
$$0.360 \text{ m} = 0.840 \text{ m} + (11.2 \text{ m/s})\sin 18.5° t - \frac{1}{2}(9.80 \text{ m/s}^2)t^2$$

to the equations for the coordinates of the final position of a projectile

$$x_f = x_i + v_{xi}t$$
$$y_f = y_i + v_{yi}t - \frac{1}{2}gt^2$$

For the equations to represent the same functions of time, all coefficients must agree: $x_i = 0$, $y_i = 0.840$ m, $v_{xi} = (11.2 \text{ m/s})\cos 18.5°$, $v_{yi} = (11.2 \text{ m/s})\sin 18.5°$ and $g = 9.80 \text{ m/s}^2$

(a) Then the original position of the athlete's center of mass is the point with coordinates $(x_i, y_i) = \boxed{(0, 0.840 \text{ m})}$. That is, his original position has position vector $\vec{r} = 0\hat{i} + 0.840 \text{ m}\hat{j}$.

(b) His original velocity is $\vec{v}_i = (11.2 \text{ m/s})\cos 18.5°\hat{i} + (11.2 \text{ m/s})\sin 18.5°\hat{j} = \boxed{11.2 \text{ m/s at } 18.5°}$ above the x-axis.

(c) From $4.90 \text{ m/s}^2 t^2 - 3.55 \text{ m/s}\, t - 0.48 \text{ m} = 0$ we find the time of flight, which must be positive

$$t = \frac{+3.55 \text{ m/s} + \sqrt{(3.55 \text{ m/s})^2 - 4(4.9 \text{ m/s}^2)(-0.48 \text{ m})}}{2(4.9 \text{ m/s}^2)} = 0.842 \text{ s}. \text{ Then}$$

$x_f = (11.2 \text{ m/s})\cos 18.5°(0.8425) = \boxed{8.94 \text{ m}}$.

(d)

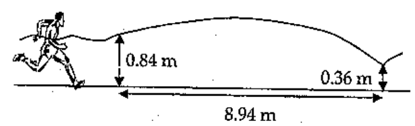


FIG. P3.18

P3.19 The horizontal kick gives zero vertical velocity to the rock. Then its time of flight follows from

$$y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2$$
$$-40.0 \text{ m} = 0 + 0 + \frac{1}{2}\left(-9.80 \text{ m/s}^2\right)t^2$$
$$t = 2.86 \text{ s}.$$

The extra time $3.00 \text{ s} - 2.86 \text{ s} = 0.143 \text{ s}$ is the time required for the sound she hears to travel straight back to the player.

It covers distance

$$(343 \text{ m/s})0.143 \text{ s} = 49.0 \text{ m} = \sqrt{x^2 + (40.0 \text{ m})^2}$$

where x represents the horizontal distance the rock travels.

$$x = 28.3 \text{ m} = v_{xi}t + 0t^2$$
$$\therefore v_{xi} = \frac{28.3 \text{ m}}{2.86 \text{ s}} = \boxed{9.91 \text{ m/s}}$$

P3.20 From the instant he leaves the floor until just before he lands, the basketball star is a projectile. His vertical velocity and vertical displacement are related by the equation $v_{yf}^2 = v_{yi}^2 + 2a_y(y_f - y_i)$. Applying this to the upward part of his flight gives $0 = v_{yi}^2 + 2\left(-9.80 \text{ m/s}^2\right)(1.85 - 1.02) \text{ m}$. From this, $v_{yi} = 4.03 \text{ m/s}$. [Note that this is the answer to part (c) of this problem.]

For the downward part of the flight, the equation gives $v_{yf}^2 = 0 + 2\left(-9.80 \text{ m/s}^2\right)(0.900 - 1.85) \text{ m}$. Thus the vertical velocity just before he lands is

$$v_{yf} = -4.32 \text{ m/s}.$$

(a) His hang time may then be found from $v_{yf} = v_{yi} + a_y t$:

$$-4.32 \text{ m/s} = 4.03 \text{ m/s} + \left(-9.80 \text{ m/s}^2\right)t$$

or $t = \boxed{0.852 \text{ s}}$.

(b) Looking at the total horizontal displacement during the leap, $x = v_{xi}t$ becomes

$$2.80 \text{ m} = v_{xi}(0.852 \text{ s})$$

which yields $v_{xi} = \boxed{3.29 \text{ m/s}}$.

(c) $v_{yi} = \boxed{4.03 \text{ m/s}}$. See above for proof.

(d) The takeoff angle is: $\theta = \tan^{-1}\left(\frac{v_{yi}}{v_{xi}}\right) = \tan^{-1}\left(\frac{4.03 \text{ m/s}}{3.29 \text{ m/s}}\right) = \boxed{50.8°}$.

continued on next page

(e) Similarly for the deer, the upward part of the flight gives $v_{yf}^2 = v_{yi}^2 + 2a_y(y_f - y_i)$:

$$0 = v_{yi}^2 + 2\left(-9.80 \text{ m/s}^2\right)(2.50 - 1.20) \text{ m}$$

so $v_{yi} = 5.04 \text{ m/s}$.

For the downward part, $v_{yf}^2 = v_{yi}^2 + 2a_y(y_f - y_i)$ yields $v_{yf}^2 = 0 + 2\left(-9.80 \text{ m/s}^2\right)(0.700 - 2.50) \text{ m}$ and $v_{yf} = -5.94 \text{ m/s}$.

The hang time is then found as $v_{yf} = v_{yi} + a_y t$: $-5.94 \text{ m/s} = 5.04 \text{ m/s} + \left(-9.80 \text{ m/s}^2\right)t$ and

$$\boxed{t = 1.12 \text{ s}}.$$

P3.21 For the smallest impact angle

$$\theta = \tan^{-1}\left(\frac{v_{yf}}{v_{xf}}\right),$$

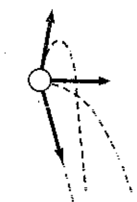


FIG. P3.21

we want to minimize v_{yf} and maximize $v_{xf} = v_{xi}$. The final y-component of velocity is related to v_{yi} by $v_{yf}^2 = v_{yi}^2 + 2gh$, so we want to minimize v_{yi} and maximize v_{xi}. Both are accomplished by making the initial velocity horizontal. Then $v_{xi} = v$, $v_{yi} = 0$, and $v_{yf} = \sqrt{2gh}$. At last, the impact angle is

$$\theta = \tan^{-1}\left(\frac{v_{yf}}{v_{xf}}\right) = \boxed{\tan^{-1}\left(\frac{\sqrt{2gh}}{v}\right)}.$$

Section 3.4 The Particle in Uniform Circular Motion

P3.22 $a = \frac{v^2}{R}$, $T = 24 \text{ h}(3\,600 \text{ s/h}) = 86\,400 \text{ s}$

$$v = \frac{2\pi R}{T} = \frac{2\pi(6.37 \times 10^6 \text{ m})}{86\,400 \text{ s}} = 463 \text{ m/s}$$

$$a = \frac{(463 \text{ m/s})^2}{6.37 \times 10^6 \text{ m}} = \boxed{0.033\,7 \text{ m/s}^2 \text{ directed toward the center of Earth}}.$$

P3.23 $a_c = \frac{v^2}{r} = \frac{(20.0 \text{ m/s})^2}{1.06 \text{ m}} = \boxed{377 \text{ m/s}^2}$

The mass is unnecessary information.

*P3.24 $a_r = \frac{v^2}{r}$. Let f represent the rotation rate. Each revolution carries each bit of metal through distance $2\pi r$, so $v = 2\pi r f$ and

$$a_r = \frac{v^2}{r} = 4\pi^2 r f^2 = 100\,g.$$

A smaller radius implies smaller acceleration. To meet the criterion for each bit of metal we consider the minimum radius:

$$f = \left(\frac{100\,g}{4\pi^2 r}\right)^{1/2} = \left(\frac{100 \cdot 9.8 \text{ m/s}^2}{4\pi^2(0.021 \text{ m})}\right)^{1/2} = 34.4\,\frac{1}{\text{s}}\left(\frac{60 \text{ s}}{1 \text{ min}}\right) = \boxed{2.06\times10^3 \text{ rev/min}}.$$

P3.25 $r = 0.500$ m;

$$v_t = \frac{2\pi r}{T} = \frac{2\pi(0.500 \text{ m})}{\frac{60.0 \text{ s}}{200 \text{ rev}}} = 10.47 \text{ m/s} = \boxed{10.5 \text{ m/s}}$$

$$a = \frac{v^2}{R} = \frac{(10.47)^2}{0.5} = \boxed{219 \text{ m/s}^2 \text{ inward}}$$

P3.26 $a_r = \frac{v^2}{r}$

$$v = \sqrt{a_r r} = \sqrt{3(9.8 \text{ m/s}^2)(9.45 \text{ m})} = 16.7 \text{ m/s}$$

Each revolution carries the astronaut over a distance of $2\pi r = 2\pi(9.45 \text{ m}) = 59.4$ m. Then the rotation rate is

$$16.7 \text{ m/s}\left(\frac{1 \text{ rev}}{59.4 \text{ m}}\right) = \boxed{0.281 \text{ rev/s}}.$$

P3.27 The satellite is in free fall. Its acceleration is due to gravity and is by effect a centripetal acceleration.

$$a_r = g$$

so $\frac{v^2}{r} = g$. Solving for the velocity, $v = \sqrt{rg} = \sqrt{(6\,400+600)(10^3 \text{ m})(8.21 \text{ m/s}^2)} = \boxed{7.58\times10^3 \text{ m/s}}$.

$v = \frac{2\pi r}{T}$ and

$$T = \frac{2\pi r}{v} = \frac{2\pi(7\,000\times10^3 \text{ m})}{7.58\times10^3 \text{ m/s}} = \boxed{5.80\times10^3 \text{ s}}$$

$$T = 5.80\times10^3 \text{ s}\left(\frac{1 \text{ min}}{60 \text{ s}}\right) = 96.7 \text{ min}.$$

Section 3.5 Tangential and Radial Acceleration

P3.28 (b) We do part (b) first. The tangential speed is described by $v_f = v_i + a_t t$

$0.7 \text{ m/s} = 0 + a_t(1.75 \text{ s})$ so $\boxed{a_t = 0.400 \text{ m/s}^2 \text{ forward}}$

(a) Now at $t = 1.25$ s, $v_f = v_i + a_t t = 0 + (0.4 \text{ m/s}^2)1.25 \text{ s}$

$v_f = 0.5$ m/s

so $a_r = \frac{v^2}{r} = \frac{(0.5 \text{ m/s})^2}{0.2 \text{ m}} = \boxed{1.25 \text{ m/s}^2 \text{ toward the center}}$

(c) $\vec{a} = \vec{a}_r + \vec{a}_t = 0.4 \text{ m/s}^2$ forward $+ 1.25 \text{ m/s}^2$ inward

$\vec{a} = \sqrt{0.4^2 + 1.25^2}$ forward and inward at $\theta = \tan^{-1}\left(\frac{1.25}{0.4}\right)$

$\vec{a} = \boxed{1.31 \text{ m/s}^2 \text{ forward and } 72.3° \text{ inward}}$

P3.29 We assume the train is still slowing down at the instant in question.

$$a_r = \frac{v^2}{r} = 1.29 \text{ m/s}^2$$

$$a_t = \frac{\Delta v}{\Delta t} = \frac{(-40.0 \text{ km/h})(10^3 \text{ m/km})\left(\frac{1 \text{ h}}{3\,600 \text{ s}}\right)}{15.0 \text{ s}} = -0.741 \text{ m/s}^2$$

$$a = \sqrt{a_r^2 + a_t^2} = \sqrt{(1.29 \text{ m/s}^2)^2 + (-0.741 \text{ m/s}^2)^2}$$

at an angle of $\tan^{-1}\left(\frac{|a_t|}{a_c}\right) = \tan^{-1}\left(\frac{0.741}{1.29}\right)$

$$\vec{a} = \boxed{1.48 \text{ m/s}^2 \text{ inward and } 29.9° \text{ backward}}$$

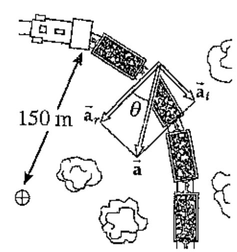


FIG. P3.29

P3.30 (a) See figure to the right.

(b) The components of the 20.2 and the 22.5 m/s² along the rope together constitute the centripetal acceleration:

$$a_c = (22.5 \text{ m/s}^2)\cos(90.0° - 36.9°) + (20.2 \text{ m/s}^2)\cos 36.9° = \boxed{29.7 \text{ m/s}^2}$$

(c) $a_c = \frac{v^2}{r}$ so $v = \sqrt{a_c r} = \sqrt{29.7 \text{ m/s}^2 (1.50 \text{ m})} = 6.67$ m/s tangent to circle

$\vec{v} = \boxed{6.67 \text{ m/s at } 36.9° \text{ above the horizontal}}$

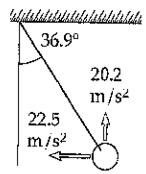


FIG. P3.30

P3.31 $r = 2.50\ \text{m},\ a = 15.0\ \text{m/s}^2$

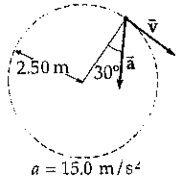


FIG. P3.31

(a) $a_c = a\cos 30.0° = (15.0\ \text{m/s}^2)(\cos 30°) = \boxed{13.0\ \text{m/s}^2}$

(b) $a_c = \dfrac{v^2}{r}$

so $v^2 = ra_c = 2.50\ \text{m}(13.0\ \text{m/s}^2) = 32.5\ \text{m}^2/\text{s}^2$

$v = \sqrt{32.5}\ \text{m/s} = \boxed{5.70\ \text{m/s}}$

(c) $a^2 = a_t^2 + a_r^2$

so $a_t = \sqrt{a^2 - a_r^2} = \sqrt{(15.0\ \text{m/s}^2)^2 - (13.0\ \text{m/s}^2)^2} = \boxed{7.50\ \text{m/s}^2}$

Section 3.6 Relative Velocity

P3.32 The bumpers are initially 100 m = 0.100 km apart. After time t the bumper of the leading car travels 40.0 t, while the bumper of the chasing car travels 60.0t.

Since the cars are side by side at time t, we have

$$0.100 + 40.0t = 60.0t,$$

yielding

$$t = 5.00 \times 10^{-3}\ \text{h} = \boxed{18.0\ \text{s}}.$$

P3.33 Total time in still water $t = \dfrac{d}{v} = \dfrac{2\,000}{1.20} = 1.67 \times 10^3\ \text{s}$.

Total time = time upstream plus time downstream:

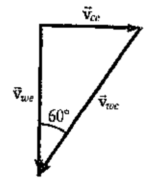
$$t_{\text{up}} = \frac{1\,000}{(1.20 - 0.500)} = 1.43 \times 10^3\ \text{s}$$
$$t_{\text{down}} = \frac{1\,000}{1.20 + 0.500} = 588\ \text{s}.$$

Therefore, $t_{\text{total}} = 1.43 \times 10^3 + 588 = \boxed{2.02 \times 10^3\ \text{s, 21.0\% more than if the water were still.}}$.

***P3.34** $\vec{v}_{ce}$ = the velocity of the car relative to the earth.

$\vec{v}_{wc}$ = the velocity of the water relative to the car.

$\vec{v}_{we}$ = the velocity of the water relative to the earth.



$\vec{v}_{we} = \vec{v}_{ce} + \vec{v}_{wc}$

FIG. P3.34

These velocities are related as shown in the diagram at the right.

(a) Since $\vec{v}_{we}$ is vertical, $v_{wc}\sin 60.0° = v_{ce} = 50.0\ \text{km/h}$ or
$\vec{v}_{wc} = \boxed{57.7\ \text{km/h at } 60.0°\ \text{west of vertical}}$

(b) Since $\vec{v}_{ce}$ has zero vertical component,

$$v_{we} = v_{wc}\cos 60.0° = (57.7\ \text{km/h})\cos 60.0° = \boxed{28.9\ \text{km/h downward}}.$$

P3.35 $v = \sqrt{150^2 + 30.0^2} = \boxed{153\ \text{km/h}}$

$\theta = \tan^{-1}\left(\dfrac{30.0}{150}\right) = \boxed{11.3°\ \text{north of west}}$

P3.36 For Alan, his speed downstream is $c + v$, while his speed upstream is $c - v$.

Therefore, the total time for Alan is

$$t_1 = \frac{L}{c+v} + \frac{L}{c-v} = \frac{\frac{2L}{c}}{1 - \frac{v^2}{c^2}}.$$

For Beth, her cross-stream speed (both ways) is

$$\sqrt{c^2 - v^2}$$

Thus, the total time for Beth is $t_2 = \dfrac{2L}{\sqrt{c^2 - v^2}} = \dfrac{\frac{2L}{c}}{\sqrt{1 - \frac{v^2}{c^2}}}$.

Since $1 - \dfrac{v^2}{c^2} < 1$, $t_1 > t_2$, or $\boxed{\text{Beth, who swims cross-stream, returns first}}$.

P3.37 Identify the student as the S' observer and the professor as the S observer. For the initial motion in S', we have

$$\frac{v'_y}{v'_x} = \tan 60.0° = \sqrt{3}.$$

Let u represent the speed of S' relative to S. Then because there is no x-motion in S, we can write $v_x = v'_x + u = 0$ so that $v'_x = -u = -10.0$ m/s. Hence the ball is thrown backwards in S'. Then,

$$v_y = v'_y = \sqrt{3}\left|v'_x\right| = 10.0\sqrt{3} \text{ m/s}.$$

Using $v_y^2 = 2gh$ we find

$$h = \frac{\left(10.0\sqrt{3} \text{ m/s}\right)^2}{2\left(9.80 \text{ m/s}^2\right)} = \boxed{15.3 \text{ m}}$$

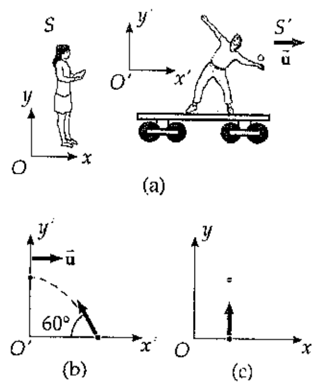


FIG. P3.37

The motion of the ball as seen by the student in S' is shown in diagram (b). The view of the professor in S is shown in diagram (c).

***P3.38** Choose the x-axis along the 20-km distance. The y-components of the displacements of the ship and the speedboat must agree:

$$(26 \text{ km/h})t \sin(40° - 15°) = (50 \text{ km/h})t \sin\alpha$$

$$\alpha = \sin^{-1}\frac{11.0}{50} = 12.7°$$

The speedboat should head

$$15° + 12.7° = \boxed{27.7° \text{ east of north}}.$$

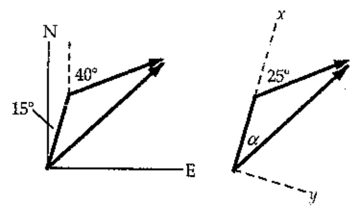


FIG. P3.38

Section 3.7 Context Connection—Lateral Acceleration of Automobiles

***P3.39** $a = \frac{v^2}{r} = \frac{\left[139 \text{ km/h}(1 \text{ h}/3\,600 \text{ s})(1\,000 \text{ m}/1 \text{ km})\right]^2}{156 \text{ m}} = 9.56 \text{ m/s}^2 = 9.56 \text{ m/s}^2\left(\frac{1g}{9.80 \text{ m/s}^2}\right) = \boxed{0.975g}$

This agrees to two digits with the tabulated $0.98g$.

***P3.40** We assume that in both cornering processes the vehicle can have the same maximum acceleration.

Then $a = \frac{v_1^2}{r_1} = \frac{v_2^2}{r_2}$. $\frac{(32 \text{ m/s})^2}{150 \text{ m}} = \frac{v_2^2}{75 \text{ m}}$ and $v_2 = \sqrt{0.5}(32 \text{ m/s}) = \boxed{22.6 \text{ m/s}}$.

Additional Problems

***P3.41** (a) The speed at the top is $v_x = v_i \cos\theta_i = (143 \text{ m/s})\cos 45° = \boxed{101 \text{ m/s}}$.

(b) In free fall the plane reaches altitude given by

$$v_{yf}^2 = v_{yi}^2 + 2a_y(y_f - y_i)$$

$$0 = (143 \text{ m/s} \sin 45°)^2 + 2(-9.8 \text{ m/s}^2)(y_f - 31\,000 \text{ ft})$$

$$y_f = 31\,000 \text{ ft} + 522 \text{ m}\left(\frac{3.28 \text{ ft}}{1 \text{ m}}\right) = \boxed{3.27 \times 10^4 \text{ ft}}.$$

(c) For the whole free fall motion $v_{yf} = v_{yi} + a_y t$

$$-101 \text{ m/s} = +101 \text{ m/s} - (9.8 \text{ m/s}^2)t$$

$$t = \boxed{20.6 \text{ s}}$$

(d) $a_c = \frac{v^2}{r}$

$$v = \sqrt{a_c r} = \sqrt{0.8(9.8 \text{ m/s}^2)4{,}130 \text{ m}} = \boxed{180 \text{ m/s}}$$

***P3.42** The time of flight of a water drop is given by $y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2$

$$0 = 2.35 \text{ m} + 0 - \frac{1}{2}(9.8 \text{ m/s}^2)t_1^2$$

For $t_1 > 0$, the root is $t_1 = \sqrt{\frac{2(2.35 \text{ m})}{9.8 \text{ m/s}^2}} = 0.693$ s.

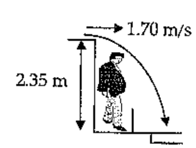


FIG. P3.42

(a) The horizontal range of the font is

$$x_{f1} = x_i + v_{xi}t + \frac{1}{2}a_x t^2$$

$$= 0 + 1.70 \text{ m/s}(0.693 \text{ s}) + 0 = 1.18 \text{ m}.$$

This is about the width of a town sidewalk, so $\boxed{\text{there is}}$ space for a walkway behind the waterfall. Unless the lip of the channel is well designed, water may drip on the visitors. A tall or inattentive person may get his head wet.

(b) Now the flight time t_2 is given by $0 = y_2 + 0 - \frac{1}{2}gt_2^2$. $t_2 = \sqrt{\frac{2y_2}{g}} = \sqrt{\frac{2y_1}{g(12)}} = \frac{1}{\sqrt{12}}\sqrt{\frac{2y_1}{g}} = \frac{t_1}{\sqrt{12}}$.

From the same equation as in part (a) for horizontal range, $x_2 = v_2 t_2$.

$$\frac{x_1}{12} = v_2\frac{t_1}{\sqrt{12}} \qquad v_2 = \frac{x_1}{t_1\sqrt{12}} = \frac{v_1}{\sqrt{12}} = \frac{1.70 \text{ m/s}}{\sqrt{12}} = \boxed{0.491 \text{ m/s}}$$

The rule that the scale factor for speed is the square root of the scale factor for distance is Froude's law, published in 1870.

P3.43 After the string breaks the ball is a projectile, and reaches the ground at time t: $y_f = v_{yi}t + \frac{1}{2}a_y t^2$

$$-1.20\text{ m} = 0 + \frac{1}{2}\left(-9.80\text{ m/s}^2\right)t^2$$

so $t = 0.495$ s.

Its constant horizontal speed is $v_x = \frac{x}{t} = \frac{2.00\text{ m}}{0.495\text{ s}} = 4.04\text{ m/s}$

so before the string breaks $a_c = \frac{v_x^2}{r} = \frac{(4.04\text{ m/s})^2}{0.300\text{ m}} = \boxed{54.4\text{ m/s}^2}$.

P3.44 (a) $y_f = (\tan\theta_i)(x_f) - \frac{g}{2v_i^2\cos^2\theta_i}x_f^2$

Setting $x_f = d\cos\phi$, and $y_f = d\sin\phi$, we have

$$d\sin\phi = (\tan\theta_i)(d\cos\phi) - \frac{g}{2v_i^2\cos^2\theta_i}(d\cos\phi)^2.$$

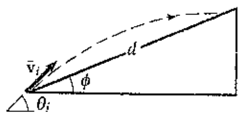


FIG. P3.44

Solving for d yields, $d = \frac{2v_i^2\cos\theta_i\left[\sin\theta_i\cos\phi - \sin\phi\cos\theta_i\right]}{g\cos^2\phi}$

or $d = \boxed{\frac{2v_i^2\cos\theta_i\sin(\theta_i - \phi)}{g\cos^2\phi}}$

(b) Setting $\frac{dd}{d\theta_i} = 0$ leads to $\boxed{\theta_i = 45° + \frac{\phi}{2}}$ and $\boxed{d_{max} = \frac{v_i^2(1-\sin\phi)}{g\cos^2\phi}}$.

P3.45 Refer to the sketch:

(b) $\Delta x = v_{xi}t$; substitution yields $130 = (v_i\cos 35.0°)t$.

$\Delta y = v_{yi}t + \frac{1}{2}at^2$; substitution yields

$$20.0 = (v_i\sin 35.0°)t + \frac{1}{2}(-9.80)t^2$$

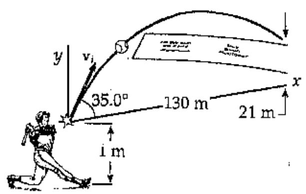


FIG. P3.45

Solving gives $t^2 = \frac{130\tan 35° - 20}{4.9} \Rightarrow t = \boxed{3.81\text{ s}}$

(a) $v_i = \boxed{41.7\text{ m/s}}$

continued on next page

(c) $v_{yf} = v_i\sin\theta_i - gt$, $v_x = v_i\cos\theta_i$

At $t = 3.81$ s, $v_{yf} = 41.7\sin 35.0° - (9.80)(3.81) = \boxed{-13.4\text{ m/s}}$

$$v_x = (41.7\cos 35.0°) = \boxed{34.1\text{ m/s}}$$

$$v_f = \sqrt{v_x^2 + v_{yf}^2} = \boxed{36.7\text{ m/s}}.$$

P3.46 (a) The moon's gravitational acceleration is the probe's centripetal acceleration: (For the moon's radius, see end papers of text.)

$$a = \frac{v^2}{r}$$

$$\frac{1}{6}\left(9.80\text{ m/s}^2\right) = \frac{v^2}{1.74\times10^6\text{ m}}$$

$$v = \sqrt{2.84\times10^6\text{ m}^2/\text{s}^2} = \boxed{1.69\text{ km/s}}$$

(b) $v = \frac{2\pi r}{T}$

$$T = \frac{2\pi r}{v} = \frac{2\pi(1.74\times10^6\text{ m})}{1.69\times10^3\text{ m/s}} = 6.47\times10^3\text{ s} = \boxed{1.80\text{ h}}$$

P3.47 $x_f = v_{xi}t = v_i t\cos 40.0°$

Thus, when $x_f = 10.0$ m, $t = \frac{10.0\text{ m}}{v_i\cos 40.0°}$

At this time, y_f should be $3.05\text{ m} - 2.00\text{ m} = 1.05$ m.

Thus, $1.05\text{ m} = \frac{(v_i\sin 40.0°)10.0\text{ m}}{v_i\cos 40.0°} + \frac{1}{2}\left(-9.80\text{ m/s}^2\right)\left[\frac{10.0\text{ m}}{v_i\cos 40.0°}\right]^2$.

From this, $v_i = \boxed{10.7\text{ m/s}}$.

P3.48 The special conditions allowing use of the horizontal range equation apply.
For the ball thrown at 45°,

$$D = R_{45} = \frac{v_i^2\sin 90}{g}.$$

For the bouncing ball,

$$D = R_1 + R_2 = \frac{v_i^2\sin 2\theta}{g} + \frac{\left(\frac{v_i}{2}\right)^2\sin 2\theta}{g}$$

where θ is the angle it makes with the ground when thrown and when bouncing.

continued on next page

(a) We require;

$$\frac{v_i^2}{g} = \frac{v_i^2 \sin 2\theta}{g} + \frac{v_i^2 \sin 2\theta}{4g}$$

$$\sin 2\theta = \frac{4}{5}$$

$$\boxed{\theta = 26.6°}$$

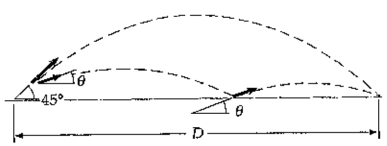


FIG. P3.48

(b) The time for any symmetric parabolic flight is given by

$$y_f = v_{yi}t - \frac{1}{2}gt^2$$
$$0 = v_i \sin\theta_i t - \frac{1}{2}gt^2$$

If $t = 0$ is the time the ball is thrown, then $t = \frac{2v_i \sin\theta_i}{g}$ is the time at landing.

So for the ball thrown at 45.0°

$$t_{45} = \frac{2v_i \sin 45.0°}{g}$$

For the bouncing ball,

$$t = t_1 + t_2 = \frac{2v_i \sin 26.6°}{g} + \frac{2\left(\frac{v_i}{2}\right)\sin 26.6°}{g} = \frac{3v_i \sin 26.6°}{g}$$

The ratio of this time to that for no bounce is

$$\frac{\frac{3v_i \sin 26.6°}{g}}{\frac{2v_i \sin 45.0°}{g}} = \frac{1.34}{1.41} = \boxed{0.949}.$$

P3.49 (a) $\Delta y = -\frac{1}{2}gt^2$; $\Delta x = v_i t$

Combine the equations eliminating t:

$$\Delta y = -\frac{1}{2}g\left(\frac{\Delta x}{v_i}\right)^2.$$

From this, $(\Delta x)^2 = \left(\frac{-2\Delta y}{g}\right)v_i^2$

thus $\Delta x = v_i\sqrt{\frac{-2\Delta y}{g}} = 275\sqrt{\frac{-2(-3\,000)}{9.80}} = 6.80\times10^3 = \boxed{6.80\text{ km}}$.

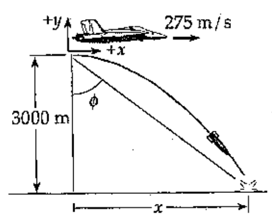


FIG. P3.49

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(b) The plane has the same velocity as the bomb in the x direction. Therefore, the plane will be $\boxed{\text{3 000 m directly above the bomb}}$ when it hits the ground.

(c) When ϕ is measured from the vertical, $\tan\phi = \frac{\Delta x}{\Delta y}$

therefore, $\phi = \tan^{-1}\left(\frac{\Delta x}{\Delta y}\right) = \tan^{-1}\left(\frac{6\,800}{3\,000}\right) = \boxed{66.2°}$.

P3.50 Measure heights above the level ground. The elevation y_b of the ball follows

$$y_b = R + 0 - \frac{1}{2}gt^2$$

with $x = v_i t$ so $y_b = R - \frac{gx^2}{2v_i^2}$.

(a) The elevation y_r of points on the rock is described by

$$y_r^2 + x^2 = R^2.$$

We will have $y_b = y_r$ at $x = 0$, but for all other x we require the ball to be above the rock surface as in $y_b > y_r$. Then $y_b^2 + x^2 > R^2$

$$\left(R - \frac{gx^2}{2v_i^2}\right)^2 + x^2 > R^2$$

$$R^2 - \frac{gx^2R}{v_i^2} + \frac{g^2x^4}{4v_i^4} + x^2 > R^2$$

$$\frac{g^2x^4}{4v_i^4} + x^2 > \frac{gx^2R}{v_i^2}$$

If this inequality is satisfied for x approaching zero, it will be true for all x. If the ball's parabolic trajectory has large enough radius of curvature at the start, the ball will clear the whole rock: $1 > \frac{gR}{v_i^2}$

$$\boxed{v_i > \sqrt{gR}}.$$

(b) With $v_i = \sqrt{gR}$ and $y_b = 0$, we have $0 = R - \frac{gx^2}{2gR}$

or $x = R\sqrt{2}$.

The distance from the rock's base is

$$x - R = \boxed{\left(\sqrt{2} - 1\right)R}.$$

P3.51 (a) While on the incline

$$v_f^2 - v_i^2 = 2a\Delta x$$
$$v_f - v_i = at$$
$$v_f^2 - 0 = 2(4.00)(50.0)$$
$$20.0 - 0 = 4.00t$$
$$v_f = \boxed{20.0\text{ m/s}}$$
$$t = \boxed{5.00\text{ s}}$$

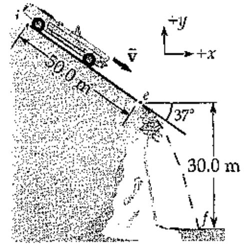

FIG. P3.51

(b) Initial free-flight conditions give us

$$v_{xi} = 20.0\cos 37.0° = 16.0\text{ m/s}$$

and

$v_{yi} = -20.0\sin 37.0° = -12.0$ m/s

$v_{xf} = v_{xi}$ since $a_x = 0$

$v_{yf} = -\sqrt{2a_y\Delta y + v_{yi}^2} = -\sqrt{2(-9.80)(-30.0) + (-12.0)^2} = -27.1$ m/s

$v_f = \sqrt{v_{xf}^2 + v_{yf}^2} = \sqrt{(16.0)^2 + (-27.1)^2} = \boxed{31.5\text{ m/s at }59.4°\text{ below the horizontal}}$

(c) $t_1 = 5\text{ s};\ t_2 = \dfrac{v_{yf} - v_{yi}}{a_y} = \dfrac{-27.1 + 12.0}{-9.80} = 1.53\text{ s}$

$t = t_1 + t_2 = \boxed{6.53\text{ s}}$

(d) $\Delta x = v_{xi}t_2 = 16.0(1.53) = \boxed{24.5\text{ m}}$

P3.52 Equation of bank: $y^2 = 16x$ (1)

Equations of motion: $x = v_i t$ (2)

$y = -\dfrac{1}{2}gt^2$ (3)

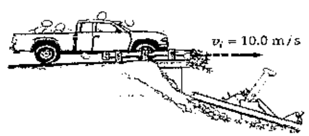

FIG. P3.52

Substitute for t from (2) into (3) $y = -\dfrac{1}{2}g\left(\dfrac{x^2}{v_i^2}\right)$. Equate y from the bank equation to y from the equations of motion:

$$16x = \left[-\frac{1}{2}g\left(\frac{x^2}{v_i^2}\right)\right]^2 \Rightarrow \frac{g^2x^4}{4v_i^4} - 16x = x\left(\frac{g^2x^3}{4v_i^4} - 16\right) = 0.$$

continued on next page

From this, $x = 0$ or $x^3 = \dfrac{64v_i^4}{g^2}$ and $x = 4\left(\dfrac{10^4}{9.80^2}\right)^{1/3} = \boxed{18.8\text{ m}}$. Also,

$$y = -\frac{1}{2}g\left(\frac{x^2}{v_i^2}\right) = -\frac{1}{2}\frac{(9.80)(18.8)^2}{(10.0)^2} = \boxed{-17.3\text{ m}}.$$

P3.53 (a) Coyote: $\Delta x = \dfrac{1}{2}at^2;\quad 70.0 = \dfrac{1}{2}(15.0)t^2$

Roadrunner: $\Delta x = v_i t;\quad 70.0 = v_i t$

Solving the above, we get $v_i = \boxed{22.9\text{ m/s}}$ and $t = 3.06$ s.

(b) At the edge of the cliff, $v_{xi} = at = (15.0)(3.06) = 45.8$ m/s.

Substituting into $\Delta y = \dfrac{1}{2}a_y t^2$, we find

$$-100 = \frac{1}{2}(-9.80)t^2$$
$$t = 4.52\text{ s}$$
$$\Delta x = v_{xi}t + \frac{1}{2}a_x t^2 = (45.8)(4.52\text{ s}) + \frac{1}{2}(15.0)(4.52\text{ s})^2$$

Solving, $\Delta x = \boxed{360\text{ m}}$.

(c) For the Coyote's motion through the air

$$v_{xf} = v_{xi} + a_x t = 45.8 + 15(4.52) = \boxed{114\text{ m/s}}$$
$$v_{yf} = v_{yi} + a_y t = 0 - 9.80(4.52) = \boxed{-44.3\text{ m/s}}.$$

P3.54 Equation 3.15: $h = \dfrac{v_i^2\sin^2\theta_i}{2g}$

Equation 3.16: $R = \dfrac{v_i^2\sin 2\theta_i}{g} = \dfrac{2v_i^2\sin\theta_i\cos\theta_i}{g}$

If $h = \dfrac{R}{6}$, Equation 3.15 yields $v_i\sin\theta_i = \sqrt{\dfrac{gR}{3}}$ (1)

Substituting Equation (1) above into Equation 3.16 gives $R = \dfrac{2\left(\sqrt{gR/3}\right)v_i\cos\theta_i}{g}$

which reduces to $v_i\cos\theta_i = \dfrac{1}{2}\sqrt{3gR}$ (2)

continued on next page

(a) From $v_{yf} = v_{yi} + a_y t$, the time to reach the peak of the path (where $v_{yf} = 0$) is found to be

$$t_{peak} = v_i \sin\theta_i / g$$

Using Equation (1), this gives $t_{peak} = \sqrt{\dfrac{R}{3g}}$

The total time of the ball's flight is then $\boxed{t_{flight} = 2t_{peak} = 2\sqrt{\dfrac{R}{3g}}}$

(b) At the path's peak, the ball moves horizontally with speed $v_{peak} = v_{xi} = v_i \cos\theta_i$

Using Equation (1), this becomes $v_{peak} = \boxed{\dfrac{1}{2}\sqrt{3gR}}$

(c) The initial vertical component of velocity is $v_{yi} = v_i \sin\theta_i$

From Equation (1), $v_{yi} = \boxed{\sqrt{\dfrac{gR}{3}}}$

(d) Squaring Eq. (1) and (2) and adding the results, $v_i^2\left(\sin^2\theta_i + \cos^2\theta_i\right) = \dfrac{gR}{3} + \dfrac{3gR}{4} = \dfrac{13gR}{12}$

Thus, the initial speed is $v_i = \boxed{\sqrt{\dfrac{13gR}{12}}}$

(e) Dividing Equation (1) by (2) yields $\tan\theta_i = \dfrac{v_i \sin\theta_i}{v_i \cos\theta_i} = \left[\dfrac{\left(\sqrt{gR/3}\right)}{\left(\frac{1}{2}\sqrt{3gR}\right)}\right] = \dfrac{2}{3}$

Therefore, $\theta_i = \tan^{-1}\left(\dfrac{2}{3}\right) = \boxed{33.7°}$

(f) For a given initial speed, the projection angle yielding maximum peak height is $\theta_i = 90.0°$. With the speed found in (d), Equation 3.15 then yields

$$h_{max} = \frac{(13gR/12)\sin^2 90.0°}{2g} = \boxed{\frac{13}{24}R}$$

(g) For a given initial speed, the projection angle yielding maximum range is $\theta_i = 45.0°$. With the speed found in (d), Equation 3.16 then gives

$$R_{max} = \frac{(13gR/12)\sin 90.0°}{g} = \boxed{\frac{13}{12}R}$$

P3.55 Consider the rocket's trajectory in 3 parts as shown in the sketch. Our initial conditions give:

$a_y = 30.0\sin 53.0° = 24.0 \text{ m/s}^2$

$a_x = 30.0\cos 53.0° = 18.1 \text{ m/s}^2$

$v_{yi} = 100\sin 53.0° = 79.9 \text{ m/s}$

$v_{xi} = 100\cos 53.0° = 60.2 \text{ m/s}$

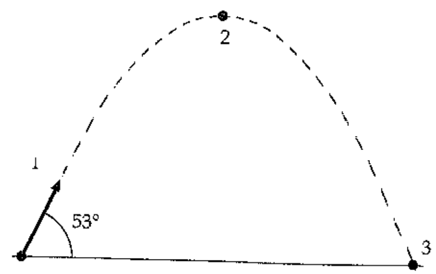

FIG. P3.55

The distances traveled during each phase of the motion are given in the table.

Path #1: $v_{yf} - 79.9 = 24.0(3.00)$ or $v_{yf} = 152 \text{ m/s}$

$v_{xf} - 60.2 = 18.1(3.00)$ or $v_{xf} = 114 \text{ m/s}$

$\Delta y = 79.9(3.00) + \frac{1}{2}(24.0)(3.00)^2 = 347 \text{ m}$

$\Delta x = 60.2(3.00) + \frac{1}{2}(18.1)(3.00)^2 = 262 \text{ m}$

Path #2: $a_x = 0$, $v_{xf} = v_{xi} = 114 \text{ m/s}$

$0 - 152 = -(9.80)t$ or $t = 15.5 \text{ s}$

$\Delta x = 114(15.5) = 1.77\times 10^3 \text{ m};$

$\Delta y = 152(15.5) - \frac{1}{2}(9.80)(15.5)^2 = 1.17\times 10^3$

Path #3: $\left(v_{yf}\right)^2 - 0 = 2(-9.80)\left(-1.52\times 10^3\right)$

$v_{yf} = -173 \text{ m/s}$ $v_{xf} = v_{xi} = 114 \text{ m/s, since } a_x = 0$

$-173 - 0 = -(9.80)t$ or $t = 17.6 \text{ s}$

$\Delta x = 114(17.6) = 2.02\times 10^3 \text{ m}$

continued on next page

(a) $\Delta y(\max) = \boxed{1.52 \times 10^{3} \text{ m}}$

(b) $t(\text{net}) = 3.00 + 15.5 + 17.6 = \boxed{36.1 \text{ s}}$

(c) $\Delta x(\text{net}) = 262 + 1.77 \times 10^{3} + 2.02 \times 10^{3}$

$\Delta x(\text{net}) = \boxed{4.05 \times 10^{3} \text{ m}}$

	Path Part #1	#2	#3
a_y	24.0	−9.80	−9.80
a_x	18.1	0.0	0.00
v_{yf}	152	0.0	−173
v_{xf}	114	114	114
v_{yi}	79.9	152	0.00
v_{xi}	60.2	114	114
Δy	347	1.17×10^{3}	-1.52×10^{3}
Δx	262	1.77×10^{3}	2.02×10^{3}
t	3.00	15.5	17.6

P3.56 Think of shaking down the mercury in an old fever thermometer. Swing your hand through a circular arc, quickly reversing direction at the bottom end. Suppose your hand moves through one-quarter of a circle of radius 60 cm in 0.1 s. Its speed is

$$\frac{\frac{1}{4}(2\pi)(0.6 \text{ m})}{0.1 \text{ s}} \approx 9 \text{ m/s}$$

and its centripetal acceleration is $\frac{v^2}{r} \approx \frac{(9 \text{ m/s})^2}{0.6 \text{ m}} \boxed{\sim 10^2 \text{ m/s}^2}$.

The tangential acceleration of stopping and reversing the motion will make the total acceleration somewhat larger, but will not affect its order of magnitude.

P3.57 (a) $\Delta x = v_{xi} t, \; \Delta y = v_{yi} t + \frac{1}{2} g t^2$

$$d \cos 50.0° = (10.0 \cos 15.0°) t$$

and

$$-d \sin 50.0° = (10.0 \sin 15.0°) t + \frac{1}{2}(-9.80) t^2.$$

Solving, $10 \cos 15 \tan 50 + 10 \sin 15 = 4.9t$,

$t = 2.88$ s and $d = \boxed{43.2 \text{ m}}$.

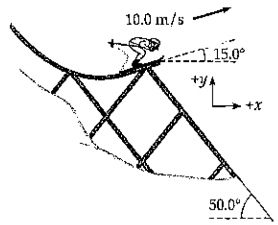

FIG. P3.57

(b) Since $a_x = 0$,

$$v_{xf} = v_{xi} = 10.0 \cos 15.0° = \boxed{9.66 \text{ m/s}}$$

$$v_{yf} = v_{yi} + a_y t = 10.0 \sin 15.0° - 9.80(2.88) = \boxed{-25.6 \text{ m/s}}.$$

Air resistance would decrease the values of the range and maximum height. As an airfoil, he can get some lift and increase his distance.

P3.58 For one electron, we have

$$y = v_{iy} t, \; D = v_{ix} t + \frac{1}{2} a_x t^2 \approx \frac{1}{2} a_x t^2, \; v_{yf} = v_{yi}, \text{ and } v_{xf} = v_{xi} + a_x t \approx a_x t.$$

The angle its direction makes with the x-axis is given by

$$\theta = \tan^{-1}\frac{v_{yf}}{v_{xf}} = \tan^{-1}\frac{v_{yi}}{a_x t} = \tan^{-1}\frac{v_{yi} t}{a_x t^2} = \tan^{-1}\frac{y}{2D}.$$

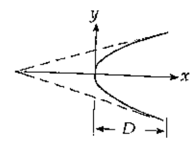

FIG. P3.58

Thus the horizontal distance from the aperture to the virtual source is $2D$. The source is at coordinate $\boxed{x = -D}$.

P3.59 (a) The ice chest floats downstream 2 km in time t, so that $2 \text{ km} = v_w t$. The upstream motion of the boat is described by $d = (v - v_w)15$ min. The downstream motion is described by $d + 2 \text{ km} = (v + v_w)(t - 15 \text{ min})$. We eliminate $t = \frac{2 \text{ km}}{v_w}$ and d by substitution:

$$(v - v_w)15 \text{ min} + 2 \text{ km} = (v + v_w)\left(\frac{2 \text{ km}}{v_w} - 15 \text{ min}\right)$$

$$v(15 \text{ min}) - v_w(15 \text{ min}) + 2 \text{ km} = \frac{v}{v_w} 2 \text{ km} + 2 \text{ km} - v(15 \text{ min}) - v_w(15 \text{ min})$$

$$v(30 \text{ min}) = \frac{v}{v_w} 2 \text{ km}$$

$$v_w = \frac{2 \text{ km}}{30 \text{ min}} = \boxed{4.00 \text{ km/h}}.$$

(b) In the reference frame of the water, the chest is motionless. The boat travels upstream for 15 min at speed v, and then downstream at the same speed, to return to the same point. Thus it travels for 30 min. During this time, the falls approach the chest at speed v_w, traveling 2 km. Thus

$$v_w = \frac{\Delta x}{\Delta t} = \frac{2 \text{ km}}{30 \text{ min}} = \boxed{4.00 \text{ km/h}}.$$

***P3.60** Let the river flow in the x direction.

(a) To minimize time, $\boxed{\text{swim perpendicular to the banks}}$ in the y direction. You are in the water for time t in $\Delta y = v_y t$, $t = \frac{80 \text{ m}}{1.5 \text{ m/s}} = 53.3$ s.

(b) The water carries you downstream by $\Delta x = v_x t = (2.50 \text{ m/s})53.3 \text{ s} = \boxed{133 \text{ m}}$.

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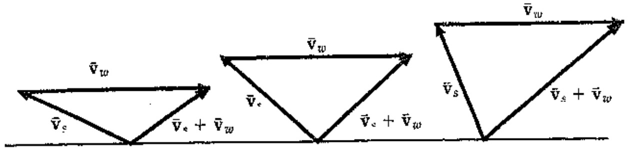
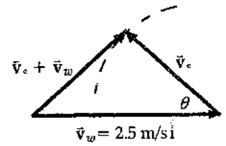
(c)



To minimize downstream drift, you should swim so that your resultant velocity $\vec{v}_s + \vec{v}_w$ is perpendicular to your swimming velocity $\vec{v}_s$ relative to the water. This condition is shown in the middle picture. It maximizes the angle between the resultant velocity and the shore. The angle between $\vec{v}_s$ and the shore is given by $\cos\theta = \frac{1.5 \text{ m/s}}{2.5 \text{ m/s}}$, $\boxed{\theta = 53.1°}$.



(d) Now $v_y = v_s \sin\theta = 1.5 \text{ m/s} \sin 53.1° = 1.20 \text{ m/s}$

$$t = \frac{\Delta y}{v_y} = \frac{80 \text{ m}}{1.2 \text{ m/s}} = 66.7 \text{ s}$$

$$\Delta x = v_x t = (2.5 \text{ m/s} - 1.5 \text{ m/s} \cos 53.1°)66.7 \text{ s} = \boxed{107 \text{ m}}.$$

P3.61 Find the highest firing angle θ_H for which the projectile will clear the mountain peak; this will yield the range of the closest point of bombardment. Next find the lowest firing angle; this will yield the maximum range under these conditions if both θ_H and θ_L are $> 45°$; $x = 2500$ m, $y = 1800$ m, $v_i = 250$ m/s.

$$y_f = v_{yi}t - \frac{1}{2}gt^2 = v_i(\sin\theta)t - \frac{1}{2}gt^2$$
$$x_f = v_{xi}t = v_i(\cos\theta)t$$

Thus $t = \frac{x_f}{v_i \cos\theta}$. Substitute into the expression for y_f

$$y_f = v_i(\sin\theta)\frac{x_f}{v_i\cos\theta} - \frac{1}{2}g\left(\frac{x_f}{v_i\cos\theta}\right)^2 = x_f\tan\theta - \frac{gx_f^2}{2v_i^2\cos^2\theta}$$

but $\frac{1}{\cos^2\theta} = \tan^2\theta + 1$ so $y_f = x_f\tan\theta - \frac{gx_f^2}{2v_i^2}(\tan^2\theta + 1)$ and $0 = \frac{gx_f^2}{2v_i^2}\tan^2\theta - x_f\tan\theta + \frac{gx_f^2}{2v_i^2} + y_f$.

Substitute values, use the quadratic formula and find

$$\tan\theta = 3.905 \text{ or } 1.197, \text{ which gives } \theta_H = 75.6° \text{ and } \theta_L = 50.1°.$$

$$\text{Range (at } \theta_H) = \frac{v_i^2 \sin 2\theta_H}{g} = 3.07 \times 10^3 \text{ m from enemy ship}$$

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$$3.07 \times 10^3 - 2\,500 - 300 = 270 \text{ m from shore.}$$

$$\text{Range (at } \theta_L) = \frac{v_i^2 \sin 2\theta_L}{g} = 6.28 \times 10^3 \text{ m from enemy ship}$$

$$6.28 \times 10^3 - 2\,500 - 300 = 3.48 \times 10^3 \text{ from shore.}$$

Therefore, safe distance is $\boxed{< 270 \text{ m}}$ or $\boxed{> 3.48 \times 10^3 \text{ m}}$ from the shore.

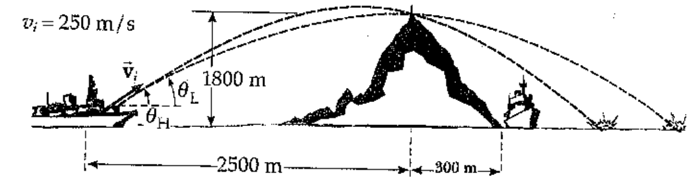

FIG. P3.61

ANSWERS TO EVEN PROBLEMS

P3.2 (a) $(1.00\hat{i} + 0.750\hat{j})$ m/s; (b) $(1.00\hat{i} + 0.500\hat{j})$ m/s, 1.12 m/s

P3.4 (a) $(2.00\hat{i} + 3.00\hat{j})$ m/s^2; (b) $(3.00t + t^2)\hat{i}$ m $+ (1.50t^2 - 2.00t)\hat{j}$ m

P3.6 (a) $\vec{r}_f = (10.0\,\hat{i} + 0.241\,\hat{j})$ mm; (b) $(1.84 \times 10^7 \text{ m/s})\hat{i} + (8.78 \times 10^5 \text{ m/s})\hat{j}$; (c) 1.85×10^7 m/s; (d) 2.73°

P3.8 (a) $d\sqrt{\frac{g}{2h}}$ horizontally; (b) $\tan^{-1}\left(\frac{2h}{d}\right)$ below the horizontal

P3.10 0.600 m/s^2 down

P3.12 (a) 22.6 m; (b) 52.3 m; (c) 1.18 s

P3.14 25.8 m/s

P3.16 $d\tan\theta_i - \frac{gd^2}{2v_i^2\cos^2\theta_i}$

P3.18 (a) (0, 0.840 m); (b) 11.2 m/s at 18.5°; (c) 8.94 m; (d) see the solution

P3.20 (a) 0.852 s; (b) 3.29 m/s; (c) 4.03 m/s; (d) 50.8°; (e) 1.12 s

P3.22 0.0337 m/s^2 toward the center of the Earth

P3.24 2.06×10^3 rev/min

P3.26 0.281 rev/s

P3.28 (a) 1.25 m/s^2 toward the center; (b) 0.400 m/s^2 forward; (c) 1.31 m/s^2 forward and 72.3° inward