Chapter 18

Heat Engines, Entropy, and the Second Law of Thermodynamics

CHAPTER OUTLINE

ANSWERS TO QUESTIONS

Q18.1 First, the efficiency of the automobile engine cannot exceed the Carnot efficiency: it is limited by the temperature of burning fuel and the temperature of the environment into which the exhaust is dumped. Second, the engine block cannot be allowed to go over a certain temperature. Third, any practical engine has friction, incomplete burning of fuel, and limits set by timing and energy transfer by heat.

Q18.2 No. Any heat engine takes in energy by heat and must also put out energy by heat. The energy that is dumped as exhaust into the low-temperature sink will always be thermal pollution in the outside environment. So-called "steady growth" in human energy use cannot continue.

Q18.3 A higher steam temperature means that more energy can be extracted from the steam. For a constant temperature heat sink at T_c, and steam at T_h, the efficiency of the power plant is determined by $\frac{T_h - T_c}{T_h} = 1 - \frac{T_c}{T_h}$ and is maximized for a high T_h.

Q18.4 No. The first law of thermodynamics is a statement about energy conservation, while the second is a statement about stable thermal equilibrium. They are by no means mutually exclusive. For the particular case of a cycling heat engine, the first law implies $|Q_h| = W_{eng} + |Q_c|$, and the second law implies $|Q_c| > 0$.

Q18.5 Take an automobile as an example. According to the first law or the idea of energy conservation, it must take in all the energy it puts out. Its energy source is chemical energy in gasoline. During the combustion process, some of that energy goes into moving the pistons and eventually into the mechanical motion of the car. Clearly much of the energy goes into heat, which, through the cooling system, is transferred into the atmosphere. Moreover, there are numerous places where friction, both mechanical and fluid, turns mechanical energy into heat. In even the most efficient internal combustion engine cars, less than 30% of the energy from the fuel actually goes into moving the car. The rest ends up as useless heat put into the atmosphere.

Q18.6 Suppose the ambient temperature is 20°C. A gas can be heated to the temperature of the bottom of the pond, and allowed to cool as it blows through a turbine. The Carnot efficiency of such an engine is about $e_c = \frac{\Delta T}{T_h} = \frac{80}{373} = 22\%$

Q18.7 No, because the work done to run the heat pump represents energy transferred into the house by heat.

Q18.8 A slice of hot pizza cools off. Road friction brings a skidding car to a stop. A cup falls to the floor and shatters. Your cat dies. Any process is irreversible if it looks funny or frightening when shown in a videotape running backwards. The free flight of a projectile is nearly reversible.

Q18.9 (a) When the two sides of the semiconductor are at different temperatures, an electric potential (voltage) is generated across the material, which can drive electric current through an external circuit. The two cups at 50°C contain the same amount of internal energy as the pair of hot and cold cups. But no energy flows by heat through the converter bridging between them and no voltage is generated across the semiconductors.

(b) A heat engine must put out exhaust energy by heat. The cold cup provides a sink to absorb output or wasted energy by heat, which has nowhere to go between two cups of equally warm water.

Q18.10 The rest of the Universe must have an entropy change of +8.0 J/K, or more.

Q18.11 (a) For an expanding ideal gas at constant temperature, $\Delta S = \frac{\Delta Q}{T} = nR\ln\left(\frac{V_2}{V_1}\right)$

(b) For a reversible adiabatic expansion $\Delta Q = 0$, and $\Delta S = 0$. An ideal gas undergoing an irreversible adiabatic expansion can have any positive value for ΔS up to the value given in part (a).

Q18.12 To increase its entropy, raise its temperature. To decrease its entropy, lower its temperature. "Remove energy from it by heat" is not such a good answer, for if you hammer on it or rub it with a blunt file and at the same time remove energy from it by heat into a constant temperature bath, its entropy can stay constant.

Q18.13 An analogy used by Carnot is instructive: A waterfall continuously converts mechanical energy into internal energy. It continuously creates entropy as the organized motion of the falling water turns into disorganized molecular motion. We humans put turbines into the waterfall, diverting some of the energy stream to our use. Water flows spontaneously from high to low elevation and energy spontaneously flows by heat from high to low temperature. Into the great flow of solar radiation from Sun to Earth, living things put themselves. They live on energy flow, more than just on energy. A basking snake diverts energy from a high-temperature source (the Sun) through itself temporarily, before the energy inevitably is radiated from the body of the snake to a low-temperature sink (outer space). A tree builds organized cellulose molecules and we build libraries and babies who look like their grandmothers, all out of a thin diverted stream in the universal flow of energy crashing down to disorder. We do not violate the second law, for we build local reductions in the entropy of one thing within the inexorable increase in the total entropy of the Universe. Your roommate's exercise puts energy into the room by heat.

Q18.14 Either statement can be considered an instructive analogy. We choose to take the first view. All processes require energy, either as energy content or as energy input. The kinetic energy that it possessed at its formation continues to make the Earth go around. Energy released by nuclear reactions in the core of the Sun drives weather on the Earth and essentially all processes in the biosphere. The energy intensity of sunlight controls how lush a forest or jungle can be and how warm a planet is. Continuous energy input is not required for the motion of the planet. Continuous energy input is required for life because energy tends to be continuously degraded, as heat flows into lower-temperature sinks. The continuously increasing entropy of the Universe is the index to energy-transfers completed.

Q18.15 Shaking opens up spaces between jellybeans. The smaller ones more often can fall down into spaces below them. The accumulation of larger candies on top and smaller ones on the bottom implies a small increase in order, a small decrease in one contribution to the total entropy, but the second law is not violated. The total entropy increases as the system warms up, its increase in internal energy coming from the work put into shaking the box and also from a bit of gravitational energy loss as the beans settle compactly together.

SOLUTIONS TO PROBLEMS

Section 18.1 Heat Engines and the Second Law of Thermodynamics

P18.1 (a) $$e=\frac{W_{\text{eng}}}{|Q_h|}=\frac{25.0\text{ J}}{360\text{ J}}=\boxed{0.069\,4}\text{ or }\boxed{6.94\%}$$

(b) $$|Q_c|=|Q_h|-W_{\text{eng}}=360\text{ J}-25.0\text{ J}=\boxed{335\text{ J}}$$

***P18.2** (a) The input energy each hour is

$$\left(7.89\times10^{3}\text{ J/revolution}\right)(2\,500\text{ rev/min})\frac{60\text{ min}}{1\text{ h}}=1.18\times10^{9}\text{ J/h}$$

implying fuel input $\left(1.18\times10^{9}\text{ J/h}\right)\left(\frac{1\text{ L}}{4.03\times10^{7}\text{ J}}\right)=\boxed{29.4\text{ L/h}}$

(b) $Q_h=W_{\text{eng}}+|Q_c|$. For a continuous-transfer process we may divide by time to have

$$\frac{Q_h}{\Delta t}=\frac{W_{\text{eng}}}{\Delta t}+\frac{|Q_c|}{\Delta t}$$

$$\text{Useful power output}=\frac{W_{\text{eng}}}{\Delta t}=\frac{Q_h}{\Delta t}-\frac{|Q_c|}{\Delta t}$$

$$=\left(\frac{7.89\times10^{3}\text{ J}}{\text{revolution}}-\frac{4.58\times10^{3}\text{ J}}{\text{revolution}}\right)\frac{2\,500\text{ rev}}{1\text{ min}}\frac{1\text{ min}}{60\text{ s}}=1.38\times10^{5}\text{ W}$$

$$\mathcal{P}_{\text{eng}}=1.38\times10^{5}\text{ W}\left(\frac{1\text{ hp}}{746\text{ W}}\right)=\boxed{185\text{ hp}}$$

(c) $$\mathcal{P}_{\text{eng}}=\tau\omega\Rightarrow\tau=\frac{\mathcal{P}_{\text{eng}}}{\omega}=\frac{1.38\times10^{5}\text{ J/s}}{(2\,500\text{ rev/60 s})}\left(\frac{1\text{ rev}}{2\pi\text{ rad}}\right)=\boxed{527\text{ N}\cdot\text{m}}$$

(d) $$\frac{|Q_c|}{\Delta t}=\frac{4.58\times10^{3}\text{ J}}{\text{revolution}}\left(\frac{2\,500\text{ rev}}{60\text{ s}}\right)=\boxed{1.91\times10^{5}\text{ W}}$$

P18.3 (a) We have $e=\frac{W_{\text{eng}}}{|Q_h|}=\frac{|Q_h|-|Q_c|}{|Q_h|}=1-\frac{|Q_c|}{|Q_h|}=0.250$

with $|Q_c|=8\,000\text{ J}$, we have $|Q_h|=\boxed{10.7\text{ kJ}}$

(b) $W_{\text{eng}}=|Q_h|-|Q_c|=2\,667\text{ J}$

and from $\mathcal{P}=\frac{W_{\text{eng}}}{\Delta t}$, we have $\Delta t=\frac{W_{\text{eng}}}{\mathcal{P}}=\frac{2\,667\text{ J}}{5\,000\text{ J/s}}=\boxed{0.533\text{ s}}$.

***P18.4** The engine's output work we identify with the kinetic energy of the bullet:

$$W_{\text{eng}}=K=\frac{1}{2}mv^2=\frac{1}{2}0.002\,4\text{ kg}(320\text{ m/s})^2=123\text{ J}$$

$$e=\frac{W_{\text{eng}}}{Q_h}$$

$$Q_h=\frac{W_{\text{eng}}}{e}=\frac{123\text{ J}}{0.011}=1.12\times10^4\text{ J}$$

$$Q_h=W_{\text{eng}}+|Q_c|$$

The energy exhaust is

$$|Q_c|=Q_h-W_{\text{eng}}=1.12\times10^4\text{ J}-123\text{ J}=1.10\times10^4\text{ J}$$

$$Q=mc\Delta T$$

$$\Delta T=\frac{Q}{mc}=\frac{1.10\times10^4\text{ J kg}^\circ\text{C}}{1.80\text{ kg }448\text{ J}}=\boxed{13.7^\circ\text{C}}$$

Section 18.2 Reversible and Irreversible Processes

Section 18.3 The Carnot Engine

P18.5 $T_c=703\text{ K}$ $T_h=2\,143\text{ K}$

(a) $e_c=\frac{\Delta T}{T_h}=\frac{1\,440}{2\,143}=\boxed{67.2\%}$

(b) $|Q_h|=1.40\times10^5\text{ J}$, $W_{\text{eng}}=0.420|Q_h|$

$$\mathcal{P}=\frac{W_{\text{eng}}}{\Delta t}=\frac{5.88\times10^4\text{ J}}{1\text{ s}}=\boxed{58.8\text{ kW}}$$

P18.6 When $e = e_c$, $1 - \frac{T_c}{T_h} = \frac{W_{\text{eng}}}{|Q_h|}$ and $\frac{\frac{W_{\text{eng}}}{\Delta t}}{\frac{|Q_h|}{\Delta t}} = 1 - \frac{T_c}{T_h}$

(a) $$|Q_h| = \frac{\left(\frac{W_{\text{eng}}}{\Delta t}\right)\Delta t}{1 - \frac{T_c}{T_h}} = \frac{(1.50 \times 10^5 \text{ W})(3\,600 \text{ s})}{1 - \frac{293}{773}}$$

$$|Q_h| = 8.70 \times 10^8 \text{ J} = \boxed{870 \text{ MJ}}$$

(b) $$|Q_c| = |Q_h| - \left(\frac{W_{\text{eng}}}{\Delta t}\right)\Delta t = 8.70 \times 10^8 - (1.50 \times 10^5)(3\,600) = 3.30 \times 10^8 \text{ J} = \boxed{330 \text{ MJ}}$$

P18.7 Isothermal expansion at $T_h = 523$ K

Isothermal compression at $T_c = 323$ K

Gas absorbs 1 200 J during expansion.

(a) $$|Q_c| = |Q_h|\left(\frac{T_c}{T_h}\right) = 1\,200 \text{ J}\left(\frac{323}{523}\right) = \boxed{741 \text{ J}}$$

(b) $$W_{\text{eng}} = |Q_h| - |Q_c| = (1\,200 - 741) \text{ J} = \boxed{459 \text{ J}}$$

P18.8 We use $e_c = 1 - \frac{T_c}{T_h}$

as $0.300 = 1 - \frac{573 \text{ K}}{T_h}$

From which, $T_h = 819 \text{ K} = \boxed{546°\text{C}}$

P18.9 The Carnot summer efficiency is $e_{c,s} = 1 - \frac{T_c}{T_h} = 1 - \frac{(273 + 20) \text{ K}}{(273 + 350) \text{ K}} = 0.530$

And in winter, $e_{c,w} = 1 - \frac{283}{623} = 0.546$

Then the actual winter efficiency is $0.320\left(\frac{0.546}{0.530}\right) = \boxed{0.330}$ or $\boxed{33.0\%}$

P18.10 (a) $$e_{\text{max}} = 1 - \frac{T_c}{T_h} = 1 - \frac{278}{293} = 5.12\times10^{-2} = \boxed{5.12\%}$$

(b) $$\mathcal{P} = \frac{W_{\text{eng}}}{\Delta t} = 75.0\times10^{6}\ \text{J/s}$$

Therefore, $$W_{\text{eng}} = \left(75.0\times10^{6}\ \text{J/s}\right)\left(3\,600\ \text{s/h}\right) = 2.70\times10^{11}\ \text{J/h}$$

From $e = \dfrac{W_{\text{eng}}}{|Q_h|}$ we find $$|Q_h| = \frac{W_{\text{eng}}}{e} = \frac{2.70\times10^{11}\ \text{J/h}}{5.12\times10^{-2}} = 5.27\times10^{12}\ \text{J/h} = \boxed{5.27\ \text{TJ/h}}$$

(c) As fossil-fuel prices rise, this way to use solar energy will become a good buy.

***P18.11** (a) $$e = \frac{W_{\text{eng1}} + W_{\text{eng2}}}{Q_{h1}} = \frac{e_1Q_{1h} + e_2Q_{2h}}{Q_{h1}}$$

Now $Q_{2h} = Q_{1c} = Q_{1h} - W_{\text{eng1}} = Q_{h1} - e_1Q_{1h}$.

So $$e = \frac{e_1Q_{1h} + e_2\left(Q_{1h} - e_1Q_{1h}\right)}{Q_{1h}} = \boxed{e_1 + e_2 - e_1e_2}.$$

(b) $$e = e_1 + e_2 - e_1e_2 = 1 - \frac{T_i}{T_h} + 1 - \frac{T_c}{T_i} - \left(1 - \frac{T_i}{T_h}\right)\left(1 - \frac{T_c}{T_i}\right) = 2 - \frac{T_i}{T_h} - \frac{T_c}{T_i} - 1 + \frac{T_i}{T_h} + \frac{T_c}{T_i} - \frac{T_c}{T_h} = \boxed{1 - \frac{T_c}{T_h}}$$

The combination of reversible engines is itself a reversible engine so it has the Carnot efficiency.

(c) With $W_{\text{eng2}} = W_{\text{eng1}}$, $e = \dfrac{W_{\text{eng1}} + W_{\text{eng2}}}{Q_{1h}} = \dfrac{2W_{\text{eng1}}}{Q_{1h}} = 2e_1$

$$1 - \frac{T_c}{T_h} = 2\left(1 - \frac{T_i}{T_h}\right)$$
$$0 - \frac{T_c}{T_h} = 1 - \frac{2T_i}{T_h}$$
$$2T_i = T_h + T_c$$
$$\boxed{T_i = \frac{1}{2}\left(T_h + T_c\right)}$$

(d) $$e_1 = e_2 = 1 - \frac{T_i}{T_h} = 1 - \frac{T_c}{T_i}$$
$$T_i^2 = T_cT_h$$
$$\boxed{T_i = \left(T_hT_c\right)^{1/2}}$$

P18.12 (a) First, consider the adiabatic process $D \to A$:

$P_D V_D^\gamma = P_A V_A^\gamma$ so $$P_D = P_A\left(\frac{V_A}{V_D}\right)^\gamma = 1\,400\text{ kPa}\left(\frac{10.0\text{ L}}{15.0\text{ L}}\right)^{5/3} = \boxed{712\text{ kPa}}$$

Also $$\left(\frac{nRT_D}{V_D}\right)V_D^\gamma = \left(\frac{nRT_A}{V_A}\right)V_A^\gamma$$

or $$T_D = T_A\left(\frac{V_A}{V_D}\right)^{\gamma-1} = 720\text{ K}\left(\frac{10.0}{15.0}\right)^{2/3} = \boxed{549\text{ K}}$$

Now, consider the isothermal process $C \to D$: $T_C = T_D = \boxed{549\text{ K}}$.

$$P_C = P_D\left(\frac{V_D}{V_C}\right) = \left[P_A\left(\frac{V_A}{V_D}\right)^\gamma\right]\left(\frac{V_D}{V_C}\right) = \frac{P_A V_A^\gamma}{V_C V_D^{\gamma-1}}$$

$$P_C = \frac{1\,400\text{ kPa}(10.0\text{ L})^{5/3}}{24.0\text{ L}(15.0\text{ L})^{2/3}} = \boxed{445\text{ kPa}}$$

Next, consider the adiabatic process $B \to C$: $P_B V_B^\gamma = P_C V_C^\gamma$.

But, $P_C = \frac{P_A V_A^\gamma}{V_C V_D^{\gamma-1}}$ from above. Also considering the isothermal process, $P_B = P_A\left(\frac{V_A}{V_B}\right)$.

Hence, $P_A\left(\frac{V_A}{V_B}\right)V_B^\gamma = \left(\frac{P_A V_A^\gamma}{V_C V_D^{\gamma-1}}\right)V_C^\gamma$ which reduces to $V_B = \frac{V_A V_C}{V_D} = \frac{10.0\text{ L}(24.0\text{ L})}{15.0\text{ L}} = \boxed{16.0\text{ L}}$.

Finally, $P_B = P_A\left(\frac{V_A}{V_B}\right) = 1\,400\text{ kPa}\left(\frac{10.0\text{ L}}{16.0\text{ L}}\right) = \boxed{875\text{ kPa}}$.

State	P(kPa)	V(L)	T(K)
A	1 400	10.0	720
B	875	16.0	720
C	445	24.0	549
D	712	15.0	549

(b) For the isothermal process $A \to B$: $\Delta E_{\text{int}} = nC_V\Delta T = \boxed{0}$

so $Q = -W = nRT\ln\left(\frac{V_B}{V_A}\right) = 2.34\text{ mol}(8.314\text{ J/mol}\cdot\text{K})(720\text{ K})\ln\left(\frac{16.0}{10.0}\right) = \boxed{+6.58\text{ kJ}}$.

For the adiabatic process $B \to C$: $Q = \boxed{0}$

$$\Delta E_{\text{int}} = nC_V(T_C - T_B) = 2.34\text{ mol}\left[\frac{3}{2}(8.314\text{ J/mol}\cdot\text{K})\right](549 - 720)\text{ K} = \boxed{-4.98\text{ kJ}}$$

and $W = -Q + \Delta E_{\text{int}} = 0 + (-4.98\text{ kJ}) = \boxed{-4.98\text{ kJ}}$.

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For the isothermal process $C \to D$: $\Delta E_{\text{int}} = nC_V\Delta T = \boxed{0}$

and $Q = -W = nRT\ln\left(\frac{V_D}{V_C}\right) = 2.34\text{ mol}(8.314\text{ J/mol K})(549\text{ K})\ln\left(\frac{15.0}{24.0}\right) = \boxed{-5.02\text{ kJ}}$

Finally, for the adiabatic process $D \to A$: $Q = \boxed{0}$

$$\Delta E_{\text{int}} = nC_V(T_A - T_D) = 2.34\text{ mol}\left[\frac{3}{2}(8.314\text{ J/mol K})\right](720 - 549)\text{ K} = \boxed{+4.98\text{ kJ}}$$

and $W = -Q + \Delta E_{\text{int}} = 0 + 4.98\text{ kJ} = \boxed{+4.98\text{ kJ}}$

Process	Q(kJ)	W(kJ)	ΔE_{int} (kJ)
$A \to B$	+6.58	−6.58	0
$B \to C$	0	−4.98	−4.98
$C \to D$	−5.02	+5.02	0
$D \to A$	0	+4.98	+4.98
ABCDA	+1.56	−1.56	0

The work done *by* the engine is the negative of the work input. The output work W_{eng} is given by the work column in the table with all signs reversed.

(c) $$e = \frac{W_{\text{eng}}}{|Q_h|} = \frac{-W_{ABCD}}{Q_{A\to B}} = \frac{1.56\text{ kJ}}{6.58\text{ kJ}} = 0.237 \text{ or } \boxed{23.7\%}$$

$$e_c = 1 - \frac{T_c}{T_h} = 1 - \frac{549}{720} = 0.237 \text{ or } \boxed{23.7\%}$$

Section 18.4 Heat Pumps and Refrigerators

***P18.13** $\text{COP(refrigerator)} = \frac{Q_c}{W}$

(a) If $Q_c = 120\text{ J}$ and $\text{COP} = 5.00$, then $\boxed{W = 24.0\text{ J}}$

(b) Heat expelled = Heat removed + Work done.

$$Q_h = Q_c + W = 120\text{ J} + 24\text{ J} = \boxed{144\text{ J}}$$

***P18.14** $\text{COP} = 3.00 = \frac{Q_c}{W}$. Therefore, $W = \frac{Q_c}{3.00}$.

The heat removed each minute is

$$\frac{Q_C}{t} = (0.0300\text{ kg})(4\,186\text{ J/kg}\,^\circ\text{C})(22.0^\circ\text{C}) + (0.0300\text{ kg})\left(3.33\times10^5\text{ J/kg}\right)$$
$$+(0.0300\text{ kg})(2\,090\text{ J/kg}\,^\circ\text{C})(20.0^\circ\text{C}) = 1.40\times10^4\text{ J/min}$$

or, $\frac{Q_c}{t} = 233\text{ J/s}$.

Thus, the work done per sec $= \mathcal{P} = \frac{233\text{ J/s}}{3.00} = \boxed{77.8\text{ W}}$.

P18.15 (a) $\left(10.0\,\frac{\text{Btu}}{\text{h W}}\right)\left(\frac{1055\text{ J}}{1\text{ Btu}}\right)\left(\frac{1\text{ h}}{3\,600\text{ s}}\right)\left(\frac{1\text{ W}}{1\text{ J/s}}\right) = \boxed{2.93}$

(b) Coefficient of performance for a refrigerator: $\boxed{(\text{COP})_{\text{refrigerator}}}$

(c) With EER 5, $5\,\frac{\text{Btu}}{\text{h W}} = \frac{10\,000\text{ Btu/h}}{\mathcal{P}}$: $\mathcal{P} = \frac{10\,000\text{ Btu/h}}{\frac{5\text{ Btu}}{\text{h}\cdot\text{W}}} = 2\,000\text{ W} = 2.00\text{ kW}$

Energy purchased is $\mathcal{P}\Delta t = (2.00\text{ kW})(1\,500\text{ h}) = 3.00\times10^3\text{ kWh}$

$\text{Cost} = \left(3.00\times10^3\text{ kWh}\right)(0.100\text{ \$/kWh}) = \$300$

With EER 10, $10\,\frac{\text{Btu}}{\text{h}\cdot\text{W}} = \frac{10\,000\text{ Btu/h}}{\mathcal{P}}$: $\mathcal{P} = \frac{10\,000\text{ Btu/h}}{\frac{10\text{ Btu}}{\text{h}\cdot\text{W}}} = 1\,000\text{ W} = 1.00\text{ kW}$

Energy purchased is $\mathcal{P}\Delta t = (1.00\text{ kW})(1\,500\text{ h}) = 1.50\times10^3\text{ kWh}$

$\text{Cost} = \left(1.50\times10^3\text{ kWh}\right)(0.100\text{ \$/kWh}) = \$150$

Thus, the cost for air conditioning is $\boxed{\text{half as much with EER 10}}$

P18.16 $(\text{COP})_{\text{refrig}} = \frac{T_c}{\Delta T} = \frac{270}{30.0} = \boxed{9.00}$

P18.17 (a) For a complete cycle, $\Delta E_{\text{int}} = 0$ and $W = |Q_h| - |Q_c| = |Q_c|\left[\frac{|(Q_h)|}{|Q_c|} - 1\right]$.

We have already shown that for a Carnot cycle (and only for a Carnot cycle) $\frac{|Q_h|}{|Q_c|} = \frac{T_h}{T_c}$.

Therefore, $W = \boxed{|Q_c|\left[\frac{T_h - T_c}{T_c}\right]}$

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(b) We have the definition of the coefficient of performance for a refrigerator, $\text{COP} = \dfrac{|Q_c|}{W}$

Using the result from part (a), this becomes $\text{COP} = \boxed{\dfrac{T_c}{T_h - T_c}}$

P18.18 $(\text{COP})_{\text{heat pump}} = \dfrac{|Q_c| + W}{W} = \dfrac{T_h}{\Delta T} = \dfrac{295}{25} = \boxed{11.8}$

P18.19 $\text{COP} = 0.100\,\text{COP}_{\text{Carnot cycle}}$

or $\dfrac{|Q_h|}{W} = 0.100\left(\dfrac{|Q_h|}{W}\right)_{\text{Carnot cycle}} = 0.100\left(\dfrac{1}{\text{Carnot efficiency}}\right)$

$$\frac{|Q_h|}{W} = 0.100\left(\frac{T_h}{T_h - T_c}\right) = 0.100\left(\frac{293\text{ K}}{293\text{ K} - 268\text{ K}}\right) = 1.17$$

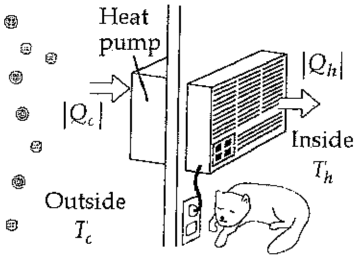

FIG. P18.19

Thus, $\boxed{\text{1.17 joules of energy enter the room by heat for each joule of work done.}}$

***P18.20** $e = \dfrac{W}{Q_h} = 0.350 \qquad W = 0.350Q_h$

$Q_h = W + Q_c \qquad Q_c = 0.650Q_h$

$\text{COP(refrigerator)} = \dfrac{Q_c}{W} = \dfrac{0.650Q_h}{0.350Q_h} = \boxed{1.86}$

P18.21 $(\text{COP})_{\text{Carnot refrig}} = \dfrac{T_c}{\Delta T} = \dfrac{4.00}{289} = 0.0138 = \dfrac{|Q_c|}{W}$

$\therefore W = \boxed{72.2\text{ J}}$ per 1 J energy removed by heat

***P18.22** (a) The TV set uses energy at the rate $400\text{ J/s}\left(\dfrac{\$0.15}{\text{kWh}}\right)\left(\dfrac{1\text{ kWh}}{3.6\times10^6\text{ J}}\right)\left(\dfrac{60\text{ s}}{1\text{ min}}\right) = \boxed{\$1.00\times10^{-3}/\text{min}}$

(b) An air conditioner is a refrigerator:

$$COP = \frac{|Q_c|}{W} = \frac{\frac{|Q_c|}{\Delta t}}{\frac{W}{\Delta t}}$$

$$\frac{W}{\Delta t} = \frac{\frac{|Q_c|}{\Delta t}}{COP} = \frac{400\text{ J/s}}{4.50} = 88.9\text{ J/s}$$

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This is the rate at which extra electric energy is required by the air conditioner to keep the room cool, with cost

$$88.9W\left(\frac{\$0.15}{\text{kwh}}\right)\left(\frac{\text{k}}{1\,000}\right)\left(\frac{1\text{ h}}{60\text{ min}}\right)=\boxed{\$2.22\times10^{-4}/\text{min}}$$

(c) Your father blames his extra output of 170 W − 120 W = 50 W on the TV set. It must be pumped out of the room by the air conditioner, with power expenditure

$$\frac{W}{\Delta t}=\frac{|Q_c|\Delta t}{COP}=\frac{50\text{ W}}{4.50}\left(\frac{\$0.15}{\text{kWh}}\right)\left(\frac{\text{k}}{1\,000}\right)\left(\frac{1\text{ h}}{60\text{ min}}\right)=\boxed{\$2.78\times10^{-5}/\text{min}}$$

Section 18.5 An Alternative Statement of the Second Law

No problems in this section

Section 18.6 Entropy

Section 18.7 Entropy and the Second Law of Thermodynamics

P18.23 $$\Delta S=\int_i^f\frac{dQ}{T}=\int_{T_i}^{T_f}\frac{mcdT}{T}=mc\ln\left(\frac{T_f}{T_i}\right)$$

$$\Delta S=250\text{ g}(1.00\text{ cal/g}\cdot{}^\circ\text{C})\ln\left(\frac{353}{293}\right)=46.6\text{ cal/K}=\boxed{195\text{ J/K}}$$

P18.24 For a freezing process,

$$\Delta S=\frac{\Delta Q}{T}=\frac{-(0.500\text{ kg})(3.33\times10^5\text{ J/kg})}{273\text{ K}}=\boxed{-610\text{ J/K}}$$

***P18.25** (a) The process is $\boxed{\text{isobaric}}$ because it takes place under constant atmospheric pressure. As described by Newton's third law, the stewing syrup must exert the same force on the air as the air exerts on it. The heating process is not adiabatic (energy goes in by heat), isothermal (T goes up), isovolumetic (it likely expands a bit), cyclic (it is different at the end), or isentropic (entropy increases). It could be made as nearly reversible as you wish, by not using a kitchen stove but a heater kept always just incrementally higher in temperature than the syrup. The process would then also be eternal, and impractical for food production.

(b) The final temperature is

$$220^\circ\text{F}=212^\circ\text{F}+8^\circ\text{F}=100^\circ\text{C}+8^\circ\text{F}\left(\frac{100-0^\circ\text{C}}{212-32^\circ\text{F}}\right)=104^\circ\text{C}$$

continued on next page

For the mixture,

$$Q = m_1c_1\Delta T + m_2c_2\Delta T = (900\text{ g }1\text{ cal/g}\cdot{}^\circ\text{C} + 930\text{ g }0.299\text{ cal/g}\cdot{}^\circ\text{C})(104.4^\circ\text{C} - 23^\circ\text{C})$$
$$= 9.59\times10^4\text{ cal} = \boxed{4.02\times10^5\text{ J}}$$

(c) Consider the reversible heating process described in part (a):

$$\Delta S = \int_i^f \frac{dQ}{T} = \int_i^f \frac{(m_1c_1 + m_2c_2)dT}{T} = (m_1c_1 + m_2c_2)\ln\frac{T_f}{T_i}$$
$$= [900(1) + 930(0.299)](\text{cal/}^\circ\text{C})\left(\frac{4.186\text{ J}}{1\text{ cal}}\right)\left(\frac{1^\circ\text{C}}{1\text{ K}}\right)\ln\left(\frac{273+104}{273+23}\right)$$
$$= (4\,930\text{ J/K})0.243 = \boxed{1.20\times10^3\text{ J/K}}$$

P18.26 We take data from Tables 17.1 and 17.2, and we assume a constant specific heat for each phase. As the ice is warmed from −12°C to 0°C, its entropy increases by

$$\Delta S = \int_i^f \frac{dQ}{T} = \int_{261\text{ K}}^{273\text{ K}} \frac{mc_{\text{ice}}dT}{T} = mc_{\text{ice}}\int_{261\text{ K}}^{273\text{ K}} T^{-1}dT = mc_{\text{ice}}\ln T\big|_{261\text{ K}}^{273\text{ K}}$$

$$\Delta S = 0.027\,9\text{ kg}(2\,090\text{ J/kg}\cdot{}^\circ\text{C})(\ln 273\text{ K} - \ln 261\text{ K}) = 0.027\,9\text{ kg}(2\,090\text{ J/kg}\cdot{}^\circ\text{C})\left(\ln\left(\frac{273}{261}\right)\right)$$

$$\Delta S = 2.62\text{ J/K}$$

As the ice melts its entropy change is

$$\Delta S = \frac{Q}{T} = \frac{mL_f}{T} = \frac{0.027\,9\text{ kg}(3.33\times10^5\text{ J/kg})}{273\text{ K}} = 34.0\text{ J/K}$$

As liquid water warms from 273 K to 373 K,

$$\Delta S = \int_i^f \frac{mc_{\text{liquid}}dT}{T} = mc_{\text{liquid}}\ln\left(\frac{T_f}{T_i}\right) = 0.027\,9\text{ kg}(4\,186\text{ J/kg}\cdot{}^\circ\text{C})\ln\left(\frac{373}{273}\right) = 36.5\text{ J/K}$$

As the water boils and the steam warms,

$$\Delta S = \frac{mL_v}{T} + mc_{\text{steam}}\ln\left(\frac{T_f}{T_i}\right)$$

$$\Delta S = \frac{0.027\,9\text{ kg}(2.26\times10^6\text{ J/kg})}{373\text{ K}} + 0.027\,9\text{ kg}(2\,010\text{ J/kg}\cdot{}^\circ\text{C})\ln\left(\frac{388}{373}\right) = 169\text{ J/K} + 2.21\text{ J/K}$$

The total entropy change is

$$(2.62 + 34.0 + 36.5 + 169 + 2.21)\text{ J/K} = \boxed{244\text{ J/K}}$$

For steam at constant pressure, the molar specific heat in Table 17.3 implies a specific heat of $(35.4\text{ J/mol}\cdot\text{K})\left(\frac{1\text{ mol}}{0.018\text{ kg}}\right) = 1\,970\text{ J/kg}\cdot\text{K}$, nearly agreeing with $2\,010\text{ J/kg}\cdot\text{K}$.

P18.27 (a) A 12 can only be obtained $\boxed{\text{one}}$ way: $6+6$

(b) A 7 can be obtained $\boxed{\text{six}}$ ways: $6+1$, $5+2$, $4+3$, $3+4$, $2+5$, $1+6$

P18.28 (a) The table is shown below. On the basis of the table, the most probable result of a toss is $\boxed{\text{2 heads and 2 tails}}$

(b) The most ordered state is the least likely state. Thus, on the basis of the table this is $\boxed{\text{either all heads or all tails}}$.

(c) The most disordered is the most likely state. Thus, this is $\boxed{\text{2 heads and 2 tails}}$

Result	Possible Combinations	Total
All heads	HHHH	1
3H, 1T	THHH, HTHH, HHTH, HHHT	4
2H, 2T	TTHH, THTH, THHT, HTTH, HTHT, HHTT	6
1H, 3T	HTTT, THTT, TTHT, TTTH	4
All tails	TTTT	1

P18.29 (a)

Result	Possible Combinations	Total
All red	RRR	1
2R, 1G	RRG, RGR, GRR	3
1R, 2G	RGG, GRG, GGR	3
All green	GGG	1

(b)

Result	Possible Combinations	Total
All red	RRRRR	1
4R, 1G	RRRRG, RRRGR, RRGRR, RGRRR, GRRRR	5
3R, 2G	RRRGG, RRGRG, RGRRG, GRRRG, RRGGR, RGRGR, GRRGR, RGGRR, GRGRR, GGRRR	10
2R, 3G	GGGRR, GGRGR, GRGGR, RGGGR, GGRRG, GRGRG, RGGRG, GRRGG, RGRGG, RRGGG	10
1R, 4G	RGGGG, GRGGG, GGRGG, GGGRG, GGGGR	5
All green	GGGGG	1

Section 18.8 Entropy Changes in Irreversible Processes

P18.30 $$\Delta S = \frac{Q_2}{T_2} - \frac{Q_1}{T_1} = \left(\frac{1\,000}{290} - \frac{1\,000}{5\,700}\right)\text{ J/K} = \boxed{3.27\text{ J/K}}$$

P18.31 The car ends up in the same thermodynamic state as it started, so it undergoes zero changes in entropy. The original kinetic energy of the car is transferred by heat to the surrounding air, adding to the internal energy of the air. Its change in entropy is

$$\Delta S = \frac{\frac{1}{2}mv^2}{T} = \frac{750(20.0)^2}{293}\text{ J/K} = \boxed{1.02\text{ kJ/K}}$$

P18.32 $c_{\text{iron}} = 448\ \text{J/kg}\cdot{}^\circ\text{C}$; $c_{\text{water}} = 4\,186\ \text{J/kg}\cdot{}^\circ\text{C}$

$Q_{\text{cold}} = -Q_{\text{hot}}$: $4.00\ \text{kg}(4\,186\ \text{J/kg}\cdot{}^\circ\text{C})(T_f - 10.0^\circ\text{C}) = -(1.00\ \text{kg})(448\ \text{J/kg}\cdot{}^\circ\text{C})(T_f - 900^\circ\text{C})$

which yields $T_f = 33.2^\circ\text{C} = 306.2\ \text{K}$

$$\Delta S = \int_{283\ \text{K}}^{306.2\ \text{K}} \frac{c_{\text{water}} m_{\text{water}}\, dT}{T} + \int_{1\,173\ \text{K}}^{306.2\ \text{K}} \frac{c_{\text{iron}} m_{\text{iron}}\, dT}{T}$$

$$\Delta S = c_{\text{water}} m_{\text{water}} \ln\left(\frac{306.2}{283}\right) + c_{\text{iron}} m_{\text{iron}} \ln\left(\frac{306.2}{1\,173}\right)$$

$$\Delta S = (4\,186\ \text{J/kg}\cdot\text{K})(4.00\ \text{kg})(0.078\,8) + (448\ \text{J/kg}\cdot\text{K})(1.00\ \text{kg})(-1.34)$$

$$\Delta S = \boxed{718\ \text{J/K}}$$

P18.33 Sitting here writing, I convert chemical energy, in ordered molecules in food, into internal energy that leaves my body by heat into the room-temperature surroundings. My rate of energy output is equal to my metabolic rate,

$$2\,500\ \text{kcal/d} = \frac{2\,500 \times 10^3\ \text{cal}}{86\,400\ \text{s}}\left(\frac{4.186\ \text{J}}{1\ \text{cal}}\right) = 120\ \text{W}.$$

My body is in steady state, changing little in entropy, as the environment increases in entropy at the rate

$$\frac{\Delta S}{\Delta t} = \frac{Q/T}{\Delta t} = \frac{Q/\Delta t}{T} = \frac{120\ \text{W}}{293\ \text{K}} = 0.4\ \text{W/K} \sim \boxed{1\ \text{W/K}}\ .$$

When using powerful appliances or an automobile, my personal contribution to entropy production is much greater than the above estimate, based only on metabolism.

P18.34 $\Delta S = nR\ln\left(\frac{V_f}{V_i}\right) = R\ln 2 = \boxed{5.76\ \text{J/K}}$

There is $\boxed{\text{no change in temperature}}$.

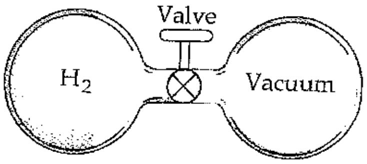

FIG. P18.34

P18.35 $\Delta S = nR\ln\left(\frac{V_f}{V_i}\right) = (0.044\,0)(2)R\ln 2$

$\Delta S = 0.088\,0(8.314)\ln 2 = \boxed{0.507\ \text{J/K}}$

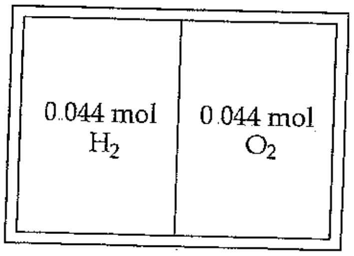

FIG. P18.35

Section 18.9 Context Connection—The Atmosphere as a Heat Engine

P18.36 The Earth presents a circular projected area of $\pi\left(6.37\times10^{6}\text{ m}\right)^{2}$ perpendicular to the direction of energy flow in sunlight. The power incident on the Earth is then

$$\mathcal{P}=IA=\left(1\,370\text{ W/m}^{2}\right)\left[\pi\left(6.37\times10^{6}\text{ m}\right)^{2}\right]=1.75\times10^{17}\text{ W}$$

The power absorbed in the atmosphere is $0.64\left(1.75\times10^{17}\text{ W}\right)=1.1\times10^{17}\text{ W}$. The mechanical wind power is $0.008\left(1.1\times10^{17}\text{ W}\right)=\boxed{9\times10^{14}\text{ W}}$.

P18.37 (a) The volume of the air is $\pi r^{2}h$ and its mass is $m=\rho V=\rho\pi r^{2}h$. Then its kinetic energy is

$$K=\frac{1}{2}mv^{2}=\frac{1}{2}\rho\pi r^{2}hv^{2}$$

$$K=\frac{1}{2}\left(1.20\text{ kg/m}^{3}\right)\left[\pi(300\,000\text{ m})^{2}(11\,000\text{ m})\right]\left(\frac{60\,000\text{ m}}{3\,600\text{ s}}\right)^{2}$$

$$K=\boxed{5.2\times10^{17}\text{ J}}$$

(b) We can suppose the light shines perpendicularly down onto the hurricane. Its intensity is

$$I=\frac{\mathcal{P}}{A}=\frac{E}{A\Delta t}:\quad \Delta t=\frac{E}{AI}=\frac{5.2\times10^{17}\text{ J m}^{2}\text{ s}}{\pi\left(3\times10^{5}\text{ m}\right)^{2}\left(10^{3}\text{ J}\right)}=\boxed{1.8\times10^{3}\text{ s}}$$

Additional Problems

P18.38 For the Carnot engine, $e_c=1-\dfrac{T_c}{T_h}=1-\dfrac{300\text{ K}}{750\text{ K}}=0.600$.

Also, $e_c=\dfrac{W_{\text{eng}}}{|Q_h|}$

so $|Q_h|=\dfrac{W_{\text{eng}}}{e_c}=\dfrac{150\text{ J}}{0.600}=250\text{ J}$.

and $|Q_c|=|Q_h|-W_{\text{eng}}=250\text{ J}-150\text{ J}=100\text{ J}$.

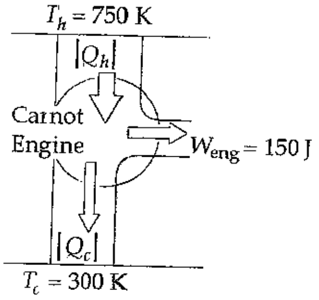

FIG. P18.38

(a) $|Q_h|=\dfrac{W_{\text{eng}}}{e_S}=\dfrac{150\text{ J}}{0.700}=\boxed{214\text{ J}}$

$|Q_c|=|Q_h|-W_{\text{eng}}=214\text{ J}-150\text{ J}=\boxed{64.3\text{ J}}$

(b) $|Q_{h,\text{net}}|=214\text{ J}-250\text{ J}=\boxed{-35.7\text{ J}}$

$|Q_{c,\text{net}}|=64.3\text{ J}-100\text{ J}=\boxed{-35.7\text{ J}}$

The net flow of energy by heat from the cold to the hot reservoir without work input, is impossible.

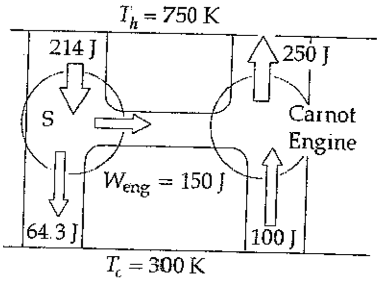

FIG. P18.38(b)

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(c) For engine S: $|Q_c| = |Q_h| - W_{\text{eng}} = \frac{W_{\text{eng}}}{e_S} - W_{\text{eng}}$

so $$W_{\text{eng}} = \frac{|Q_c|}{\frac{1}{e_S}-1} = \frac{100\text{ J}}{\frac{1}{0.700}-1} = \boxed{233\text{ J}}$$

and $$|Q_h| = |Q_c| + W_{\text{eng}} = 233\text{ J} + 100\text{ J} = \boxed{333\text{ J}}$$

(d) $|Q_{h,\text{net}}| = 333\text{ J} - 250\text{ J} = \boxed{83.3\text{ J}}$

$W_{\text{net}} = 233\text{ J} - 150\text{ J} = \boxed{83.3\text{ J}}$

$|Q_{c,\text{net}}| = \boxed{0}$

The output of 83.3 J of energy from the heat engine by work in a cyclic process without any exhaust by heat is impossible.

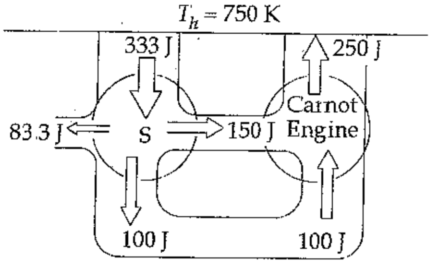

FIG. P18.38(d)

(e) Both engines operate in cycles, so $\Delta S_S = \Delta S_{\text{Carnot}} = 0$.

For the reservoirs, $\Delta S_h = -\frac{|Q_h|}{T_h}$ and $\Delta S_c = +\frac{|Q_c|}{T_c}$

Thus, $$\Delta S_{\text{total}} = \Delta S_S + \Delta S_{\text{Carnot}} + \Delta S_h + \Delta S_c = 0 + 0 - \frac{83.3\text{ J}}{750\text{ K}} + \frac{0}{300\text{ K}} = \boxed{-0.111\text{ J/K}}$$

A decrease in total entropy is impossible.

P18.39 As in Section 6.6, let T_{ET} represent the energy transferred by electrical transmission. Then

(a) $\mathcal{P}_{\text{electric}} = \frac{T_{ET}}{\Delta t}$ so if all the electric energy is converted into internal energy, the steady-state condition of the house is described by $T_{ET} = |Q|$

Therefore, $$\mathcal{P}_{\text{electric}} = \frac{Q}{\Delta t} = \boxed{5\,000\text{ W}}$$

(b) For a heat pump, $$(\text{COP})_{\text{Carnot}} = \frac{T_h}{\Delta T} = \frac{295\text{ K}}{27\text{ K}} = 10.92$$

$$\text{Actual COP} = 0.6(10.92) = 6.55 = \frac{|Q_h|}{W} = \frac{|Q_h|/\Delta t}{W/\Delta t}$$

Therefore, to bring 5 000 W of energy into the house only requires input power

$$\mathcal{P}_{\text{heat pump}} = \frac{W}{\Delta t} = \frac{|Q_h|/\Delta t}{\text{COP}} = \frac{5\,000\text{ W}}{6.56} = \boxed{763\text{ W}}$$

P18.40 The conversion of gravitational potential energy into kinetic energy as the water falls is reversible. But the subsequent conversion into internal energy is not. We imagine arriving at the same final state by adding energy by heat, in amount mgy, to the water from a stove at a temperature infinitesimally above 20.0°C. Then,

$$\Delta S=\int\frac{dQ}{T}=\frac{Q}{T}=\frac{mgy}{T}=\frac{5\,000\text{ m}^3\left(1\,000\text{ kg/m}^3\right)\left(9.80\text{ m/s}^2\right)(50.0\text{ m})}{293\text{ K}}=\boxed{8.36\times10^{6}\text{ J/K}}.$$

P18.41 $|Q_c|=mc\Delta T+mL+mc\Delta T=$

$$|Q_c|=0.500\text{ kg}(4\,186\text{ J/kg}\,^\circ\text{C})(10^\circ\text{C})+0.500\text{ kg}\left(3.33\times10^{5}\text{ J/kg}\right)+0.500\text{ kg}(2\,090\text{ J/kg}\,^\circ\text{C})(20^\circ\text{C})$$

$$|Q_c|=2.08\times10^{5}\text{ J}$$

$$\frac{|Q_c|}{W}=\text{COP}_c\,(\text{refrigerator})=\frac{T_c}{T_h-T_c}$$

$$W=\frac{|Q_c|(T_h-T_c)}{T_c}=\frac{\left(2.08\times10^{5}\text{ J}\right)\left[20.0^\circ\text{C}-(-20.0^\circ\text{C})\right]}{(273-20.0)\text{ K}}=\boxed{32.9\text{ kJ}}$$

***P18.42** (a) Let state i represent the gas before its compression and state f afterwards, $V_f=\frac{V_i}{8}$. For a diatomic ideal gas, $C_v=\frac{5}{2}R$, $C_p=\frac{7}{2}R$, and $\gamma=\frac{C_p}{C_V}=1.40$. Next,

$$P_iV_i^{\gamma}=P_fV_f^{\gamma}$$

$$P_f=P_i\left(\frac{V_i}{V_f}\right)^{\gamma}=P_i8^{1.40}=18.4P_i$$

$$P_iV_i=nRT_i$$

$$P_fV_f=\frac{18.4P_iV_i}{8}=2.30P_iV_i=2.30nRT_i=nRT_f$$

so $T_f=2.30T_i$

$$\Delta E_{\text{int}}=nC_V\Delta T=n\frac{5}{2}R\left(T_f-T_i\right)=\frac{5}{2}nR1.30T_i=\frac{5}{2}1.30P_iV_i$$

$$=\frac{5}{2}1.30\left(1.013\times10^{5}\text{ N/m}^2\right)0.12\times10^{-3}\text{ m}^3=39.4\text{ J}$$

Since the process is adiabatic, $Q=0$ and $\Delta E_{\text{int}}=Q+W$ gives $W=\boxed{39.4\text{ J}}$.

(b) The moment of inertia of the wheel is $I=\frac{1}{2}MR^2=\frac{1}{2}5.1\text{ kg}(0.085\text{ m})^2=0.018\,4\text{ kg}\cdot\text{m}^2$. We want the flywheel to do work 39.4 J, so the work on the flywheel should be −39.4 J:

continued on next page

$$K_{\text{rot}\,i}+W=K_{\text{rot}\,f}$$

$$\frac{1}{2}I\omega_i^2-39.4\text{ J}=0$$

$$\omega_i=\left(\frac{2(39.4\text{ J})}{0.018\,4\text{ kg}\cdot\text{m}^2}\right)^{1/2}=\boxed{65.4\text{ rad/s}}$$

(c) Now we want $W=0.05K_{\text{rot}\,i}$

$$39.4J=0.05\frac{1}{2}0.018\,4\text{ kg}\cdot\text{m}^2\omega_i^2$$

$$\omega=\left(\frac{2(789\text{ J})}{0.018\,4\text{ kg}\cdot\text{m}^2}\right)^{1/2}=\boxed{293\text{ rad/s}}$$

P18.43 (a) For an isothermal process, $Q=nRT\ln\left(\frac{V_2}{V_1}\right)$

Therefore, $Q_1=nR(3T_i)\ln 2$

and $Q_3=nR(T_i)\ln\left(\frac{1}{2}\right)$

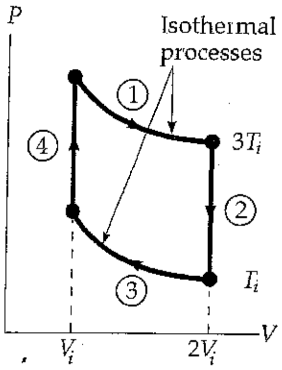

FIG. P18.43

For the constant volume processes, $Q_2=\Delta E_{\text{int},\,2}=\frac{3}{2}nR(T_i-3T_i)$

and $Q_4=\Delta E_{\text{int},\,4}=\frac{3}{2}nR(3T_i-T_i)$

The net energy by heat transferred is then

$$Q=Q_1+Q_2+Q_3+Q_4$$

or $Q=\boxed{2nRT_i\ln 2}$

(b) A positive value for heat represents energy transferred into the system.

Therefore, $|Q_h|=Q_1+Q_4=3nRT_i(1+\ln 2)$

Since the change in temperature for the complete cycle is zero,

$$\Delta E_{\text{int}}=0\text{ and }W_{\text{eng}}=Q$$

Therefore, the efficiency is $e_c=\frac{W_{\text{eng}}}{|Q_h|}=\frac{Q}{|Q_h|}=\frac{2\ln 2}{3(1+\ln 2)}=\boxed{0.273}$

P18.44 $\Delta S_{\text{hot}}=\frac{-1\,000\text{ J}}{600\text{ K}}$

$\Delta S_{\text{cold}}=\frac{+750\text{ J}}{350\text{ K}}$

(a) $\Delta S_U=\Delta S_{\text{hot}}+\Delta S_{\text{cold}}=\boxed{0.476\text{ J/K}}$

continued on next page

(b) $$e_c = 1 - \frac{T_1}{T_2} = 0.417$$

$$W_{\text{eng}} = e_c|Q_h| = 0.417(1\,000\text{ J}) = \boxed{417\text{ J}}$$

(c) $$W_{\text{net}} = 417\text{ J} - 250\text{ J} = 167\text{ J}$$

$$T_1\Delta S_U = 350\text{ K}(0.476\text{ J/K}) = \boxed{167\text{ J}}$$

P18.45 $$e_c = 1 - \frac{T_c}{T_h} = \frac{W_{\text{eng}}}{|Q_h|} = \frac{\frac{W_{\text{eng}}}{\Delta t}}{\frac{|Q_h|}{\Delta t}}: \qquad \frac{|Q_h|}{\Delta t} = \frac{\mathcal{P}}{\left(1 - \frac{T_c}{T_h}\right)} = \frac{\mathcal{P}T_h}{T_h - T_c}$$

$$|Q_h| = W_{\text{eng}} + |Q_c|: \qquad \frac{|Q_c|}{\Delta t} = \frac{|Q_h|}{\Delta t} - \frac{W_{\text{eng}}}{\Delta t}$$

$$\frac{|Q_c|}{\Delta t} = \frac{\mathcal{P}T_h}{T_h - T_c} - \mathcal{P} = \frac{\mathcal{P}T_c}{T_h - T_c}$$

$$|Q_c| = mc\Delta T: \qquad \frac{|Q_c|}{\Delta t} = \left(\frac{\Delta m}{\Delta t}\right)c\Delta T = \frac{\mathcal{P}T_c}{T_h - T_c}$$

$$\frac{\Delta m}{\Delta t} = \frac{\mathcal{P}T_c}{(T_h - T_c)c\Delta T}$$

$$\frac{\Delta m}{\Delta t} = \frac{(1.00\times10^9\text{ W})(300\text{ K})}{200\text{ K}(4\,186\text{ J/kg }^\circ\text{C})(6.00^\circ\text{C})} = \boxed{5.97\times10^4\text{ kg/s}}$$

P18.46 $$e_c = 1 - \frac{T_c}{T_h} = \frac{W_{\text{eng}}}{|Q_h|} = \frac{\frac{W_{\text{eng}}}{\Delta t}}{\frac{|Q_h|}{\Delta t}} \qquad \frac{|Q_h|}{\Delta t} = \frac{\mathcal{P}}{\left(1 - \frac{T_c}{T_h}\right)} = \frac{\mathcal{P}T_h}{T_h - T_c}$$

$$\frac{|Q_c|}{\Delta t} = \left(\frac{|Q_h|}{\Delta t}\right) - \mathcal{P} = \frac{\mathcal{P}T_c}{T_h - T_c}$$

$|Q_c| = mc\Delta T$, where c is the specific heat of water.

Therefore, $$\frac{|Q_c|}{\Delta t} = \left(\frac{\Delta m}{\Delta t}\right)c\Delta T = \frac{\mathcal{P}T_c}{T_h - T_c}$$

and $$\frac{\Delta m}{\Delta t} = \boxed{\frac{\mathcal{P}T_c}{(T_h - T_c)c\Delta T}}$$

We test for dimensional correctness by identifying the units of the right-hand side: $\frac{\text{W}\cdot{}^\circ\text{C}}{{}^\circ\text{C}(\text{J/kg }^\circ\text{C})^\circ\text{C}} = \frac{(\text{J/s})\text{kg}}{\text{J}} = \text{kg/s}$, as on the left hand side. Think of yourself as a power-company engineer arranging to have enough cooling water to carry off your thermal pollution. If the plant power $\mathcal{P}$ increases, the required flow rate increases in direct proportion. If environmental regulations require a smaller temperature change, then the required flow rate increases again, now in inverse proportion. If you can run the reactor core or firebox hotter, since T_h is in the bottom of the fraction, the required flow rate decreases! If the turbines take in steam at higher temperature, they can be made more efficient to reduce waste heat output.

P18.47 We want to evaluate $\frac{dP}{dV}$ for the function implied by $PV = nRT =$ constant, and also for the different function implied by $PV^{\gamma} =$ constant. We can use implicit differentiation:

From $PV =$ constant $\qquad P\frac{dV}{dV} + V\frac{dP}{dV} = 0 \qquad \left(\frac{dP}{dV}\right)_{\text{isotherm}} = -\frac{P}{V}$

From $PV^{\gamma} =$ constant $\qquad P\gamma V^{\gamma-1} + V^{\gamma}\frac{dP}{dV} = 0 \qquad \left(\frac{dP}{dV}\right)_{\text{adiabat}} = -\frac{\gamma P}{V}$

Therefore, $\left(\frac{dP}{dV}\right)_{\text{adiabat}} = \gamma\left(\frac{dP}{dV}\right)_{\text{isotherm}}$. The theorem is proved.

***P18.48** (a) $$35.0°\text{F} = \frac{5}{9}(35.0 - 32.0)°\text{C} = (1.67 + 273.15)\text{ K} = 274.82\text{ K}$$

$$98.6°\text{F} = \frac{5}{9}(98.6 - 32.0)°\text{C} = (37.0 + 273.15)\text{ K} = 310.15\text{ K}$$

$$\Delta S_{\text{ice water}} = \int\frac{dQ}{T} = (453.6\text{ g})(1.00\text{ cal/g K})\times\int_{274.82}^{310.15}\frac{dT}{T} = 453.6\ln\left(\frac{310.15}{274.82}\right) = 54.86\text{ cal/K}$$

$$\Delta S_{\text{body}} = -\frac{|Q|}{T_{\text{body}}} = -(453.6)(1.00)\frac{(310.15 - 274.82)}{310.15} = -51.67\text{ cal/K}$$

$$\Delta S_{\text{system}} = 54.86 - 51.67 = \boxed{3.19\text{ cal/K}}$$

(b) $$(453.6)(1)(T_F - 274.82) = \left(70.0\times10^3\right)(1)(310.15 - T_F)$$

Thus,

$$(70.0 + 0.4536)\times10^3\,T_F = \left[(70.0)(310.15) + (0.4536)(274.82)\right]\times10^3$$

and $T_F = 309.92\text{ K} = 36.77°\text{C} = \boxed{98.19°\text{F}}$

$$\Delta S'_{\text{ice water}} = 453.6\ln\left(\frac{309.92}{274.82}\right) = 54.52\text{ cal/K}$$

$$\Delta S'_{\text{body}} = -\left(70.0\times10^3\right)\ln\left(\frac{310.15}{309.92}\right) = -51.93\text{ cal/K}$$

$\Delta S'_{\text{sys}} = 54.52 - 51.93 = \boxed{2.59\text{ cal/K}}$ which is less than the estimate in part (a).

P18.49 (a) For the isothermal process AB, the work on the gas is

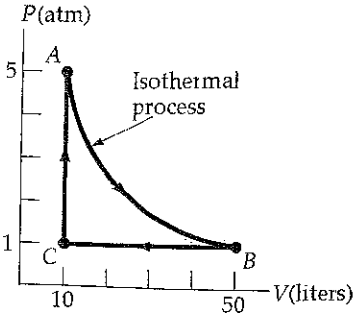

FIG. P18.49

$$W_{AB} = -P_A V_A \ln\left(\frac{V_B}{V_A}\right)$$

$$W_{AB} = -5\left(1.013\times10^5\ \text{Pa}\right)\left(10.0\times10^{-3}\ \text{m}^3\right)\ln\left(\frac{50.0}{10.0}\right)$$

$$W_{AB} = -8.15\times10^3\ \text{J}$$

where we have used $1.00\ \text{atm} = 1.013\times10^5\ \text{Pa}$

and $1.00\ \text{L} = 1.00\times10^{-3}\ \text{m}^3$

$$W_{BC} = -P_B\Delta V = -\left(1.013\times10^5\ \text{Pa}\right)\left[(10.0-50.0)\times10^{-3}\right]\text{m}^3 = +4.05\times10^3\ \text{J}$$

$$W_{CA} = 0 \text{ and } W_{\text{eng}} = -W_{AB} - W_{BC} = 4.11\times10^3\ \text{J} = \boxed{4.11\ \text{kJ}}$$

(b) Since AB is an isothermal process, $\Delta E_{\text{int},\,AB} = 0$

and $Q_{AB} = -W_{AB} = 8.15\times10^3\ \text{J}$

For an ideal monatomic gas, $C_V = \dfrac{3R}{2}$ and $C_P = \dfrac{5R}{2}$

$$T_B = T_A = \frac{P_B V_B}{nR} = \frac{\left(1.013\times10^5\right)\left(50.0\times10^{-3}\right)}{R} = \frac{5.05\times10^3}{R}$$

Also,

$$T_C = \frac{P_C V_C}{nR} = \frac{\left(1.013\times10^5\right)\left(10.0\times10^{-3}\right)}{R} = \frac{1.01\times10^3}{R}$$

$$Q_{CA} = nC_V\Delta T = 1.00\left(\frac{3}{2}R\right)\left(\frac{5.05\times10^3 - 1.01\times10^3}{R}\right) = 6.08\ \text{kJ}$$

so the total energy absorbed by heat is $Q_{AB} + Q_{CA} = 8.15\ \text{kJ} + 6.08\ \text{kJ} = \boxed{14.2\ \text{kJ}}$.

(c)

$$Q_{BC} = nC_P\Delta T = \frac{5}{2}(nR\Delta T) = \frac{5}{2}P_B\Delta V_{BC}$$

$$Q_{BC} = \frac{5}{2}\left(1.013\times10^5\right)\left[(10.0-50.0)\times10^{-3}\right] = -1.01\times10^4\ \text{J} = \boxed{-10.1\ \text{kJ}}$$

(d)

$$e = \frac{W_{\text{eng}}}{|Q_h|} = \frac{W_{\text{eng}}}{Q_{AB}+Q_{CA}} = \frac{4.11\times10^3\ \text{J}}{1.42\times10^4\ \text{J}} = 0.289 \text{ or } \boxed{28.9\%}$$

P18.50 Like a refrigerator, an air conditioner has as its purpose the removal of energy by heat from the cold reservoir.

Its ideal COP is $$\mathrm{COP}_{\text{Carnot}} = \frac{T_c}{T_h - T_c} = \frac{280\ \text{K}}{20\ \text{K}} = 14.0$$

(a) Its actual COP is

$$0.400(14.0) = 5.60 = \frac{|Q_c|}{|Q_h| - |Q_c|} = \frac{|Q_c/\Delta t|}{|Q_h/\Delta t| - |Q_c/\Delta t|}$$

$$5.60\left|\frac{Q_h}{\Delta t}\right| - 5.60\left|\frac{Q_c}{\Delta t}\right| = \left|\frac{Q_c}{\Delta t}\right|$$

$$5.60(10.0\ \text{kW}) = 6.60\left|\frac{Q_c}{\Delta t}\right| \text{ and } \left|\frac{Q_c}{\Delta t}\right| = \boxed{8.48\ \text{kW}}$$

(b) $|Q_h| = W_{\text{eng}} + |Q_c|$:

$$\frac{W_{\text{eng}}}{\Delta t} = \left|\frac{Q_h}{\Delta t}\right| - \left|\frac{Q_c}{\Delta t}\right| = 10.0\ \text{kW} - 8.48\ \text{kW} = \boxed{1.52\ \text{kW}}$$

(c) The air conditioner operates in a cycle, so the entropy of the working fluid does not change. The hot reservoir increases in entropy by

$$\frac{|Q_h|}{T_h} = \frac{(10.0 \times 10^3\ \text{J/s})(3\,600\ \text{s})}{300\ \text{K}} = 1.20 \times 10^5\ \text{J/K}$$

The cold room decreases in entropy by

$$\Delta S = -\frac{|Q_c|}{T_c} = -\frac{(8.48 \times 10^3\ \text{J/s})(3\,600\ \text{s})}{280\ \text{K}} = -1.09 \times 10^5\ \text{J/K}$$

The net entropy change is positive, as it must be:

$$+1.20 \times 10^5\ \text{J/K} - 1.09 \times 10^5\ \text{J/K} = \boxed{1.09 \times 10^4\ \text{J/K}}$$

(d) The new ideal COP is $$\mathrm{COP}_{\text{Carnot}} = \frac{T_c}{T_h - T_c} = \frac{280\ \text{K}}{25\ \text{K}} = 11.2$$

We suppose the actual COP is $0.400(11.2) = 4.48$

As a fraction of the original 5.60, this is $\dfrac{4.48}{5.60} = 0.800$, so the fractional change is to $\boxed{\text{drop by } 20.0\%}$

P18.51 At point A, $P_iV_i = nRT_i$ and $n = 1.00\text{ mol}$

At point B, $3P_iV_i = nRT_B$ so $T_B = 3T_i$

At point C, $(3P_i)(2V_i) = nRT_C$ and $T_C = 6T_i$

At point D, $P_i(2V_i) = nRT_D$ so $T_D = 2T_i$

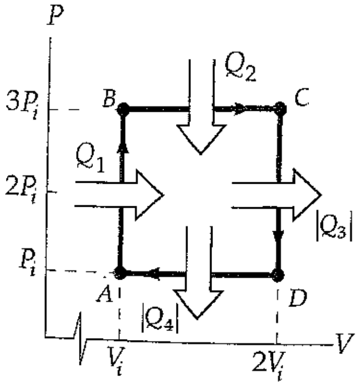

FIG. P18.51

The heat for each step in the cycle is found using $C_V = \dfrac{3R}{2}$ and $C_P = \dfrac{5R}{2}$:

$$Q_{AB} = nC_V(3T_i - T_i) = 3nRT_i$$
$$Q_{BC} = nC_P(6T_i - 3T_i) = 7.50nRT_i$$
$$Q_{CD} = nC_V(2T_i - 6T_i) = -6nRT_i$$
$$Q_{DA} = nC_P(T_i - 2T_i) = -2.50nRT_i$$

(a) Therefore, $Q_{\text{entering}} = |Q_h| = Q_{AB} + Q_{BC} = \boxed{10.5nRT_i}$

(b) $Q_{\text{leaving}} = |Q_c| = |Q_{CD} + Q_{DA}| = \boxed{8.50nRT_i}$

(c) Actual efficiency, $e = \dfrac{|Q_h| - |Q_c|}{|Q_h|} = \boxed{0.190}$

(d) Carnot efficiency, $e_c = 1 - \dfrac{T_c}{T_h} = 1 - \dfrac{T_i}{6T_i} = \boxed{0.833}$

P18.52 $\Delta S = \displaystyle\int_i^f \frac{dQ}{T} = \int_i^f \frac{nC_P dT}{T} = nC_P \int_i^f T^{-1}dT = nC_P \ln T\big|_{T_i}^{T_f} = nC_P(\ln T_f - \ln T_i) = nC_P \ln\left(\frac{T_f}{T_i}\right)$

$$\Delta S = nC_P \ln\left(\frac{PV_f}{nR}\frac{nR}{PV_i}\right) = \boxed{nC_P \ln 3}$$

P18.53 (a) The ideal gas at constant temperature keeps constant internal energy. As it puts out energy by work in expanding it must take in an equal amount of energy by heat. Thus its entropy increases. Let P_i, V_i, T_i represent the state of the gas before the isothermal expansion. Let P_C, V_C, T_i represent the state after this process, so that $P_iV_i = P_CV_C$. Let P_i, $3V_i$, T_f represent the state after the adiabatic compression.

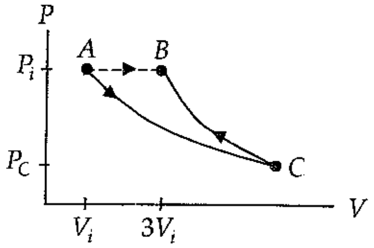

FIG. P18.53

Then $P_CV_C^\gamma = P_i(3V_i)^\gamma$

Substituting $P_C = \dfrac{P_iV_i}{V_C}$

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gives $$P_iV_iV_C^{\gamma-1}=P_i\left(3^\gamma V_i^\gamma\right)$$

Then $$V_C^{\gamma-1}=3^\gamma V_i^{\gamma-1} \text{ and } \frac{V_C}{V_i}=3^{\gamma/(\gamma-1)}$$

The work output in the isothermal expansion is

$$W=\int_i^C P\,dV=nRT_i\int_i^C V^{-1}dV=nRT_i\ln\left(\frac{V_C}{V_i}\right)=nRT_i\ln\left(3^{\gamma/(\gamma-1)}\right)=nRT_i\left(\frac{\gamma}{\gamma-1}\right)\ln 3$$

This is also the input heat, so the entropy change is

$$\Delta S=\frac{Q}{T}=nR\left(\frac{\gamma}{\gamma-1}\right)\ln 3$$

Since $$C_P=\gamma C_V=C_V+R$$

we have $$(\gamma-1)C_V=R,\ C_V=\frac{R}{\gamma-1}$$

and $$C_P=\frac{\gamma R}{\gamma-1}$$

Then the result is $$\boxed{\Delta S=nC_P\ln 3}$$

(b) The pair of processes considered here carry the gas from the initial state in Problem 18.52 to the final state there. Entropy is a function of state. Entropy change does not depend on path. Therefore the entropy change in Problem 18.52 equals $\Delta S_{\text{isothermal}}+\Delta S_{\text{adiabatic}}$ in this problem. Since $\Delta S_{\text{adiabatic}}=0$, the answers to Problems 18.52 and 18.53 (a) must be the same.

***P18.54** Simply evaluate the maximum (Carnot) efficiency.

$$e_C=\frac{\Delta T}{T_h}=\frac{4.00\text{ K}}{277\text{ K}}=\boxed{0.014\,4}$$

The proposal does not merit serious consideration.

P18.55 (a) Use the equation of state for an ideal gas

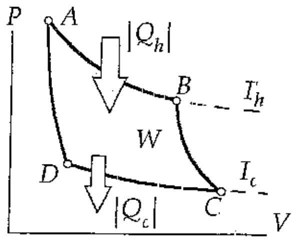

FIG. P18.55

$$V=\frac{nRT}{P}$$

$$V_A=\frac{1.00(8.314)(600)}{25.0\left(1.013\times10^5\right)}=\boxed{1.97\times10^{-3}\text{ m}^3}$$

$$V_C=\frac{1.00(8.314)(400)}{1.013\times10^5}=\boxed{32.8\times10^{-3}\text{ m}^3}$$

Since AB is isothermal, $$P_AV_A=P_BV_B$$

and since BC is adiabatic, $$P_BV_B^\gamma=P_CV_C^\gamma$$

continued on next page

Combining these expressions, $V_B = \left[\left(\frac{P_C}{P_A}\right)\frac{V_C^{\gamma}}{V_A}\right]^{1/(\gamma-1)} = \left[\left(\frac{1.00}{25.0}\right)\frac{\left(32.8\times10^{-3}\text{ m}^3\right)^{1.40}}{1.97\times10^{-3}\text{ m}^3}\right]^{(1/0.400)}$

$$V_B = \boxed{11.9\times10^{-3}\text{ m}^3}$$

Similarly, $V_D = \left[\left(\frac{P_A}{P_C}\right)\frac{V_A^{\gamma}}{V_C}\right]^{1/(\gamma-1)} = \left[\left(\frac{25.0}{1.00}\right)\frac{\left(1.97\times10^{-3}\text{ m}^3\right)^{1.40}}{32.8\times10^{-3}\text{ m}^3}\right]^{(1/0.400)}$

or $V_D = \boxed{5.44\times10^{-3}\text{ m}^3}$

Since AB is isothermal, $P_AV_A = P_BV_B$

and $P_B = P_A\left(\frac{V_A}{V_B}\right) = 25.0\text{ atm}\left(\frac{1.97\times10^{-3}\text{ m}^3}{11.9\times10^{-3}\text{ m}^3}\right) = \boxed{4.14\text{ atm}}$

Also, CD is an isothermal and $P_D = P_C\left(\frac{V_C}{V_D}\right) = 1.00\text{ atm}\left(\frac{32.8\times10^{-3}\text{ m}^3}{5.44\times10^{-3}\text{ m}^3}\right) = \boxed{6.03\text{ atm}}$

Solving part (c) before part (b):

(c) For this Carnot cycle, $e_c = 1-\frac{T_c}{T_h} = 1-\frac{400\text{ K}}{600\text{ K}} = \boxed{0.333}$

(b) Energy is added by heat to the gas during the process AB. For the isothermal process, $\Delta E_{\text{int}} = 0$.

and the first law gives $Q_{AB} = -W_{AB} = nRT_h\ln\left(\frac{V_B}{V_A}\right)$

or $|Q_h| = Q_{AB} = 1.00\text{ mol}(8.314\text{ J/mol K})(600\text{ K})\ln\left(\frac{11.9}{1.97}\right) = 8.97\text{ kJ}$

Then, from $e = \frac{W_{\text{eng}}}{|Q_h|}$

the net work done per cycle is $W_{\text{eng}} = e_c|Q_h| = 0.333(8.97\text{ kJ}) = \boxed{2.99\text{ kJ}}$.

P18.56 $\frac{|Q_c|}{W} = \text{COP}_C\text{ (refrigerator)} = \frac{T_c}{T_h-T_c} = \frac{\frac{|Q_c|}{\Delta t}}{\frac{W}{\Delta t}}$

$$\frac{0.150\text{ W}}{\frac{W}{\Delta t}} = \frac{260\text{ K}}{40.0\text{ K}}$$

$$\mathcal{P} = \frac{W}{\Delta t} = 0.150\text{ W}\left(\frac{40.0\text{ K}}{260\text{ K}}\right) = \boxed{23.1\text{ mW}}$$

P18.57 Define $T_1 = \text{Temp Cream} = 5.00°\text{C} = 278\ \text{K}$. Define $T_2 = \text{Temp Coffee} = 60.0°\text{C} = 333\ \text{K}$. The final temperature of the mixture is: $T_f = \dfrac{(20.0\ \text{g})T_1 + (200\ \text{g})T_2}{220\ \text{g}} = 55.0°\text{C} = 328\ \text{K}$. The entropy change due to this mixing is $\Delta S = (20.0\ \text{g})\int_{T_1}^{T_f}\dfrac{c_V dT}{T} + (200\ \text{g})\int_{T_2}^{T_f}\dfrac{c_V dT}{T}$

$$\Delta S = (84.0\ \text{J/K})\ln\left(\frac{T_f}{T_1}\right) + (840\ \text{J/K})\ln\left(\frac{T_f}{T_2}\right) = (84.0\ \text{K/J})\ln\left(\frac{328}{278}\right) + (840\ \text{J/K})\ln\left(\frac{328}{333}\right)$$

$$\Delta S = \boxed{+1.18\ \text{J/K}}$$

P18.58 No energy is transferred by heat during the adiabatic processes. Energy Q_h is added by heat during process BC and energy $|Q_h|$ is exhausted during process DA. We have $Q_h = nC_V(T_C - T_B)$ and $|Q_C| = nC_V(T_D - T_A)$. So $e = 1 - \dfrac{|Q_C|}{Q_h} = 1 - \dfrac{T_D - T_A}{T_C - T_B}$. For the adiabatic processes, $P_C V_2^{\gamma} = P_D V_1^{\gamma}$ and $P_B V_2^{\gamma} = P_A V_1^{\gamma}$.

From the gas law $P_C = \dfrac{nRT_C}{V_2}$ $P_D = \dfrac{nRT_D}{V_1}$

$$P_B = \frac{nRT_B}{V_2} \qquad P_A = \frac{nRT_A}{V_1}$$

Then $nRT_C V_2^{\gamma-1} = nRT_D V_1^{\gamma-1}$ and $T_B V_2^{\gamma-1} = T_A V_1^{\gamma-1}$ Thus, $\dfrac{T_C}{T_D} = \left(\dfrac{V_1}{V_2}\right)^{\gamma-1} = r^{\gamma-1}$ and $\dfrac{T_B}{T_A} = \left(\dfrac{V_1}{V_2}\right)^{\gamma-1} = r^{\gamma-1}$ Then $e = 1 - \dfrac{T_D - T_A}{T_D r^{\gamma-1} - T_A r^{\gamma-1}} = 1 - \dfrac{1}{r^{\gamma-1}} = 1 - r^{1-\gamma}$.

ANSWERS TO EVEN PROBLEMS

P18.2 (a) 29.4 L/h; (b) 185 hp; (c) 527 N m; (d) 1.91×10^5 W

P18.4 13.7°C

P18.6 (a) 870 MJ; (b) 330 MJ

P18.8 546°C

P18.10 (a) 5.12%; (b) 5.27 TJ/h; (c) As conventional energy sources become more expensive, or as their true costs are recognized, alternative sources become economically viable.

P18.12 (a)

State	P (kPa)	V (L)	T (K)
A	1 400	10.0	720
B	875	16.0	720
C	445	24.0	549
D	712	15.0	549

(b)

Process	Q (kJ)	W (kJ)	ΔE_{int} (kJ)
$A\to B$	6.58	−6.58	0
$B\to C$	0	−4.98	−4.98
$C\to D$	−5.02	5.02	0
$D\to A$	0	4.98	4.98
$ABCDA$	1.56	−1.56	0

P18.14 77.8 W

P18.16 9.00

P18.18 11.8

P18.20 1.86

P18.22 (a) $\$1.00 \times 10^{-3}$/min; (b) $\$2.22 \times 10^{-4}$/min; (c) $\$2.78 \times 10^{-5}$/min

P18.24 −610 J/K