P26.12 (a) 2.22 cm; (b) 10.0

P26.14 (a) 160 mm; (b) −267 mm

P26.16 (a) convex; (b) at the 30.0 cm mark; (c) −20.0 cm

P26.18 (a) 15.0 cm; (b) 60.0 cm

P26.20 (a) see the solution; (b) at 0.639 s and 0.782 s

P26.22 4.82 cm

P26.24 see the solution, the image is real, inverted, and diminished

P26.26 1.50 cm/s

P26.28 20.0 cm

P26.30 (a) $q = 40.0$ cm, $M = -1.00$, real and inverted; (b) $q \to \infty$, no image is formed; (c) $q = -20.0$ cm, $M = +2.00$, virtual and upright

P26.32 (a) 6.40 cm; (b) −0.250; (c) converging

P26.34 (a) $f = 39.0$ mm; (b) $p = 39.5$ mm

P26.36 (a) 5.36 cm; (b) −18.8 cm; (c) virtual, right side up, enlarged; (d) A magnifying glass with focal length 7.50 cm is used to form an image of a stamp, enlarged 3.50 times. Find the object distance. Locate and describe the image.

P26.38 (a) $f = 13.3$ cm; (b) see the solution; (c) 224 cm^4

P26.40 2.18 mm away from the film plane

P26.42 (a) the bottom half; (b) 1.50 mm; (c) 48.2°; (d) see the solution, 96.4°; (e) see the solution

P26.44 160 cm to the left of the lens, inverted, $M = -0.800$

P26.46 see the solution

P26.48 25.3 cm to the right of the mirror, virtual, upright, enlarged 8.05 times

P26.50 (a) 1.40 kW/m^2; (b) 6.91 mW/m^2; (c) 0.164 cm; (d) 58.1 W/m^2

P26.52 11.7 cm

P26.54 (a) 0.334 m or larger; (b) R_n must be at least 0.025 5 R

P26.56 Align the lenses on the same axis and 9.00 cm apart. Let the light pass first through the diverging lens and then through the converging lens. The diameter increases by a factor of 1.75.

P26.58 (a) 1.99; (b) 10.0 cm to the left of the lens, −2.50; (c) inverted

P26.60 (a) 20.0 cm to the right of the second lens, −6.00; (b) inverted; (c) 6.67 to the right of the second lens, −2.00, inverted

Chapter 27
Wave Optics

CHAPTER OUTLINE

27.1	Conditions for Interference
27.2	Young's Double-Slit Experiment
27.3	Light Waves in Interference
27.4	Change of Phase Due to Reflection
27.5	Interference in Thin Films
27.6	Diffraction Patterns
27.7	Resolution of Single-Slit and Circular Apertures
27.8	The Diffraction Grating
27.9	Diffraction of X-Rays by Crystals
27.10	Context Connection—Holography

ANSWERS TO QUESTIONS

Q27.1 (a) Two waves interfere constructively if their path difference is zero, or an integral multiple of the wavelength, according to $\delta = m\lambda$, with $m = 0, 1, 2, 3, \ldots$.

(b) Two waves interfere destructively if their path difference is a half wavelength, or an odd multiple of $\frac{\lambda}{2}$, described by

$$\delta = \left(m + \frac{1}{2}\right)\lambda, \text{ with } m = 0, 1, 2, 3, \ldots$$

Q27.2 The light from the flashlights consists of many different wavelengths (that's why it's white) with random time differences between the light waves. There is no *coherence* between the two sources. The light from the two flashlights does not maintain a constant phase relationship over time. These three equivalent statements mean no possibility of an interference pattern.

Q27.3 Every color produces its own pattern, with a spacing between the maxima that is characteristic of the wavelength. With several colors, the patterns are superimposed and it can be difficult to pick out a single maximum. Using monochromatic light can eliminate this problem.

Q27.4 Underwater, the wavelength of the light would decrease, $\lambda_{water} = \frac{\lambda_{air}}{n_{water}}$. Since the positions of light and dark bands are proportional to λ, (according to Equations 27.2 and 27.3), the underwater fringe separations will decrease.

Q27.5 The threads that are woven together to make the cloth have small meshes between them. These bits of space act as pinholes through which the light diffracts. Since the cloth is a grid of such pinholes, an interference pattern is formed, as when you look through a diffraction grating.

Q27.6 If the oil film is brightest where it is thinnest, then $n_{air} < n_{oil} < n_{water}$. With this condition, light reflecting from both the top and the bottom surface of the oil film will undergo phase reversal. Then these two beams will be in phase with each other where the film is very thin. This is the condition for constructive interference as the thickness of the oil film decreases toward zero.

Q27.7 As water evaporates from the 'soap' bubble, the thickness of the bubble wall approaches zero. Since light reflecting from the front of the water surface is phase-shifted 180° and light reflecting from the back of the soap film is phase-shifted 0°, the reflected light meets the conditions for a minimum. Thus the soap film appears black, as in the illustration accompanying textbook Example 27.4, "Interference in a Wedge-Shaped Film."

Q27.8 If the film is more than a few wavelengths thick, the interference fringes are so close together that you cannot resolve them.

Q27.9 Suppose the coating is intermediate in index of refraction between vacuum and the glass. When the coating is very thin, light reflected from its top and bottom surfaces will interfere constructively, so you see the surface white and brighter. As the thickness reaches one quarter of the wavelength of violet light in the coating, destructive interference for violet will make the surface look red or perhaps orange. Next to interfere destructively are blue, green, yellow, orange, and red, making the surface look red, purple, and then blue. As the coating gets still thicker, we can get constructive interference for violet and then for other colors in spectral order. Still thicker coating will give constructive and destructive interference for several visible wavelengths, so the reflected light will start to look white again.

Q27.10 The wavelength of light is extremely small in comparison to the dimensions of your hand, so the diffraction of light around an obstacle the size of your hand is totally negligible. However, sound waves have wavelengths that are comparable to the dimensions of the hand or even larger. Therefore, significant diffraction of sound waves occurs around hand-sized obstacles.

Q27.11 Audible sound has wavelengths on the order of meters or centimeters, while visible light has a wavelength on the order of half a micrometer. In this world of breadbox-sized objects, $\frac{\lambda}{a}$ is large for sound. When sound goes through a doorway, the sound diffracts around behind the adjoining walls. But $\frac{\lambda}{a}$ is a tiny fraction for visible light passing ordinary-size objects or apertures, so light changes its direction by only very small angles when it diffracts.

Another way of phrasing the answer: We can see by a small angle around a small obstacle or around the edge of a small opening. The side fringes in Figure 27.12 and the Arago spot in the center of Figure 27.13 show this diffraction. We cannot always hear around corners. Out-of-doors, away from reflecting surfaces, have someone a few meters distant face away from you and whisper. The high-frequency, short-wavelength, information-carrying components of the sound do not diffract around his head enough for you to understand his words.

Q27.12 Yes, but no diffraction effects are observed because the separation distance between adjacent ribs is so much greater than the wavelength of x-rays. Diffraction does not limit the resolution of an x-ray image. Diffraction might sometimes limit the resolution of an ultrasonogram.

Q27.13 The intensity of the light coming through the slit decreases, as you would expect. The central maximum increases in width as the width of the slit decreases. In the condition $\sin\theta = \frac{\lambda}{a}$ for destructive interference on each side of the central maximum, θ increases as a decreases.

Q27.14 It is shown in the correct orientation. If the horizontal width of the opening is equal to or less than the wavelength of the sound, then the equation $a\sin\theta = (1)\lambda$ has the solution $\theta = 90°$, or has no solution. The central diffraction maximum covers the whole seaward side. If the vertical height of the opening is large compared to the wavelength, then the angle in $a\sin\theta = (1)\lambda$ will be small, and the central diffraction maximum will form a thin horizontal sheet.

Q27.15 The fine hair blocks off light that would otherwise go through a fine slit and produce a diffraction pattern on a distant screen. The width of the central maximum in the pattern is inversely proportional to the distance across the slit. When the hair is in place, it subtracts the same diffraction pattern from the projected disk of laser light. The hair produces a diffraction minimum that crosses the bright circle on the screen. The width of the minimum is inversely proportional to the diameter of the hair. The central minimum is flanked by narrower maxima and minima. Measure the width $2y$ of the central minimum between the maxima bracketing it, and use Equation 27.13 in the form $\frac{y}{L} = \frac{\lambda}{a}$ to find the width a of the hair.

Q27.16 The condition for constructive interference is that the three radio signals arrive at the city in phase. We know the speed of the waves (it is the speed of light c), the angular bearing θ of the city east of north from the broadcast site, and the distance d between adjacent towers. The wave from the westernmost tower must travel an extra distance $2d\sin\theta$ to reach the city, compared to the signal from the eastern tower. For each cycle of the carrier wave, the western antenna would transmit first, the center antenna after a time delay $\frac{d\sin\theta}{c}$, and the eastern antenna after an additional equal time delay.

SOLUTIONS TO PROBLEMS

Section 27.1 Conditions for Interference

Section 27.2 Young's Double-Slit Experiment

Section 27.3 Light Waves in Interference

P27.1 $y_{\text{bright}} = \frac{\lambda L}{d} m$

For $m = 1$, $\lambda = \frac{yd}{L} = \frac{(3.40\times10^{-3}\text{ m})(5.00\times10^{-4}\text{ m})}{3.30\text{ m}} = \boxed{515\text{ nm}}$

P27.2 $\lambda = \frac{v}{f} = \frac{354\text{ m/s}}{2\,000\text{ s}^{-1}} = 0.177\text{ m}$

(a) $d\sin\theta = m\lambda$ so $(0.300\text{ m})\sin\theta = 1(0.177\text{ m})$ and $\theta = \boxed{36.2°}$

(b) $d\sin\theta = m\lambda$ so $d\sin 36.2° = 1(0.030\,0\text{ m})$ and $d = \boxed{5.08\text{ cm}}$

continued on next page

(c) $(1.00\times10^{-6}\text{ m})\sin 36.2° = (1)\lambda$ so $\lambda = 590\text{ nm}$

$$f = \frac{c}{\lambda} = \frac{3.00\times10^{8}\text{ m/s}}{5.90\times10^{-7}\text{ m}} = \boxed{508\text{ THz}}$$

P27.3 Note, with the conditions given, the small angle approximation **does not work well**. That is, $\sin\theta$, $\tan\theta$, and θ are significantly different. We treat the interference as a Fraunhofer pattern.

(a) At the $m = 2$ maximum, $\tan\theta = \frac{400\text{ m}}{1\,000\text{ m}} = 0.400$

$$\theta = 21.8°$$

so $$\lambda = \frac{d\sin\theta}{m} = \frac{(300\text{ m})\sin 21.8°}{2} = \boxed{55.7\text{ m}}$$

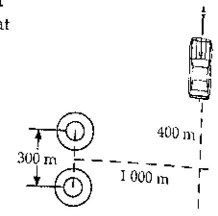

FIG. P27.3

(b) The next minimum encountered is the $m = 2$ minimum:

and at that point, $d\sin\theta = \left(m + \frac{1}{2}\right)\lambda$

which becomes $d\sin\theta = \frac{5}{2}\lambda$

or $\sin\theta = \frac{5}{2}\frac{\lambda}{d} = \frac{5}{2}\left(\frac{55.7\text{ m}}{300\text{ m}}\right) = 0.464$

and $\theta = 27.7°$

so $y = (1\,000\text{ m})\tan 27.7° = 524\text{ m}$.

Therefore, the car must travel an additional $\boxed{124\text{ m}}$.

If we considered Fresnel interference, we would more precisely find
(a) $\lambda = \frac{1}{2}\left(\sqrt{550^2 + 1\,000^2} - \sqrt{250^2 + 1\,000^2}\right) = 55.2\text{ m}$ and (b) 123 m.

P27.4 $\lambda = \frac{340\text{ m/s}}{2\,000\text{ Hz}} = 0.170\text{ m}$

Maxima are at $d\sin\theta = m\lambda$:

$m = 0$	gives	$\theta = 0°$	
$m = 1$	gives	$\sin\theta = \frac{\lambda}{d} = \frac{0.170\text{ m}}{0.350\text{ m}}$.	$\theta = 29.1°$
$m = 2$	gives	$\sin\theta = \frac{2\lambda}{d} = 0.971$	$\theta = 76.3°$
$m = 3$	gives	$\sin\theta = 1.46$	No solution.

continued on next page

Minima are at $d\sin\theta = \left(m + \frac{1}{2}\right)\lambda$:

$m = 0$	gives	$\sin\theta = \frac{\lambda}{2d} = 0.243$	$\theta = 14.1°$
$m = 1$	gives	$\sin\theta = \frac{3\lambda}{2d} = 0.729$	$\theta = 46.8°$
$m = 2$	gives	$\sin\theta = 1.21$	No solution.

$\boxed{\text{So we have maxima at } 0°,\ 29.1°, \text{ and } 76.3°;\ \text{minima at } 14.1° \text{ and } 46.8°}$.

P27.5 In the equation $d\sin\theta = \left(m + \frac{1}{2}\right)\lambda$.

The first minimum is described by $m = 0$

and the tenth by $m = 9$: $\sin\theta = \frac{\lambda}{d}\left(9 + \frac{1}{2}\right)$

Also, $\tan\theta = \frac{y}{L}$

but for small θ, $\sin\theta \approx \tan\theta$.

Thus, $d = \frac{9.5\lambda}{\sin\theta} = \frac{9.5\lambda L}{y}$

$$d = \frac{9.5(5\,890\times10^{-10}\text{ m})(2.00\text{ m})}{7.26\times10^{-3}\text{ m}} = 1.54\times10^{-3}\text{ m} = \boxed{1.54\text{ mm}}$$

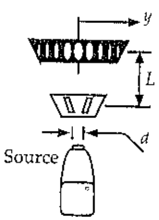

FIG. P27.5

***P27.6** Location of A = central maximum,

Location of B = first minimum.

So, $\Delta y = |y_{\min} - y_{\max}| = \frac{\lambda L}{d}\left(0 + \frac{1}{2}\right) - 0 = \frac{1}{2}\frac{\lambda L}{d} = 20.0\text{ m}$.

Thus, $d = \frac{\lambda L}{2(20.0\text{ m})} = \frac{(3.00\text{ m})(150\text{ m})}{40.0\text{ m}} = \boxed{11.3\text{ m}}$.

P27.7 At 30.0° $d\sin\theta = m\lambda$

$(3.20\times10^{-4}\text{ m})\sin 30.0° = m(500\times10^{-9}\text{ m})$ so $m = 320$

There are 320 maxima to the right, 320 to the left, and one for $m = 0$ straight ahead.

There are $\boxed{641\text{ maxima}}$

P27.8 Observe that the pilot must not only home in on the airport, but must be headed in the right direction when she arrives at the end of the runway.

(a) $\lambda = \frac{v}{f} = \frac{3\times 10^{8}\ \text{m/s}}{30\times 10^{6}\ \text{s}^{-1}} = \boxed{10.0\ \text{m}}$

(b) The first side maximum is at an angle given by $d\sin\theta = (1)\lambda$.

$(40\ \text{m})\sin\theta = 10\ \text{m}$ $\quad\theta = 14.5°\quad$ $\tan\theta = \frac{y}{L}$

$y = L\tan\theta = (2\,000\ \text{m})\tan 14.5° = \boxed{516\ \text{m}}$

(c) The signal of 10-m wavelength in parts (a) and (b) would show maxima at 0°, 14.5°, 30.0°, 48.6°, and 90°. A signal of wavelength 11.23-m would show maxima at 0°, 16.3°, 34.2°, and 57.3°. The only value in common is 0°. If λ_1 and λ_2 were related by a ratio of small integers (a just musical consonance!) in $\frac{\lambda_1}{\lambda_2} = \frac{n_1}{n_2}$, then the equations $d\sin\theta = n_2\lambda_1$ and $d\sin\theta = n_1\lambda_2$ would both be satisfied for the same nonzero angle. The pilot could come flying in with that inappropriate bearing, and run off the runway immediately after touchdown.

P27.9 $\phi = \frac{2\pi}{\lambda}d\sin\theta = \frac{2\pi}{\lambda}d\left(\frac{y}{L}\right)$

(a) $\phi = \frac{2\pi}{(5.00\times 10^{-7}\ \text{m})}(1.20\times 10^{-4}\ \text{m})\sin(0.500°) = \boxed{13.2\ \text{rad}}$

(b) $\phi = \frac{2\pi}{(5.00\times 10^{-7}\ \text{m})}(1.20\times 10^{-4}\ \text{m})\left(\frac{5.00\times 10^{-3}\ \text{m}}{1.20\ \text{m}}\right) = \boxed{6.28\ \text{rad}}$

(c) If $\phi = 0.333\ \text{rad} = \frac{2\pi d\sin\theta}{\lambda}$ $\quad\theta = \sin^{-1}\left(\frac{\lambda\phi}{2\pi d}\right) = \sin^{-1}\left[\frac{(5.00\times 10^{-7}\ \text{m})(0.333\ \text{rad})}{2\pi(1.20\times 10^{-4}\ \text{m})}\right]$

$\theta = \boxed{1.27\times 10^{-2}\ \text{deg}}$.

(d) If $d\sin\theta = \frac{\lambda}{4}$ $\quad\theta = \sin^{-1}\left(\frac{\lambda}{4d}\right) = \sin^{-1}\left[\frac{5\times 10^{-7}\ \text{m}}{4(1.20\times 10^{-4}\ \text{m})}\right]$

$\theta = \boxed{5.97\times 10^{-2}\ \text{deg}}$.

*P27.10 The path difference between rays 1 and 2 is: $\delta = d\sin\theta_1 - d\sin\theta_2$.

For constructive interference, this path difference must be equal to an integral number of wavelengths: $d\sin\theta_1 - d\sin\theta_2 = m\lambda$, or

$\boxed{d(\sin\theta_1 - \sin\theta_2) = m\lambda}$.

P27.11 $I_{av} = I_{max}\cos^2\left(\frac{\pi d\sin\theta}{\lambda}\right)$

For small θ, $\quad\sin\theta = \frac{y}{L}$

and $\quad I_{av} = 0.750 I_{max}$

$$y = \frac{\lambda L}{\pi d}\cos^{-1}\sqrt{\frac{I_{av}}{I_{max}}}$$

$$y = \frac{(6.00\times 10^{-7})(1.20\ \text{m})}{\pi(2.50\times 10^{-3}\ \text{m})}\cos^{-1}\sqrt{\frac{0.750 I_{max}}{I_{max}}} = \boxed{48.0\ \mu\text{m}}$$

P27.12 (a) The intensity of the sum of two coherent waves is described by $\frac{I}{I_{max}} = \cos^2\left(\frac{\phi}{2}\right)$. Therefore,

$$\phi = 2\cos^{-1}\sqrt{\frac{I}{I_{max}}} = 2\cos^{-1}\sqrt{0.640} = \boxed{1.29\ \text{rad}}$$

(b) $\delta = \frac{\lambda\phi}{2\pi} = \frac{(486.1\ \text{nm})(1.29\ \text{rad})}{2\pi} = \boxed{99.6\ \text{nm}}$

P27.13 $I = I_{max}\cos^2\left(\frac{\pi yd}{\lambda L}\right)$

$$\frac{I}{I_{max}} = \cos^2\left[\frac{\pi(6.00\times 10^{-3}\ \text{m})(1.80\times 10^{-4}\ \text{m})}{(656.3\times 10^{-9}\ \text{m})(0.800\ \text{m})}\right] = \boxed{0.968}$$

Section 27.4 Change of Phase Due to Reflection

Section 27.5 Interference in Thin Films

P27.14 Light reflecting from the first surface suffers phase reversal. Light reflecting from the second surface does not, but passes twice through the thickness t of the film. So, for constructive interference, we require

$$\frac{\lambda_n}{2} + 2t = \lambda_n$$

where $\quad\lambda_n = \frac{\lambda}{n}$ is the wavelength in the material.

Then $\quad 2t = \frac{\lambda_n}{2} = \frac{\lambda}{2n}$

$$\lambda = 4nt = 4(1.33)(115\ \text{nm}) = \boxed{612\ \text{nm}}$$

P27.15 (a) The light reflected from the top of the oil film undergoes phase reversal. Since $1.45 > 1.33$, the light reflected from the bottom undergoes no reversal. For constructive interference of reflected light, we then have

$$2nt = \left(m + \frac{1}{2}\right)\lambda$$

or $$\lambda_m = \frac{2nt}{m+(1/2)} = \frac{2(1.45)(280\text{ nm})}{m+(1/2)}$$

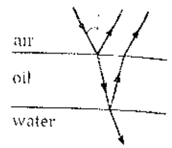

FIG. P27.15

Substituting for m gives: $m = 0$, $\lambda_0 = 1\,620$ nm (infrared)

$m = 1$, $\lambda_1 = 541$ nm (green)

$m = 2$, $\lambda_2 = 325$ nm (ultraviolet).

Both infrared and ultraviolet light are invisible to the human eye, so the dominant color in reflected light is $\boxed{\text{green}}$.

(b) The dominant wavelengths in the transmitted light are those that produce destructive interference in the reflected light. The condition for destructive interference upon reflection is

$$2nt = m\lambda$$

or $$\lambda_m = \frac{2nt}{m} = \frac{812\text{ nm}}{m}$$

Substituting for m gives: $m = 1$, $\lambda_1 = 812$ nm (near infrared)

$m = 2$, $\lambda_2 = 406$ nm (violet)

$m = 3$, $\lambda_3 = 271$ nm (ultraviolet).

Of these, the only wavelength visible to the human eye (and hence the dominate wavelength observed in the transmitted light) is 406 nm. Thus, the dominant color in the transmitted light is $\boxed{\text{violet}}$.

P27.16 Treating the anti-reflectance coating like a camera-lens coating,

$$2t = \left(m + \frac{1}{2}\right)\frac{\lambda}{n}$$

Let $m = 0$: $t = \frac{\lambda}{4n} = \frac{3.00\text{ cm}}{4(1.50)} = \boxed{0.500\text{ cm}}$.

This anti-reflectance coating could be easily countered by changing the wavelength of the radar—to 1.50 cm—now creating maximum reflection!

P27.17 $2nt = \left(m + \frac{1}{2}\right)\lambda$ so $t = \left(m + \frac{1}{2}\right)\frac{\lambda}{2n}$

Minimum $t = \left(\frac{1}{2}\right)\frac{(500\text{ nm})}{2(1.30)} = \boxed{96.2\text{ nm}}$.

***P27.18** (a) For maximum transmission, we want destructive interference in the light reflected from the front and back surfaces of the film.

If the surrounding glass has refractive index greater than 1.378, light reflected from the front surface suffers no phase reversal and light reflected from the back does undergo phase reversal. This effect by itself would produce destructive interference, so we want the distance down and back to be one whole wavelength in the film: $2t = \frac{\lambda}{n}$

$$t = \frac{\lambda}{2n} = \frac{656.3\text{ nm}}{2(1.378)} = \boxed{238\text{ nm}}$$

(b) The filter will expand. As t increases in $2nt = \lambda$, so does $\boxed{\lambda\text{ increase}}$

(c) Destructive interference for reflected light happens also for λ in $2nt = 2\lambda$,

or $\lambda = 1.378(238\text{ nm}) = \boxed{328\text{ nm}}$ (near ultraviolet).

P27.19 For destructive interference in the air,

$2t = m\lambda$.

For 30 dark fringes, including the one where the plates meet,

$$t = \frac{29(600\text{ nm})}{2} = 8.70 \times 10^{-6}\text{ m}.$$

Therefore, the *radius* of the wire is

$$r = \frac{t}{2} = \frac{8.70\ \mu\text{m}}{2} = \boxed{4.35\ \mu\text{m}}$$

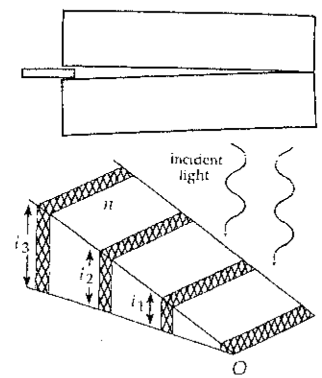

FIG. P27.19

Section 27.6 Diffraction Patterns

P27.20 $\sin\theta = \frac{\lambda}{a} = \frac{6.328 \times 10^{-7}}{3.00 \times 10^{-4}} = 2.11 \times 10^{-3}$

$$\frac{y}{1.00\text{ m}} = \tan\theta \approx \sin\theta = \theta \text{ (for small } \theta)$$

$2y = \boxed{4.22\text{ mm}}$

P27.21 $\frac{y}{L} \approx \sin\theta = \frac{m\lambda}{a}$ $\qquad \Delta y = 3.00\times10^{-3}$ m

$\Delta m = 3 - 1 = 2$ $\qquad$ and $\qquad a = \frac{\Delta m \lambda L}{\Delta y}$

$$a = \frac{2(690\times10^{-9}\ \text{m})(0.500\ \text{m})}{(3.00\times10^{-3}\ \text{m})} = \boxed{2.30\times10^{-4}\ \text{m}}$$

P27.22 The positions of the first-order minima are $\frac{y}{L} \approx \sin\theta = \pm\frac{\lambda}{a}$. Thus, the spacing between these two minima is $\Delta y = 2\left(\frac{\lambda}{a}\right)L$ and the wavelength is

$$\lambda = \left(\frac{\Delta y}{2}\right)\left(\frac{a}{L}\right) = \left(\frac{4.10\times10^{-3}\ \text{m}}{2}\right)\left(\frac{0.550\times10^{-3}\ \text{m}}{2.06\ \text{m}}\right) = \boxed{547\ \text{nm}}.$$

P27.23 For destructive interference,

$$\sin\theta = m\frac{\lambda}{a} = \frac{\lambda}{a} = \frac{5.00\ \text{cm}}{36.0\ \text{cm}} = 0.139$$

and $\qquad \theta = 7.98°$

$$\frac{d}{L} = \tan\theta$$

gives $\qquad d = L\tan\theta = (6.50\ \text{m})\tan 7.98° = 0.912\ \text{m}$

$$d = \boxed{91.2\ \text{cm}}.$$

P27.24 If the speed of sound is 340 m/s, $\qquad \lambda = \frac{v}{f} = \frac{340\ \text{m/s}}{650\ \text{s}^{-1}} = 0.523\ \text{m}$.

Diffraction minima occur at angles described by $\quad a\sin\theta = m\lambda$

$(1.10\ \text{m})\sin\theta_1 = 1(0.523\ \text{m})$ $\qquad \theta_1 = 28.4°$

$(1.10\ \text{m})\sin\theta_2 = 2(0.523\ \text{m})$ $\qquad \theta_2 = 72.0°$

$(1.10\ \text{m})\sin\theta_3 = 3(0.523\ \text{m})$ $\qquad \theta_3$ nonexistent

Maxima appear straight ahead at $\boxed{0°}$ and left and right at an angle given approximately by

$(1.10\ \text{m})\sin\theta_x = 1.5(0.523\ \text{m})$ $\qquad \theta_x \boxed{\approx 46°}$

There is no solution to $a\sin\theta = 2.5\lambda$, so our answer is already complete, with $\boxed{\text{three}}$ sound maxima.

*P27.25 The rectangular patch on the wall is wider than it is tall. The aperture will be taller than it is wide. For horizontal spreading we have

$$\tan\theta_{\text{width}} = \frac{y_{\text{width}}}{L} = \frac{0.110\ \text{m}/2}{4.5\ \text{m}} = 0.012\,2$$

$$a_{\text{width}}\sin\theta_{\text{width}} = 1\lambda$$

$$a_{\text{width}} = \frac{632.8\times10^{-9}\ \text{m}}{0.012\,2} = \boxed{5.18\times10^{-5}\ \text{m}}$$

For vertical spreading, similarly

$$\tan\theta_{\text{height}} = \frac{0.006\ \text{m}/2}{4.5\ \text{m}} = 0.000\,667$$

$$a_{\text{height}} = \frac{1\lambda}{\sin\theta_h} = \frac{632.8\times10^{-9}\ \text{m}}{0.000\,667} = \boxed{9.49\times10^{-4}\ \text{m}}$$

Section 27.7 Resolution of Single-Slit and Circular Apertures

P27.26 $\sin\theta = \frac{\lambda}{a} = \frac{5.00\times10^{-7}\ \text{m}}{5.00\times10^{-4}} = \boxed{1.00\times10^{-3}\ \text{rad}}$

P27.27 Undergoing diffraction from a circular opening, the beam spreads into a cone of half-angle

$$\theta_{\min} = 1.22\frac{\lambda}{D} = 1.22\left(\frac{632.8\times10^{-9}\ \text{m}}{0.005\,00\ \text{m}}\right) = 1.54\times10^{-4}\ \text{rad}.$$

The radius of the beam ten kilometers away is, from the definition of radian measure,

$$r_{\text{beam}} = \theta_{\min}(1.00\times10^4\ \text{m}) = 1.544\ \text{m}$$

and its diameter is $d_{\text{beam}} = 2r_{\text{beam}} = \boxed{3.09\ \text{m}}$.

*P27.28 By Rayleigh's criterion: $\theta_{\min} = \frac{d}{L} = 1.22\frac{\lambda}{D}$, where $\theta_{\min}$ is the smallest angular separation of two objects for which they are resolved by an aperture of diameter D, d is the separation of the two objects, and L is the maximum distance of the aperture from the two objects at which they can be resolved.

Two objects can be resolved if their angular separation is greater than $\theta_{\min}$. Thus, $\theta_{\min}$ should be as small as possible. Therefore, the smaller of the two given wavelengths is easier to resolve, i.e. $\boxed{\text{violet}}$.

$$L = \frac{Dd}{1.22\lambda} = \frac{(5.20\times10^{-3}\ \text{m})(2.80\times10^{-2}\ \text{m})}{1.22\lambda} = \frac{1.193\times10^{-4}\ \text{m}^2}{\lambda}$$

Thus $L = 186$ m for $\lambda = 640$ nm, and $L = 271$ m for $\lambda = 440$ nm. The viewer can resolve adjacent tubes of violet in the range $\boxed{186\ \text{m to}\ 271\ \text{m}}$, but cannot resolve adjacent tubes of red in this range.

P27.29 By Rayleigh's criterion, two dots separated center-to-center by 2.00 mm would overlap

when $\theta_{\min} = \frac{d}{L} = 1.22\frac{\lambda}{D}$.

Thus, $L = \frac{dD}{1.22\lambda} = \frac{(2.00\times10^{-3}\text{ m})(4.00\times10^{-3}\text{ m})}{1.22(500\times10^{-9}\text{ m})} = \boxed{13.1\text{ m}}$.

P27.30 The concave mirror of the spy satellite is probably about 2 m in diameter, and is surely not more than 5 m in diameter. That is the size of the largest piece of glass successfully cast to a precise shape, for the mirror of the Hale telescope on Mount Palomar. If the spy satellite had a larger mirror, its manufacture could not be kept secret, and it would be visible from the ground. Outer space is probably closer than your state capitol, but the satellite is surely above 200-km altitude, for reasonably low air friction. We find the distance between barely resolvable objects at a distance of 200 km, seen in yellow light through a 5-m aperture:

$$\frac{y}{L} = 1.22\frac{\lambda}{D}$$

$$y = (200\,000\text{ m})(1.22)\left(\frac{6\times10^{-7}\text{ m}}{5\text{ m}}\right) = 3\text{ cm}$$

(Considering atmospheric seeing caused by variations in air density and temperature, the distance between barely resolvable objects is more like $(200\,000\text{ m})(1\text{ s})\left(\frac{1°}{3\,600\text{ s}}\right)\left(\frac{\pi\text{ rad}}{180°}\right) = 97\text{ cm}$.) Thus the snooping spy satellite cannot see the difference between III and II or IV on a license plate. It cannot count coins spilled on a sidewalk, much less read the date on them.

P27.31 $1.22\frac{\lambda}{D} = \frac{d}{L}$ $\qquad \lambda = \frac{c}{f} = 0.020\,0\text{ m}$

$D = 2.10\text{ m}$ $\qquad L = 9\,000\text{ m}$

$$d = 1.22\frac{(0.020\,0\text{ m})(9\,000\text{ m})}{2.10\text{ m}} = \boxed{105\text{ m}}$$

Section 27.8 The Diffraction Grating

P27.32 The principal maxima are defined by

$d\sin\theta = m\lambda$ $\qquad m = 0, 1, 2, \ldots$

For $m = 1$, $\qquad \lambda = d\sin\theta$

where θ is the angle between the central ($m = 0$) and the first order ($m = 1$) maxima. The value of θ can be determined from the information given about the distance between maxima and the grating-to-screen distance. From the figure,

$$\tan\theta = \frac{0.488\text{ m}}{1.72\text{ m}} = 0.284$$

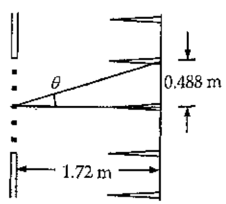

FIG. P27.32

continued on next page

so $\theta = 15.8°$

and $\sin\theta = 0.273$.

The distance between grating "slits" equals the reciprocal of the number of grating lines per centimeter

$$d = \frac{1}{5\,310\text{ cm}^{-1}} = 1.88\times10^{-4}\text{ cm} = 1.88\times10^{3}\text{ nm}.$$

The wavelength is $\lambda = d\sin\theta = (1.88\times10^{3}\text{ nm})(0.273) = \boxed{514\text{ nm}}$.

P27.33 The grating spacing is $d = \frac{1.00\times10^{-2}\text{ m}}{4\,500} = 2.22\times10^{-6}\text{ m}$.

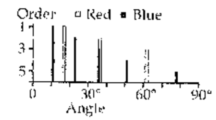

FIG. P27.33

In the 1st-order spectrum, diffraction angles are given by

$\sin\theta = \frac{\lambda}{d}$: $\qquad \sin\theta_1 = \frac{656\times10^{-9}\text{ m}}{2.22\times10^{-6}\text{ m}} = 0.295$

so that for red $\qquad \theta_1 = 17.17°$

and for blue $\qquad \sin\theta_2 = \frac{434\times10^{-9}\text{ m}}{2.22\times10^{-6}\text{ m}} = 0.195$

so that $\qquad \theta_2 = 11.26°$

The angular separation is in first-order, $\Delta\theta = 17.17° - 11.26° = \boxed{5.91°}$.

In the second-order spectrum, $\Delta\theta = \sin^{-1}\left(\frac{2\lambda_1}{d}\right) - \sin^{-1}\left(\frac{2\lambda_2}{d}\right) = \boxed{13.2°}$.

Again, in the third order, $\Delta\theta = \sin^{-1}\left(\frac{3\lambda_1}{d}\right) - \sin^{-1}\left(\frac{3\lambda_2}{d}\right) = \boxed{26.5°}$.

Since the red does not appear in the fourth-order spectrum, the answer is complete.

P27.34 $\sin\theta = 0.350$: $\qquad d = \frac{\lambda}{\sin\theta} = \frac{632.8\text{ nm}}{0.350} = 1.81\times10^{3}\text{ nm}$

Line spacing $= \boxed{1.81\ \mu\text{m}}$

P27.35 $d = \frac{1.00\times10^{-3}\text{ m/mm}}{250\text{ lines/mm}} = 4.00\times10^{-6}\text{ m} = 4\,000\text{ nm}$ $\qquad d\sin\theta = m\lambda \Rightarrow m = \frac{d\sin\theta}{\lambda}$

(a) The number of times a complete order is seen is the same as the number of orders in which the long wavelength limit is visible.

$$m_{\max} = \frac{d\sin\theta_{\max}}{\lambda} = \frac{(4\,000\text{ nm})\sin 90.0°}{700\text{ nm}} = 5.71$$

or $\boxed{5\text{ orders is the maximum}}$.

continued on next page

(b) The highest order in which the violet end of the spectrum can be seen is:

$$m_{\max} = \frac{d\sin\theta_{\max}}{\lambda} = \frac{(4\,000 \text{ nm})\sin 90.0°}{400 \text{ nm}} = 10.0$$

or $\boxed{\text{10 orders in the short-wavelength region}}$.

*P27.36 $\sin\theta = \dfrac{m\lambda}{d}$

Therefore, taking the ends of the visible spectrum to be $\lambda_v = 400$ nm and $\lambda_r = 750$ nm, the ends the different order spectra are:

End of second order: $\sin\theta_{2r} = \dfrac{2\lambda_r}{d} = \dfrac{1\,500 \text{ nm}}{d}$

Start of third order: $\sin\theta_{3v} = \dfrac{3\lambda_v}{d} = \dfrac{1\,200 \text{ nm}}{d}$

Thus, it is seen that $\boxed{\theta_{2r} > \theta_{3v} \text{ and these orders must overlap}}$ regardless of the value of the grating spacing d.

*P27.37 The sound has wavelength $\lambda = \dfrac{v}{f} = \dfrac{343 \text{ m/s}}{37.2\times10^3/\text{s}} = 9.22\times10^{-3}$ m. Each diffracted beam is described by

$$d\sin\theta = m\lambda, \quad m = 0,\ 1,\ 2,\ \ldots$$

The zero-order beam is at $m = 0$, $\theta = 0$. The beams in the first order of interference are to the left and right at $\theta = \sin^{-1}\dfrac{1\lambda}{d} = \sin^{-1}\dfrac{9.22\times10^{-3} \text{ m}}{1.3\times10^{-2} \text{ m}} = \sin^{-1} 0.709 = 45.2°$. For a second-order beam we would need $\theta = \sin^{-1}\dfrac{2\lambda}{d} = \sin^{-1} 2(0.709) = \sin^{-1} 1.42$. No angle, smaller or larger than 90° has a sine greater than 1. Then a diffracted beam does not exist for the second order or any higher order. The whole answer is then, $\boxed{\text{three beams, at } 0° \text{ and at } 45.2° \text{ to the right and to the left}}$

Section 27.9 Diffraction of X-Rays by Crystals

P27.38 $2d\sin\theta = m\lambda$: $\quad \lambda = \dfrac{2d\sin\theta}{m} = \dfrac{2(0.353\times10^{-9} \text{ m})\sin 7.60°}{1} = 9.34\times10^{-11} \text{ m} = \boxed{0.093\,4 \text{ nm}}$

P27.39 $2d\sin\theta = m\lambda$: $\quad \sin\theta = \dfrac{m\lambda}{2d} = \dfrac{1(0.140\times10^{-9} \text{ m})}{2(0.281\times10^{-9} \text{ m})} = 0.249$

and $\boxed{\theta = 14.4°}$

P27.40 Figure 27.28 of the text shows the situation.

$2d\sin\theta = m\lambda$ or $\lambda = \dfrac{2d\sin\theta}{m}$

$m = 1$: $\quad \lambda_1 = \dfrac{2(2.80 \text{ m})\sin 80.0°}{1} = \boxed{5.51 \text{ m}}$

$m = 2$: $\quad \lambda_2 = \dfrac{2(2.80 \text{ m})\sin 80.0°}{2} = \boxed{2.76 \text{ m}}$

$m = 3$: $\quad \lambda_3 = \dfrac{2(2.80 \text{ m})\sin 80.0°}{3} = \boxed{1.84 \text{ m}}$

Section 27.10 Context Connection—Holography

P27.41 (a) The several narrow parallel slits make a diffraction grating. The zeroth- and first- order maxima are separated according to

$d\sin\theta = (1)\lambda \qquad \sin\theta = \dfrac{\lambda}{d} = \dfrac{632.8\times10^{-9} \text{ m}}{1.2\times10^{-3} \text{ m}}$

$\theta = \sin^{-1}(0.000\,527) = 0.000\,527$ rad

$y = L\tan\theta = (1.40 \text{ m})(0.000\,527) = \boxed{0.738 \text{ mm}}$

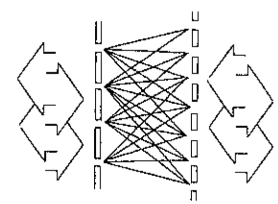

FIG. P27.41

(b) Many equally spaced transparent lines appear on the film. It is itself a diffraction grating. When the same light is sent through the film, it produces interference maxima separated according to

$d\sin\theta = (1)\lambda \qquad \sin\theta = \dfrac{\lambda}{d} = \dfrac{632.8\times10^{-9} \text{ m}}{0.738\times10^{-3} \text{ m}} = 0.000\,857$

$y = L\tan\theta = (1.40 \text{ m})(0.000\,857) = 1.20$ mm

An image of the original set of slits appears on the screen. If the screen is removed, light diverges from the real images with the same wave fronts reconstructed as the original slits produced. Reasoning from the mathematics of Fourier transforms, Gabor showed that light diverging from any object, not just a set of slits, could be used. In the picture, the slits or maxima on the left are separated by 1.20 mm. The slits or maxima on the right are separated by 0.738 mm. The length difference between any pair of lines is an integer number of wavelengths. Light can be sent through equally well toward the right or toward the left.

P27.42 (a) The light in the cavity is incident perpendicularly on the mirrors, although the diagram shows a large angle of incidence for clarity. We ignore the variation of the index of refraction with wavelength. To minimize reflection at a vacuum wavelength of 632.8 nm, the net phase difference between rays (1) and (2) should be 180°. There is automatically a 180° shift in one of the two rays upon reflection, so the extra distance traveled by ray (2) should be one whole wavelength:

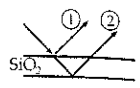

FIG. P27.42

$$2t = \frac{\lambda}{n}$$

$$t = \frac{\lambda}{2n} = \frac{632.8 \text{ nm}}{2(1.458)} = \boxed{217 \text{ nm}}$$

(b) The total phase difference should be 360°, including contributions of 180° by reflection and 180° by extra distance traveled:

$$2t = \frac{\lambda}{2n}$$

$$t = \frac{\lambda}{4n} = \frac{543 \text{ nm}}{4(1.458)} = \boxed{93.1 \text{ nm}}$$

Additional Problems

*P27.43 Along the line of length d joining the source, two identical waves moving in opposite directions add to give a standing wave. An antinode is halfway between the sources. If $\frac{d}{2} > \frac{\lambda}{2}$, there is space for two more antinodes for a total of three. If $\frac{d}{2} > \lambda$, there will be at least five antinodes, and so on. To repeat, if $\frac{d}{\lambda} > 0$, the number of antinodes is 1 or more. If $\frac{d}{\lambda} > 1$, the number of antinodes is 3 or more. If $\frac{d}{\lambda} > 2$, the number of antinodes is 5 or more. In general,

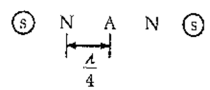

FIG. P27.43

$\boxed{\text{The number of antinodes is 1 plus 2 times the greatest integer less than or equal to } \frac{d}{\lambda}.}$

If $\frac{d}{2} < \frac{\lambda}{4}$, there will be no nodes. If $\frac{d}{2} > \frac{\lambda}{4}$, there will be space for at least two nodes, as shown in the picture. If $\frac{d}{2} > \frac{3\lambda}{4}$, there will be at least four nodes. If $\frac{d}{2} > \frac{5\lambda}{4}$ six or more nodes will fit in, and so on. To repeat, if $2d < \lambda$ the number of nodes is 0. If $2d > \lambda$ the number of nodes is 2 or more. If $2d > 3\lambda$ the number of nodes is 4 or more. If $2d > 5\lambda$ the number of nodes is 6 or more. Again, if $\left(\frac{d}{\lambda} + \frac{1}{2}\right) > 1$, the number of nodes is at least 2. If $\left(\frac{d}{\lambda} + \frac{1}{2}\right) > 2$, the number of nodes is at least 4. If $\left(\frac{d}{\lambda} + \frac{1}{2}\right) > 3$, the number of nodes is at least 6. In general,

continued on next page

$\boxed{\text{the number of nodes is 2 times the greatest nonzero integer less than } \left(\frac{d}{\lambda} + \frac{1}{2}\right).}$

Next, we enumerate the zones of constructive interference. They are described by $d \sin\theta = m\lambda$, $m = 0, 1, 2, \ldots$ with θ counted as positive both left and right of the maximum at $\theta = 0$ in the center. The number of side maxima on each side is the greatest integer satisfying $\sin\theta \le 1$, $d1 \ge m\lambda$, $m \le \frac{d}{\lambda}$. So the total

$\boxed{\text{number of bright fringes is one plus 2 times the greatest integer less than or equal to } \frac{d}{\lambda}.}$

It is equal to the number of antinodes on the line joining the sources.

The interference minima are to the left and right at angles described by $d \sin\theta = \left(m + \frac{1}{2}\right)\lambda$, $m = 0, 1, 2, \ldots$. With $\sin\theta < 1$, $d1 > \left(m_{max} + \frac{1}{2}\right)\lambda$, $m_{max} < \frac{d}{\lambda} - \frac{1}{2}$ or $m_{max} + 1 < \frac{d}{\lambda} + \frac{1}{2}$. Let $n = 1, 2, 3, \ldots$. Then the number of side minima is the greatest integer n less than $\frac{d}{\lambda} + \frac{1}{2}$. Counting both left and right, $\boxed{\text{the number of dark fringes is two times the greatest positive integer less than } \left(\frac{d}{\lambda} + \frac{1}{2}\right).}$ It is equal to the number of nodes in the standing wave between the sources.

P27.44 My middle finger has width $d = 2$ cm.

(a) Two adjacent directions of constructive interference for 600-nm light are described by

$$d \sin\theta = m\lambda$$
$$\theta_0 = 0$$
$$\left(2 \times 10^{-2} \text{ m}\right)\sin\theta_1 = 1\left(6 \times 10^{-7} \text{ m}\right)$$

Thus, $\theta_1 = 2 \times 10^{-3}$ degree

and $\theta_1 - \theta_0 \boxed{\sim 10^{-3} \text{ degree}}$

(b) Choose $\theta_1 = 20°$

$$\left(2 \times 10^{-2} \text{ m}\right)\sin 20° = (1)\lambda$$
$$\lambda = 7 \text{ mm}$$

Millimeter waves are $\boxed{\text{microwaves}}$.

$f = \frac{c}{\lambda}$ $\quad f = \frac{3 \times 10^8 \text{ m/s}}{7 \times 10^{-3} \text{ m}} \boxed{\sim 10^{11} \text{ Hz}}$

P27.45 No phase shift occurs upon reflection from the upper surface (glass to air) of the film, but there will be a shift of $\frac{\lambda}{2}$ due to the reflection at the lower surface of the film (air to metal). The total phase difference in the two reflected beams is

then $\delta = 2nt + \frac{\lambda}{2}$

For constructive interference, $\delta = m\lambda$

or $2(1.00)t + \frac{\lambda}{2} = m\lambda$.

Thus, the film thickness for the m^{th} order bright fringe is

$$t_m = \left(m - \frac{1}{2}\right)\frac{\lambda}{2} = m\left(\frac{\lambda}{2}\right) - \frac{\lambda}{4}$$

and the thickness for the $m-1$ bright fringe is:

$$t_{m-1} = (m-1)\left(\frac{\lambda}{2}\right) - \frac{\lambda}{4}$$

Therefore, the change in thickness required to go from one bright fringe to the next is

$$\Delta t = t_m - t_{m-1} = \frac{\lambda}{2}$$

To go through 200 bright fringes, the change in thickness of the air film must be:

$$200\left(\frac{\lambda}{2}\right) = 100\lambda.$$

Thus, the increase in the length of the rod is

$$\Delta L = 100\lambda = 100\left(5.00\times10^{-7}\text{ m}\right) = 5.00\times10^{-5}\text{ m}.$$

From $\Delta L = L_i \alpha \Delta T$

we have: $\alpha = \frac{\Delta L}{L_i \Delta T} = \frac{5.00\times10^{-5}\text{ m}}{(0.100\text{ m})(25.0°\text{C})} = \boxed{20.0\times10^{-6}\ °\text{C}^{-1}}$.

***P27.46** The central bright fringe is wider than the side bright fringes, so the light must have been diffracted by a $\boxed{\text{single slit}}$. For precision, we measure to the third minimum from the center

$$y = 4.0\text{ cm}$$
$$\tan\theta = \frac{y}{L} = \frac{0.04\text{ m}}{2.6\text{ m}} = 0.015\,4$$
$$\theta = 0.881° = 0.015\,4\text{ rad}$$
$$a\sin\theta = m\lambda$$
$$a = \frac{m\lambda}{\sin\theta} = \frac{3\left(632.8\times10^{-9}\text{ m}\right)}{\sin 0.881°} = \frac{3\left(632.8\times10^{-9}\text{ m}\right)}{0.015\,4} = \boxed{1.23\times10^{-4}\text{ m}}$$

P27.47 For destructive interference, the path length must differ by $m\lambda$. We may treat this problem as a double slit experiment if we remember the light undergoes a $\frac{\pi}{2}$-phase shift at the mirror. The second slit is the mirror image of the source, 1.00 cm below the mirror plane. Modifying the equation for Young's experiment, $\frac{dy}{L} = m\lambda$ we have

$$y_{\text{dark}} = \frac{m\lambda L}{d} = \frac{1\left(5.00\times10^{-7}\text{ m}\right)(100\text{ m})}{\left(2.00\times10^{-2}\text{ m}\right)} = \boxed{2.50\text{ mm}}$$

P27.48 $2\sqrt{(15.0\text{ km})^2 + h^2} = 30.175\text{ km}$

$(15.0\text{ km})^2 + h^2 = 227.63$

$h = \boxed{1.62\text{ km}}$

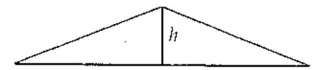

FIG. P27.48

P27.49 For dark fringes, $2nt = m\lambda$

and at the edge of the wedge, $t = \frac{84(500\text{ nm})}{2}$

When submerged in water, $2nt = m\lambda$

$$m = \frac{2(1.33)(42)(500\text{ nm})}{500\text{ nm}}$$

so $m + 1 = \boxed{113\text{ dark fringes}}$.

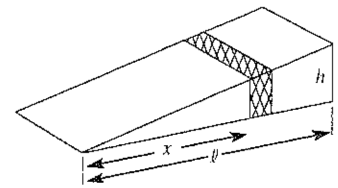

FIG. P27.49

P27.50 (a) Minimum: $2nt = m\lambda_2$ for $m = 0, 1, 2, \ldots$

Maximum: $2nt = \left(m' + \frac{1}{2}\right)\lambda_1$ for $m' = 0, 1, 2, \ldots$

for $\lambda_1 > \lambda_2$, $\left(m' + \frac{1}{2}\right) < m$

so $m' = m - 1$.

Then $2nt = m\lambda_2 = \left(m - \frac{1}{2}\right)\lambda_1$

$2m\lambda_2 = 2m\lambda_1 - \lambda_1$

so $\boxed{m = \frac{\lambda_1}{2(\lambda_1 - \lambda_2)}}$.

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(b) $m = \frac{500}{2(500-370)} = 1.92 \to 2$ (wavelengths measured to ± 5 nm)

Minimum: $2nt = m\lambda_2$

$2(1.40)t = 2(370 \text{ nm})$ $\quad t = 264$ nm

Maximum: $2nt = \left(m - 1 + \frac{1}{2}\right)\lambda = 1.5\lambda$

$2(1.40)t = 1.5(500 \text{ nm})$ $\quad t = 268$ nm

Film thickness $= \boxed{266 \text{ nm}}$

P27.51 The shift between the two reflected waves is $\delta = 2na - b - \frac{\lambda}{2}$ where a and b are as shown in the ray diagram, n is the index of refraction, and the term $\frac{\lambda}{2}$ is due to phase reversal at the top surface. For constructive interference, $\delta = m\lambda$ where m has integer values. This condition becomes

$$2na - b = \left(m + \frac{1}{2}\right)\lambda \qquad (1)$$

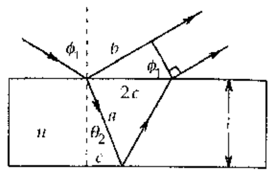

FIG. P27.51

From the figure's geometry, $a = \frac{t}{\cos\theta_2}$

$$c = a\sin\theta_2 = \frac{t\sin\theta_2}{\cos\theta_2}$$

$$b = 2c\sin\phi_1 = \frac{2t\sin\theta_2}{\cos\theta_2}\sin\phi_1$$

Also, from Snell's law, $\sin\phi_1 = n\sin\theta_2$.

Thus, $b = \frac{2nt\sin^2\theta_2}{\cos\theta_2}$

With these results, the condition for constructive interference given in Equation (1) becomes:

$$2n\left(\frac{t}{\cos\theta_2}\right) - \frac{2nt\sin^2\theta_2}{\cos\theta_2} = \frac{2nt}{\cos\theta_2}\left(1 - \sin^2\theta_2\right) = \left(m + \frac{1}{2}\right)\lambda$$

or $\boxed{2nt\cos\theta_2 = \left(m + \frac{1}{2}\right)\lambda}$.

P27.52 (a) Bright bands are observed when $2nt = \left(m + \frac{1}{2}\right)\lambda$.

Hence, the first bright band ($m = 0$) corresponds to $nt = \frac{\lambda}{4}$

Since $\frac{x_1}{x_2} = \frac{t_1}{t_2}$

we have $x_2 = x_1\left(\frac{t_2}{t_1}\right) = x_1\left(\frac{\lambda_2}{\lambda_1}\right) = (3.00 \text{ cm})\left(\frac{680 \text{ nm}}{420 \text{ nm}}\right) = \boxed{4.86 \text{ cm}}$

(b) $t_1 = \frac{\lambda_1}{4n} = \frac{420 \text{ nm}}{4(1.33)} = \boxed{78.9 \text{ nm}}$

$t_2 = \frac{\lambda_2}{4n} = \frac{680 \text{ nm}}{4(1.33)} = \boxed{128 \text{ nm}}$

(c) $\theta \approx \tan\theta = \frac{t_1}{x_1} = \frac{78.9 \text{ nm}}{3.00 \text{ cm}} = \boxed{2.63 \times 10^{-6} \text{ rad}}$

P27.53 The first minimum is at $a\sin\theta = (1)\lambda$.

This has no solution if $\frac{\lambda}{a} > 1$.

or if $a < \lambda = \boxed{632.8 \text{ nm}}$.

P27.54 Consider vocal sound moving at 340 m/s and of frequency 3 000 Hz. Its wavelength is

$$\lambda = \frac{v}{f} = \frac{340 \text{ m/s}}{3\,000 \text{ Hz}} = 0.113 \text{ m}.$$

If your mouth, for horizontal dispersion, behaves similarly to a slit 6.00 cm wide, then $a\sin\theta = m\lambda$ predicts no diffraction minima. You are a nearly isotropic source of this sound. It spreads out from you nearly equally in all directions. On the other hand, if you use a megaphone with width 60.0 cm at its wide end, then $a\sin\theta = m\lambda$ predicts the first diffraction minimum at

$$\theta = \sin^{-1}\left(\frac{m\lambda}{a}\right) = \sin^{-1}\left(\frac{0.113 \text{ m}}{0.600 \text{ m}}\right) = 10.9°$$

This suggests that the sound is radiated mostly toward the front into a diverging beam of angular diameter only about 20°. With less sound energy wasted in other directions, more is available for your intended auditors. We could check that a distant observer to the side or behind you receives less sound when a megaphone is used.

P27.55 $d = \dfrac{1}{400\text{ mm}^{-1}} = 2.50\times10^{-6}\text{ m}$

(a) $d\sin\theta = m\lambda$ $\qquad \theta_a = \sin^{-1}\left(\dfrac{2\times541\times10^{-9}\text{ m}}{2.50\times10^{-6}\text{ m}}\right) = \boxed{25.6°}$

(b) $\lambda = \dfrac{541\times10^{-9}\text{ m}}{1.33} = 4.07\times10^{-7}\text{ m}$ $\qquad \theta_b = \sin^{-1}\left(\dfrac{2\times4.07\times10^{-7}\text{ m}}{2.50\times10^{-6}\text{ m}}\right) = \boxed{19.0°}$

(c) $d\sin\theta_a = 2\lambda$ $\qquad d\sin\theta_b = \dfrac{2\lambda}{n}$

$n\sin\theta_b = (1)\sin\theta_a$

P27.56 (a) $\lambda = \dfrac{v}{f}$ $\qquad \lambda = \dfrac{3.00\times10^{8}\text{ m/s}}{1.40\times10^{9}\text{ s}^{-1}} = 0.214\text{ m}$

$\theta_{min} = 1.22\dfrac{\lambda}{D}$ $\qquad \theta_{min} = 1.22\left(\dfrac{0.214\text{ m}}{3.60\times10^{4}\text{ m}}\right) = \boxed{7.26\ \mu\text{rad}}$

$\theta_{min} = 7.26\ \mu\text{rad}\left(\dfrac{180\times60\times60\text{ s}}{\pi}\right) = \boxed{1.50\text{ arc seconds}}$

(b) $\theta_{min} = \dfrac{d}{L}$ $\qquad d = \theta_{min}L = (7.26\times10^{-6}\text{ rad})(26\,000\text{ ly}) = \boxed{0.189\text{ ly}}$

(c) $\theta_{min} = 1.22\dfrac{\lambda}{D}$ $\qquad \theta_{min} = 1.22\left(\dfrac{500\times10^{-9}\text{ m}}{12.0\times10^{-3}\text{ m}}\right) = \boxed{50.8\ \mu\text{rad}}$ (10.5 seconds of arc)

(d) $d = \theta_{min}L = (50.8\times10^{-6}\text{ rad})(30.0\text{ m}) = 1.52\times10^{-3}\text{ m} = \boxed{1.52\text{ mm}}$

P27.57 (a) $d\sin\theta = m\lambda$

or $d = \dfrac{m\lambda}{\sin\theta} = \dfrac{3(500\times10^{-9}\text{ m})}{\sin32.0°} = 2.83\ \mu\text{m}$

Therefore, lines per unit length $= \dfrac{1}{d} = \dfrac{1}{2.83\times10^{-6}\text{ m}}$

or lines per unit length $= 3.53\times10^{5}\text{ m}^{-1} = \boxed{3.53\times10^{3}\text{ cm}^{-1}}$.

continued on next page

(b) $\sin\theta = \dfrac{m\lambda}{d} = \dfrac{m(500\times10^{-9}\text{ m})}{2.83\times10^{-6}\text{ m}} = m(0.177)$

For $\sin\theta \le 1.00$, we must have $\qquad m(0.177) \le 1.00$

or $\qquad m \le 5.66$.

Therefore, the highest order observed is $\qquad m = 5$.

Total number of primary maxima observed is $\qquad 2m+1 = \boxed{11}$.

***P27.58** (a) Bragg's law applies to the space lattice of melanin rods. Consider the planes $d = 0.25\ \mu\text{m}$ apart. For light at near-normal incidence, strong reflection happens for the wavelength given by $2d\sin\theta = m\lambda$. The longest wavelength reflected strongly corresponds to $m = 1$:

$2(0.25\times10^{-6}\text{ m})\sin90° = 1\lambda$ $\qquad \lambda = 500\text{ nm}$. This is the blue-green color.

(b) For light incident at grazing angle 60°, $2d\sin\theta = m\lambda$ gives $1\lambda = 2(0.25\times10^{-6}\text{ m})\sin60° = 433\text{ nm}$. This is violet.

(c) Your two eyes receive light reflected from the feather at different angles, so they receive light incident at different angles and containing different colors reinforced by constructive interference.

(d) The longest wavelength that can be reflected with extra strength by these melanin rods is the one we computed first, 500 nm blue-green.

(e) If the melanin rods were farther apart (say 0.32 μm) they could reflect red with constructive interference.

***P27.59** A central maximum and side maxima in seven orders of interference appear. If the seventh order is just at 90°,

$d\sin\theta = m\lambda$ $\qquad d1 = 7(654\times10^{-9}\text{ m})$ $\qquad d = 4.58\ \mu\text{m}$.

If the seventh order is at less than 90°, the eighth order might be nearly ready to appear according to

$d1 = 8(654\times10^{-9}\text{ m})$ $\qquad d = 5.23\ \mu\text{m}$.

Thus $\boxed{4.58\ \mu\text{m} < d < 5.23\ \mu\text{m}}$.

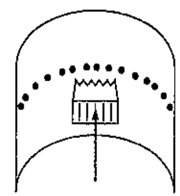

FIG. P27.59

P27.60 (a) We require $\theta_{min} = 1.22\dfrac{\lambda}{D} = \dfrac{\text{radius of diffraction disk}}{L} = \dfrac{D}{2L}$

Then $\boxed{D^2 = 2.44\lambda L}$

(b) $D = \sqrt{2.44(500\times10^{-9}\text{ m})(0.150\text{ m})} = \boxed{428\ \mu\text{m}}$

P27.61 $d\sin\theta = m\lambda$

and, differentiating, $d(\cos\theta)d\theta = md\lambda$

or $d\sqrt{1-\sin^2\theta}\Delta\theta \approx m\Delta\lambda$

$$d\sqrt{1-\frac{m^2\lambda^2}{d^2}}\Delta\theta \approx m\Delta\lambda$$

so $$\boxed{\Delta\theta \approx \frac{\Delta\lambda}{\sqrt{(d^2/m^2)-\lambda^2}}}$$

ANSWERS TO EVEN PROBLEMS

P27.2 (a) 36.2°; (b) 5.08 cm; (c) 508 THz

P27.4 maxima at 0°, 29.1°, and 76.3°; minima at 14.1° and 46.8°

P27.6 11.3 m

P27.8 (a) 10.0 m;
(b) 516 m;
(c) Only the runway centerline is a maximum for the interference patterns for both frequencies. If the frequencies were related by a ratio of small integers $\frac{k}{\ell}$, the plane could by mistake fly along the k-th side maximum of one signal where it coincides with the ℓ-th side maximum of the other.

P27.10 see the solution

P27.12 (a) 1.29 rad; (b) 99.6 nm

P27.14 612 nm

P27.16 0.500 cm

P27.18 (a) 238 nm; (b) λ increase; (c) 328 nm

P27.20 4.22 mm

P27.22 547 nm

P27.24 three maxima, at 0° and near 46° on both sides

P27.26 1.00 mrad

P27.28 she can resolve only the violet tubes if she is between 186 m and 271 m away

P27.30 neither, it can resolve objects no closer than several centimeters apart

P27.32 514 nm

P27.34 1.81 μm 1.81 m

P27.36 see the solution

P27.38 93.4 pm

P27.40 5.51 m, 2.76 m, 1.84 m

P27.42 (a) 217 nm; (b) 93.1 nm

P27.44 (a) $\sim 10^{-3}$ degree;
(b) $\sim 10^{11}$ Hz, microwave

P27.46 one slit 0.123 mm wide

P27.48 1.62 km

P27.50 (a) see the solution; (b) 266 nm

P27.52 (a) 4.86 cm from the top;
(b) 78.9 nm and 128 nm;
(c) 2.63×10^{-6} rad

P27.54 see the solution

P27.56 (a) 7.26 μrad (1.50 seconds of arc);
(b) 0.189 ly;
(c) 50.8 μrad (10.5 seconds of arc);
(d) 1.52 mm

P27.58 see the solution

P27.60 (a) $D^2 = 2.44\lambda L$; (b) 0.428 mm

CONTEXT 8 CONCLUSION SOLUTIONS TO PROBLEMS

CC8.1 Depth = one-quarter of the wavelength in plastic.

$$t = \frac{\lambda}{4n} = \frac{780\text{ nm}}{4(1.50)} = \boxed{130\text{ nm}}$$

CC8.2 For a side maximum, $\tan\theta = \frac{y}{L} = \frac{0.4\ \mu\text{m}}{6.9\ \mu\text{m}}$

$\theta = 3.32°$

$d\sin\theta = m\lambda$ $\quad d = \frac{(1)(780\times10^{-9}\text{ m})}{\sin 3.32°} = 13.5\ \mu\text{m}$.

The number of grooves per millimeter $= \frac{1\times10^{-3}\text{ m}}{13.5\times10^{-6}\text{ m}} = \boxed{74.2}$.

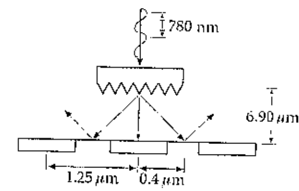

FIG. CC8.2

***CC8.3** In one second, a 1.3-m length of audio track passes the laser. This length includes 705 600 bits of information. The average length per bit is $\frac{1.3\text{ m}}{705\ 600\text{ bits}} = 1.8\times10^{-6}$ m/bit $= \boxed{1.8\ \mu\text{m/bit}}$. The average length per bit of *total* information on a CD is smaller than this because the disc includes additional information, such as error correction codes, song numbers, and timing codes. The shortest length per bit of total information is actually about 0.8 μm/bit.

***CC8.4** We convert each bit to a power of 2 and add them up:

$1\times2^{15} = 32\ 768$	$0\times2^{14} = 0$	$1\times2^{13} = 8\ 192$	$1\times2^{12} = 4\ 096$
$1\times2^{11} = 2\ 048$	$0\times2^{12} = 0$	$1\times2^{9} = 512$	$1\times2^{8} = 256$
$1\times2^{7} = 128$	$0\times2^{6} = 0$	$1\times2^{5} = 32$	$1\times2^{4} = 16$
$1\times2^{3} = 8$	$0\times2^{2} = 0$	$1\times2^{1} = 2$	$1\times2^{0} = 1$

sum $= \boxed{48\ 059}$

The CD player converts this number into a voltage, representing one of the 44 100 values that will be used to build one second of the electronic waveform that represents the recorded sound.

CC8.5 The volume of the cylinder is $\pi r^2 h = \pi(10^{-6}\text{ m})^2(10^{-6}\text{ m}) \approx 3\times10^{-18}\text{ m}^3$

The mass of the magnetic material is $\rho V = (2\times10^3\text{ kg/m}^3)(3\times10^{-18}\text{ m}^3) \approx 6\times10^{-15}$ kg

We model the disk as absorbing all of the incident light. To identify the amount of energy transfer T_{ER} by electromagnetic radiation, we compute the heat that would have the same effect:

continued on next page

$$"Q" = mc\Delta T = \left(6\times10^{-15}\text{ kg}\right)\left(\frac{300\text{ J}}{\text{kg}\cdot{}^\circ\text{C}}\right)(600\text{ K} - 300\text{ K})\left(\frac{1^\circ\text{C}}{1\text{ K}}\right)$$

$$"Q" = 6\times10^{-10}\text{ J}$$

The pit moves past in a time interval of $\Delta t = \dfrac{2\times10^{-6}\text{ m}}{1\text{ m/s}} = 2\times10^{-6}\text{ s}$.

So, the intensity is $I = \dfrac{\mathcal{P}}{A} = \dfrac{"Q"}{A\,\Delta t}$ $\quad I = \dfrac{6\times10^{-10}\text{ J}}{\left[\pi\left(10^{-6}\text{ m}\right)^2\right]\left(2\times10^{-6}\text{ s}\right)} = 9\times10^{7}\text{ W/m}^2$

$$I\ \boxed{\sim10^{8}\text{ W/m}^2}$$

ANSWERS TO EVEN CONTEXT 8 CONCLUSION PROBLEMS

CC8.2 74.2 grooves/mm

CC8.4 48 059

Chapter 28
Quantum Physics

CHAPTER OUTLINE

ANSWERS TO QUESTIONS

Q28.1 Planck made two new assumptions: (1) molecular energy is quantized and (2) molecules emit or absorb energy in discrete irreducible packets. These assumptions contradict the classical idea of energy as continuously divisible. They also imply that an atom must have a definite structure—it cannot just be a soup of electrons orbiting the nucleus.

Q28.2 (c) UV light has the highest frequency of the three, and hence each photon delivers more energy to a skin cell. This explains why you can become sunburned on a cloudy day: clouds block visible light and infrared, but not much ultraviolet. You usually do not become sunburned through window glass, even though you can see the visible light from the Sun coming through the window, because the glass absorbs much of the ultraviolet and reemits it as infrared.

Q28.3 No. The second metal may have a larger work function than the first, in which case the incident photons may not have enough energy to eject photoelectrons.

Q28.4 The Compton effect describes the *scattering* of photons from electrons, while the photoelectric effect predicts the ejection of electrons due to the *absorption* of photons by a material.

Q28.5 Wave theory predicts that the photoelectric effect should occur at any frequency, provided the light intensity is high enough. However, as seen in the photoelectric experiments, the light must have a sufficiently high frequency for the effect to occur.

Q28.6 A few photons would only give a few dots of exposure, apparently randomly scattered.

Q28.7 The x-ray photon transfers some of its energy to the electron. Thus, its frequency must decrease.

Q28.8 Light has both classical-wave and classical-particle characteristics. In single- and double-slit experiments light behaves like a wave. In the photoelectric effect light behaves like a particle. Light may be characterized as an electromagnetic wave with a particular wavelength or frequency, yet at the same time light may be characterized as a stream of photons, each carrying a discrete energy, hf. Since light displays *both* wave and particle characteristics, perhaps it would be fair to call light a "wavicle". It is customary to call a photon a quantum particle, different from a classical particle.