

$$(b) \quad t = 126 \text{ s} \left(\ln \left(\frac{4 \times 10^4 \text{ C}}{5 \times 10^4 \text{ C}} \right) \right) = \boxed{261 \text{ s}}$$

$$(c) \quad t = 126 \text{ s} \left(\ln \left(\frac{4 \times 10^4 \text{ C}}{0} \right) \right) \rightarrow \infty$$

The exponential decay goes on forever, as modeled mathematically.

$$CC6.2 \quad (a) \quad i = \frac{Q}{\Delta t}$$

$$\Delta t = \frac{Q}{i} = \frac{25 \text{ C}}{2000 \text{ C/s}} = 1.25 \times 10^{-2} \text{ s} = \boxed{0.01 \text{ s}}$$

$$(b) \quad \text{number} = i \left(\frac{1}{\frac{1}{1 \text{ d}} \left(\frac{1}{1.25 \times 10^{-2} \text{ s}} \right)} \right) = \boxed{7 \times 10^6}$$

CC6.3 (a) For radial currents, the two layers of the atmosphere are resistors in series, presenting equivalent resistance

$$\frac{\rho_1 L}{A} + \frac{\rho_2 L}{A} = \frac{2 \times 10^{13} \Omega \cdot \text{m} (2.5 \times 10^3 \text{ m})}{4\pi (6.37 \times 10^6 \text{ m})^2} + \frac{0.2 \times 10^{13} \Omega \cdot \text{m} (2.5 \times 10^3 \text{ m})}{4\pi (6.37 \times 10^6 \text{ m})^2} = 108 \Omega$$

Then the time constant for discharge is

$$RC = 0.9 \text{ F}(108 \Omega) = 97.1 \text{ s}$$

According to the argument of the context conclusion, we consider six time constants as the time for 2×10^4 lightning strikes. Then the number in a day is

$$\frac{2 \times 10^4}{6RC} = \frac{2 \times 10^4}{6(97.1 \text{ s})} \left(\frac{1}{1 \text{ d}} \right) = \boxed{3 \times 10^6}$$

(b) For radial currents through the atmosphere, the air in the two hemispheres presents resistances in parallel, with equivalent resistance

$$R_{eq} = \frac{1}{\left(\frac{\rho_1 L}{A} \right)^{-1} + \left(\frac{\rho_2 L}{A} \right)^{-1}} = \frac{1}{\frac{2\pi (6.37 \times 10^6 \text{ m})^2}{2 \times 10^{13} \Omega \cdot \text{m} (5 \times 10^3 \text{ m})} + \frac{2\pi (6.37 \times 10^6 \text{ m})^2}{0.2 \times 10^{13} \Omega \cdot \text{m} (5 \times 10^3 \text{ m})}} = 35.7 \Omega$$

$$\text{number of daily strokes} = \frac{2 \times 10^4}{6RC} = \frac{2 \times 10^4}{6(35.7 \Omega)(0.9 \text{ F})} \left(\frac{1}{1 \text{ d}} \right) = \boxed{9 \times 10^5}$$

ANSWERS TO EVEN CONTEXT 6 CONCLUSION PROBLEMS

CC6.2 (a) 0.01 s; (b) 7×10^6

Chapter 22

Magnetic Forces and Magnetic Fields

ANSWERS TO QUESTIONS

Q22.1

If they are projected in the same direction into the same magnetic field, the charges are of opposite sign.

Q22.2

Similarities: Both can alter the velocity of a charged particle moving through the field. Both exert forces directly proportional to the charge of the particle feeling the force. Positive and negative charges feel forces in opposite directions. Differences: The direction of the electric force is parallel or antiparallel to the direction of the electric field, but the direction of the magnetic force is perpendicular to the magnetic field and to the velocity of the charged particle. Electric forces can accelerate a charged particle from rest or stop a moving particle, but magnetic forces cannot.

CHAPTER OUTLINE	
22.1	Historical Overview
22.2	The Magnetic Field
22.3	Motion of a Charged Particle in a Uniform Magnetic Field
22.4	Applications Involving Charged Particles Moving in a Magnetic Field
22.5	Magnetic Force on a Current-Carrying Conductor
22.6	Torque on a Current Loop in a Uniform Magnetic Field
22.7	The Biot-Savart Law
22.8	The Magnetic Force Between Two Parallel Conductors
22.9	Ampere's Law
22.10	The Magnetic Field of a Solenoid
22.11	Magnetism in Matter
22.12	Context Connection—The Attractive Model for Magnetic Levitation

Q22.3

(a) The $q\vec{v} \times \vec{B}$ force on each electron is down. Since electrons are negative, $\vec{v} \times \vec{B}$ must be up. With $\vec{v}$ to the right, $\vec{B}$ must be into the page, away from you.

(b)

Reversing the current in the coils would reverse the direction of $\vec{B}$, making it toward you. Then $\vec{v} \times \vec{B}$ is in the direction **right $\times$ toward you = down**, and $q\vec{v} \times \vec{B}$ will make the electron beam curve up.

Q22.4

If the current is in a direction *parallel* or *antiparallel* to the magnetic field, then there is no force.

Q22.5

Yes. If the magnetic field is perpendicular to the plane of the loop, then it exerts no torque on the loop.

Q22.6

If you can hook a spring balance to the particle and measure the force on it in a known electric field, then $q = \frac{F}{E}$ will tell you its charge. You cannot hook a spring balance to an electron. Measuring the acceleration of small particles by observing their deflection in known electric and magnetic fields can tell you the charge-to-mass ratio, but not separately the charge or mass. Both an acceleration produced by an electric field and an acceleration caused by a magnetic field depend on the properties of the particle only by being proportional to the ratio $\frac{q}{m}$.

Q22.7

The Earth's magnetic field exerts force on a charged incoming cosmic ray, tending to make it spiral around a magnetic field line. If the particle energy is low enough, the spiral will be tight enough that the particle will first hit some matter as it follows a field line down into the atmosphere or to the surface at a high geographic latitude.

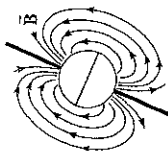


FIG. Q22.7

Q22.8

The spiral tracks are left by charged particles gradually losing kinetic energy. A straight path might be left by an uncharged particle that managed to leave a trail of bubbles, or it might be the imperceptibly curving track of a very fast charged particle, or of a very massive charged particle.

Q22.9

The magnetic field created by wire 1 at the position of wire 2 is into the paper. Hence, the magnetic force on wire 2 is in direction down $\times$ into the paper. Now wire 2 creates a magnetic field into the page at the location of wire 1, so wire 1 feels force up $\times$ into the paper = left, away from wire 2.

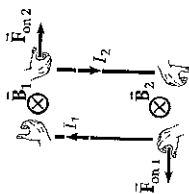


FIG. Q22.9

Q22.10

No total force, but a torque. Let wire 1 carry current in the y direction, toward the top of the page. Let wire 2 be a millimeter above the plane of the paper and carry current to the right, in the x direction. On the left-hand side of wire 1, wire 1 creates magnetic field in the z direction, which exerts force in the $\hat{i} \times \hat{k} = -\hat{j}$ direction on wire 2. On the right-hand side, wire 1 produces magnetic field in the $-\hat{k}$ direction and makes a $\hat{i} \times (-\hat{k}) = +\hat{j}$ force of equal magnitude act on wire 2. If wire 2 is free to move, its center section will twist counterclockwise and then be attracted to wire 1.

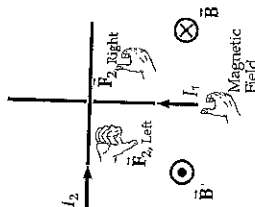


FIG. Q22.10

Apply Ampère's law to the circular path labeled 1 in the picture. Since there is no current inside this path, the magnetic field inside the tube must be zero. On the other hand, the current through path 2 is the current carried by the conductor. Therefore the magnetic field outside the tube is nonzero.

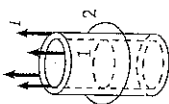


FIG. Q22.11

Q22.12

The magnetic field inside a long solenoid is given by $B = \frac{\mu_0 NI}{\ell}$

(a) If the length ℓ is doubled, the field is cut in half.

(b) If N is doubled, the magnetic field is doubled.

Q22.13 Magnetic domain alignment creates a stronger external magnetic field. The field of one piece of iron in turn can align domains in another iron sample. A nonuniform magnetic field exerts a net force of attraction on magnetic dipoles aligned with the field.

Q22.14 The geographic North Pole is a magnetic South Pole.

Q22.15 The shock misaligns the domains. Heating will also decrease magnetism.

Q22.16 The medium for any magnetic recording should be a hard ferromagnetic substance, so that thermal vibrations and stray magnetic fields will not rapidly erase the information.

Q22.17 (a) The magnets repel each other with a force equal to the weight of one of them.

(b) The pencil prevents motion to the side and prevents the magnets from rotating under their mutual torques. Its constraint changes unstable equilibrium into stable.

(c) Most likely, the disks are magnetized perpendicular to their flat faces, making one face a north pole and the other a south pole. One disk has its north pole on the top side and the other has its north pole on the bottom side.

(d) Then if either were inverted they would attract each other and stick firmly together.

SOLUTIONS TO PROBLEMS

Section 22.1 Historical Overview

No problems in this section

P22.1 (a) up

(b) out of the page, since the charge is negative.

(c) no deflection

(d) into the page

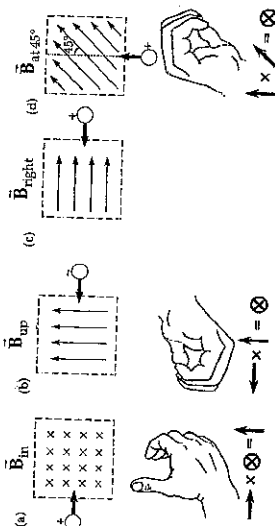


FIG. P22.1

*P22.2

At the equator, the Earth's magnetic field is horizontally north. Because an electron has negative charge, $\vec{F} = q\vec{v} \times \vec{B}$ is opposite in direction to $\vec{v} \times \vec{B}$. Figures are drawn looking down.

(a) Down $\times$ North = East, so the force is directed **West**.

(b) North $\times$ North = $\sin 0^\circ = 0$: **Zero deflection**.

(c) West $\times$ North = Down, so the force is directed **Up**.

(d) Southeast $\times$ North = Up, so the force is **Down**.

P22.3 (a) $F_B = qvB \sin \theta = (1.60 \times 10^{-19} \text{ C})(3.00 \times 10^6 \text{ m/s})(3.00 \times 10^{-1} \text{ T}) \sin 37.0^\circ$
 $F_B = \boxed{8.67 \times 10^{-14} \text{ N}}$

(b) $a = \frac{F}{m} = \frac{8.67 \times 10^{-14} \text{ N}}{9.11 \times 10^{-31} \text{ kg}} = \boxed{9.52 \times 10^{16} \text{ m/s}^2}$

P22.4 First find the speed of the electron.

$$\Delta K = \frac{1}{2}mv^2 = e\Delta V; \quad v = \sqrt{\frac{2e\Delta V}{m}} = \sqrt{\frac{2(1.60 \times 10^{-19} \text{ C})(2400 \text{ V})}{9.11 \times 10^{-31} \text{ kg}}} = \boxed{2.90 \times 10^7 \text{ m/s}}$$

(a) $F_{B, \text{max}} = qvB = (1.60 \times 10^{-19} \text{ C})(2.90 \times 10^7 \text{ m/s})(1.70 \text{ T}) = \boxed{7.90 \times 10^{-12} \text{ N}}$

(b) $F_{B, \text{min}} = \boxed{0}$ occurs when $\vec{v}$ is either parallel to or anti-parallel to $\vec{B}$.

P22.5

Gravitational force: $F_g = mg = (9.11 \times 10^{-31} \text{ kg})(9.80 \text{ m/s}^2) = \boxed{8.93 \times 10^{-30} \text{ N down}}$

Electric force: $F_e = qE = (-1.60 \times 10^{-19} \text{ C})(100 \text{ N/C down}) = \boxed{1.60 \times 10^{-17} \text{ N up}}$

Magnetic force: $\vec{F}_B = q\vec{v} \times \vec{B} = (-1.60 \times 10^{-19} \text{ C})(6.00 \times 10^6 \text{ m/s } \hat{i}) \times (50.0 \times 10^{-6} \text{ N } \cdot \text{s/C } \cdot \text{m } \hat{n})$
 $\vec{F}_B = -4.80 \times 10^{-17} \text{ N up} = \boxed{4.80 \times 10^{-17} \text{ N down}}$

P22.6

$\vec{F}_B = q\vec{v} \times \vec{B}$

$$\vec{v} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ +2 & -4 & +1 \\ +1 & +2 & -3 \end{vmatrix} = (12 - 2)\hat{i} + (1 + 6)\hat{j} + (4 + 4)\hat{k} = 10\hat{i} + 7\hat{j} + 8\hat{k}$$

$|\vec{v} \times \vec{B}| = \sqrt{10^2 + 7^2 + 8^2} = 14.6 \text{ T} \cdot \text{m/s}$

$|\vec{F}_B| = q|\vec{v} \times \vec{B}| = (1.60 \times 10^{-19} \text{ C})(14.6 \text{ T} \cdot \text{m/s}) = \boxed{2.34 \times 10^{-18} \text{ N}}$

Section 22.3 Motion of a Charged Particle in a Uniform Magnetic Field

*P22.7

For each electron, $|q|vB \sin 90.0^\circ = \frac{mv^2}{r}$ and $v = \frac{eBr}{m}$

The electrons have no internal structure to absorb energy, so the collision must be perfectly elastic:

$K = \frac{1}{2}mv_i^2 + 0 = \frac{1}{2}mv_f^2 + \frac{1}{2}mv_{2f}^2$

$K = \frac{1}{2} \left(\frac{e^2 B^2 R_1^2}{m^2} \right) + \frac{1}{2} m \left(\frac{e^2 B^2 R_2^2}{m^2} \right) = \frac{e^2 B^2}{2m} (R_1^2 + R_2^2)$

$K = \frac{e^2 (1.60 \times 10^{-19} \text{ C})^2 (0.0440 \text{ N} \cdot \text{s/C} \cdot \text{m})^2}{2(9.11 \times 10^{-31} \text{ kg})} \left[(0.0100 \text{ m})^2 + (0.0240 \text{ m})^2 \right] = \boxed{115 \text{ keV}}$

P22.8

(a) We begin with $qvB = \frac{mv^2}{R}$

or

$qRB = mv$

But $L = m\omega R = qR^2 B$.

Therefore,

$$R = \frac{L}{qB} = \frac{4.00 \times 10^{-25} \text{ J} \cdot \text{s}}{\sqrt{1.60 \times 10^{-19} \text{ C}} \sqrt{1.00 \times 10^{-3} \text{ T}}} = \boxed{0.0500 \text{ m} = 5.00 \text{ cm}}$$

(b)

Thus, $v = \frac{L}{mR} = \frac{4.00 \times 10^{-25} \text{ J} \cdot \text{s}}{(9.11 \times 10^{-31} \text{ kg})(0.0500 \text{ m})} = \boxed{8.78 \times 10^6 \text{ m/s}}$

P22.9 $E = \frac{1}{2}mv^2 = e\Delta V$

and $evB \sin 90^\circ = \frac{mv^2}{R}$

$$B = \frac{mv}{eR} = \frac{m}{eR} \sqrt{\frac{2e\Delta V}{m}} = \frac{1}{R} \sqrt{\frac{2m\Delta V}{e}}$$

$$B = \frac{1}{5.80 \times 10^{-10} \text{ m}} \sqrt{\frac{2(1.67 \times 10^{-27} \text{ kg})(10.0 \times 10^6 \text{ V})}{1.60 \times 10^{-19} \text{ C}}} = 7.88 \times 10^{-12} \text{ T}$$

Section 22.4 Applications Involving Charged Particles Moving in a Magnetic Field

P22.10 $F_g = F_c$

so $qvB = qE$

where $v = \sqrt{\frac{2K}{m}}$ and K is kinetic energy of the electron.

$$E = vB = \sqrt{\frac{2K}{m}} B = \sqrt{\frac{2(750)(1.60 \times 10^{-19})}{9.11 \times 10^{-31}}}(0.0150) = 244 \text{ kV/m}$$

P22.11 In the velocity selector:

$$v = \frac{E}{B} = \frac{2500 \text{ V/m}}{0.0350 \text{ T}} = 7.14 \times 10^4 \text{ m/s}$$

In the deflection chamber:

$$r = \frac{mv}{qB} = \frac{(2.18 \times 10^{-26} \text{ kg})(7.14 \times 10^4 \text{ m/s})}{(1.60 \times 10^{-19} \text{ C})(0.0350 \text{ T})} = 0.278 \text{ m}$$

P22.12 Note that the "cyclotron frequency" is an angular speed. The motion of the proton is described by $\sum F = m\omega$:

$$|q|vB \sin 90^\circ = \frac{mv^2}{r}$$

$$|q|B = m\frac{v}{r} = m\omega$$

(a) $\omega = \frac{|q|B}{m} = \frac{(1.60 \times 10^{-19} \text{ C})(0.8 \text{ N} \cdot \text{s/C} \cdot \text{m})}{(1.67 \times 10^{-27} \text{ kg})} \left(\frac{\text{kg} \cdot \text{m}}{\text{N} \cdot \text{s}^2} \right) = 7.66 \times 10^7 \text{ rad/s}$

(b) $v = \omega r = (7.66 \times 10^7 \text{ rad/s})(0.350 \text{ m}) \left(\frac{1}{1 \text{ rad}} \right) = 2.68 \times 10^7 \text{ m/s}$

(c) $K = \frac{1}{2}mv^2 = \frac{1}{2}(1.67 \times 10^{-27} \text{ kg})(2.68 \times 10^7 \text{ m/s})^2 \left(\frac{1 \text{ eV}}{1.6 \times 10^{-19} \text{ J}} \right) = 3.76 \times 10^6 \text{ eV}$

continued on next page

(d) The proton gains 600 eV twice during each revolution, so the number of revolutions is

$$\frac{3.76 \times 10^6 \text{ eV}}{2(600 \text{ eV})} = \frac{3.13 \times 10^3 \text{ revolutions}}{2}$$

(e) $\theta = \omega t$ $t = \frac{\theta}{\omega} = \frac{3.13 \times 10^3 \text{ rev} \left(\frac{2\pi \text{ rad}}{1 \text{ rev}} \right)}{7.66 \times 10^7 \text{ rad/s}} = 2.57 \times 10^{-4} \text{ s}$

P22.13 $\theta = \tan^{-1} \left(\frac{25.0}{10.0} \right) = 68.2^\circ$ and $R = \frac{1.00 \text{ cm}}{\sin 68.2^\circ} = 1.08 \text{ cm}$.

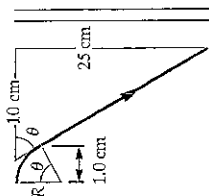
Ignoring relativistic correction, the kinetic energy of the electrons is

$$\frac{1}{2}mv^2 = q\Delta V \quad \text{so} \quad v = \sqrt{\frac{2q\Delta V}{m}} = 1.33 \times 10^8 \text{ m/s}$$

From Newton's second law $\frac{mv^2}{R} = qvB$, we find the magnetic field

$$B = \frac{mv}{|q|R} = \frac{(9.11 \times 10^{-31} \text{ kg})(1.33 \times 10^8 \text{ m/s})}{(1.60 \times 10^{-19} \text{ C})(1.08 \times 10^{-2} \text{ m})} = 70.1 \text{ mT}$$

FIG. P22.13



P22.14

(a) If the charge carriers are negative, to carry current in the x direction they move with drift velocity v_d in the $-x$ direction. The magnetic force $q\vec{v} \times \vec{B}$ is in the $(-\hat{i}) \times \hat{j} = \hat{k}$ direction, so the negative charges are deflected to the top of the ribbon, point c . Some accumulate there to make V_c negative with respect to V_d until ...

(b) ... an upward electric field $\vec{E} = \frac{V_c - V_d}{d} \hat{k}$ exerts a downward force on the other charge carriers to let them drift in equilibrium according to

$$\Sigma F_z = 0: \quad |q| \frac{V_c - V_d}{d} (-\hat{k}) + |q| \left| \frac{\Delta V_H}{d} \right| \hat{k} = 0$$

$$v_d = \frac{|\Delta V_H|}{dB}$$

Since the measured current is $I = n|q|v_d t d$

we have $I = n|q| \left| \frac{\Delta V_H}{dB} \right| t d$

$$n = \frac{IB}{|q| \Delta V_H t}$$

Since we have shown that ΔV_H is negative if q is negative, this expression simplifies to

$$n = \frac{IB}{q\Delta V_H t}$$

Section 22.5 Magnetic Force on a Current-Carrying Conductor

P22.15 $\vec{F}_B = I\vec{L} \times \vec{B} = (2.40 \text{ A})(0.750 \text{ m})\hat{i} \times (1.60 \text{ T})\hat{k} = \boxed{(-2.88\hat{j}) \text{ N}}$

P22.16 (a) $F_B = ILB \sin \theta = (5.00 \text{ A})(2.80 \text{ m})(0.390 \text{ T}) \sin 60.0^\circ = \boxed{4.73 \text{ N}}$

(b) $F_B = (5.00 \text{ A})(2.80 \text{ m})(0.390 \text{ T}) \sin 90.0^\circ = \boxed{5.46 \text{ N}}$

(c) $F_B = (5.00 \text{ A})(2.80 \text{ m})(0.390 \text{ T}) \sin 120^\circ = \boxed{4.73 \text{ N}}$

P22.17 The magnetic force on each bit of ring is

$I\vec{d}\vec{s} \times \vec{B} = IdsB$ radially inward and upward, at angle θ above the radial line. The radially inward components tend to squeeze the ring but all cancel out as forces. The upward components $I\vec{d}\vec{s} \sin \theta$ all add to $\boxed{[2\pi rB \sin \theta] \text{ up}}$.

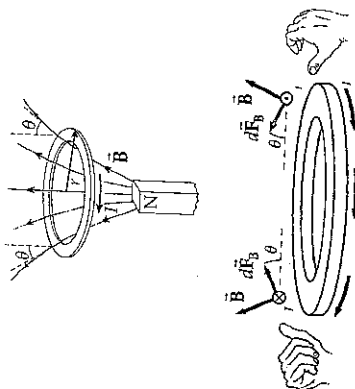


FIG. P22.17

P22.18 For each segment, $I = 5.00 \text{ A}$ and $\vec{B} = 0.0200 \text{ N/A} \cdot \text{m}$.

Segment	$\vec{z}$	$\vec{F}_B = I(\vec{L} \times \vec{B})$
ab	$-0.400 \text{ m } \hat{j}$	$\boxed{0}$
bc	$0.400 \text{ m } \hat{k}$	$\boxed{(40.0 \text{ mN})(-\hat{i})}$
cd	$-0.400 \text{ m } \hat{i} + 0.400 \text{ m } \hat{j}$	$\boxed{(40.0 \text{ mN})(-\hat{k})}$
da	$0.400 \text{ m } \hat{i} - 0.400 \text{ m } \hat{k}$	$\boxed{(40.0 \text{ mN})(\hat{k} + \hat{i})}$

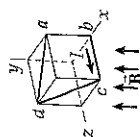


FIG. P22.18

Section 22.6 Torque on a Current Loop in a Uniform Magnetic Field

P22.19 (a) $2\pi r = 2.00 \text{ m}$

so $r = 0.318 \text{ m}$

$\mu = IA = (17.0 \times 10^{-3} \text{ A})[\pi(0.318)^2 \text{ m}^2] = \boxed{5.41 \text{ mA} \cdot \text{m}^2}$

(b) $\vec{\tau} = \vec{\mu} \times \vec{B}$

so $\tau = (5.41 \times 10^{-3} \text{ A} \cdot \text{m}^2)(0.800 \text{ T}) = \boxed{4.33 \text{ mN} \cdot \text{m}}$

P22.20 Choose $U = 0$ when the dipole moment is at $\theta = 90.0^\circ$ to the field. The field exerts torque of magnitude $\mu B \sin \theta$ on the dipole, tending to turn the dipole moment in the direction of decreasing θ . According to Equations 7.15 and 10.28, the potential energy of the dipole-field system is given by

$U - 0 = \int_{90.0^\circ}^{\theta} \mu B \sin \theta d\theta = \mu B(-\cos \theta) \Big|_{90.0^\circ}^{\theta} = -\mu B \cos \theta + 0 \quad \text{or} \quad \boxed{U = -\vec{\mu} \cdot \vec{B}}$

P22.21 $\tau = NBAI \sin \phi$

$\tau = 100(0.800 \text{ T})(0.400 \times 0.300 \text{ m}^2)(1.20 \text{ A}) \sin 60^\circ$

$\tau = \boxed{9.98 \text{ N} \cdot \text{m}}$

Note that ϕ is the angle between the magnetic moment and the $\vec{B}$ field. The loop will rotate so as to align the magnetic moment with the $\vec{B}$ field. Looking down along the y -axis, the loop will rotate in a $\boxed{\text{clockwise}}$ direction.

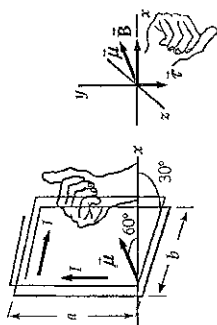


FIG. P22.21

*P22.22 (a) $|\vec{\tau}| = |\vec{\mu} \times \vec{B}| = NIAB \sin \theta$

$\tau_{\max} = 80(10^{-2} \text{ A})(0.025 \text{ m} \cdot 0.04 \text{ m})(0.8 \text{ N/A} \cdot \text{m}) \sin 90^\circ = \boxed{6.40 \times 10^{-4} \text{ N} \cdot \text{m}}$

(b) $\mathcal{P}_{\max} = \tau_{\max} \omega = 6.40 \times 10^{-4} \text{ N} \cdot \text{m} (3600 \text{ rev/min}) \left(\frac{2\pi \text{ rad}}{1 \text{ rev}} \right) \left(\frac{1 \text{ min}}{60 \text{ s}} \right) = \boxed{0.241 \text{ W}}$

(c) In one half revolution the work is

$W = U_{\max} - U_{\min} = -\mu B \cos 180^\circ - (-\mu B \cos 0^\circ) = 2\mu B$
 $= 2NIAB = 2(6.40 \times 10^{-4} \text{ N} \cdot \text{m}) = \boxed{1.28 \times 10^{-3} \text{ J}}$

In one full revolution, $W = 2(1.28 \times 10^{-3} \text{ J}) = \boxed{2.56 \times 10^{-3} \text{ J}}$

(d) $\mathcal{P}_{\text{avg}} = \frac{W}{\Delta t} = \frac{2.56 \times 10^{-3} \text{ J}}{(1/60) \text{ s}} = \boxed{0.154 \text{ W}}$

The peak power in (b) is greater by the factor $\frac{\pi}{2}$.

Section 22.7 The Biot-Savart Law

P22.23 $B = \frac{\mu_0 I}{2R} = \frac{\mu_0 q(v/2\pi R)}{2R} = \frac{12.5 \text{ T}}{2R}$

P22.24 $B = \frac{\mu_0 I}{2\pi R} = \frac{(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(1.00 \times 10^{-4} \text{ A})}{2\pi(100 \text{ m})} = 2.00 \times 10^{-5} \text{ T} = 20.0 \text{ } \mu\text{T}$

P22.25 For leg 1, $\vec{ds} \times \vec{r} = 0$, so there is no contribution to the field from this segment. For leg 2, the wire is only semi-infinite; thus,

$$B = \frac{1}{2} \left(\frac{\mu_0 I}{2\pi x} \right) = \frac{\mu_0 I}{4\pi x} \text{ into the paper}$$

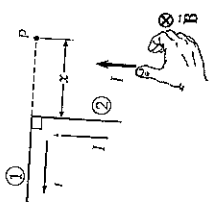


FIG. P22.25

P22.26 $B = \frac{\mu_0 I}{2\pi r} = \frac{(4\pi \times 10^{-7})(1.00 \text{ A})}{2\pi(1.00 \text{ m})} = 2.00 \times 10^{-7} \text{ T}$

P22.27 We can think of the total magnetic field as the superposition of the field due to the long straight wire (having magnitude $\frac{\mu_0 I}{2\pi R}$ and directed into the page) and the field due to the circular loop (having magnitude $\frac{\mu_0 I}{2R}$ and directed into the page). The resultant magnetic field is:

$$\vec{B} = \left(1 + \frac{1}{\pi} \right) \frac{\mu_0 I}{2R} \text{ (directed into the page)}$$

P22.28 Along the axis of a circular loop of radius R ,

$$B = \frac{\mu_0 I R^2}{2(x^2 + R^2)^{3/2}}$$

$$\text{or } \frac{B}{B_0} = \frac{1}{\left[\left(\frac{x}{R} \right)^2 + 1 \right]^{3/2}}$$

where $B_0 = \frac{\mu_0 I}{2R}$

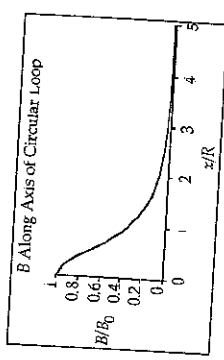


FIG. P22.28

x/R	B/B_0
0.00	1.00
1.00	0.354
2.00	0.0894
3.00	0.0316
4.00	0.0143
5.00	0.00754

*P22.29 Wire 1 creates at the origin magnetic field

$$\vec{B}_1 = \frac{\mu_0 I}{2\pi r} \text{ right hand rule} = \frac{\mu_0 I_1}{2\pi a} = \frac{\mu_0 I_1}{2\pi a} \hat{j}$$

(a) If the total field at the origin is $\frac{2\mu_0 I_1}{2\pi a} \hat{j} = \frac{\mu_0 I_1}{\pi a} \hat{j} + \vec{B}_2$ then the second wire must create field according to $\vec{B}_2 = \frac{\mu_0 I_1}{2\pi a} \hat{j} - \frac{\mu_0 I_2}{2\pi(2a)}$

Then $I_2 = 2I_1$ out of the paper

(b) The other possibility is $\vec{B}_1 + \vec{B}_2 = \frac{2\mu_0 I_1}{2\pi a} (-\hat{j}) = \frac{\mu_0 I_1}{\pi a} (-\hat{j})$. Then

$$\vec{B}_2 = \frac{3\mu_0 I_1}{2\pi a} (-\hat{j}) = \frac{\mu_0 I_2}{2\pi(2a)} \hat{j}$$

$I_2 = 6I_1$ into the paper

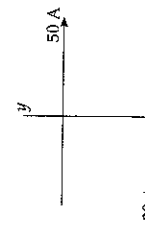


FIG. P22.30

(a) Above the pair of wires, the field out of the page of the 50 A current will be stronger than the $(-\hat{k})$ field of the 30 A current, so they cannot add to zero. Between the wires, both produce fields into the page. They can only add to zero below the wires, at coordinate $y = -|y|$. Here the total field is

$$\vec{B} = \frac{\mu_0 I}{2\pi r} + \frac{\mu_0 I}{2\pi r}$$

$$0 = \frac{\mu_0}{2\pi} \left[\frac{50 \text{ A}}{(|y| + 0.28 \text{ m})} (-\hat{k}) + \frac{30 \text{ A}}{|y|} (\hat{k}) \right]$$

$$50|y| = 30(|y| + 0.28 \text{ m})$$

$$50(-y) = 30(0.28 \text{ m} - y)$$

$$-20y = 30(0.28 \text{ m}) \text{ at } y = -0.420 \text{ m}$$

(b) At $y = 0.1 \text{ m}$ the total field is $\vec{B} = \frac{\mu_0 I}{2\pi r} + \frac{\mu_0 I}{2\pi r}$

$$\vec{B} = \frac{4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}}{2\pi} \left(\frac{50 \text{ A}}{(0.28 - 0.10) \text{ m}} (-\hat{k}) + \frac{30 \text{ A}}{0.10 \text{ m}} (-\hat{k}) \right) = 1.16 \times 10^{-4} \text{ T} (-\hat{k})$$

The force on the particle is

$$\vec{F} = q\vec{v} \times \vec{B} = (-2 \times 10^{-6} \text{ C})(150 \times 10^6 \text{ m/s})(\hat{j}) \times (1.16 \times 10^{-4} \text{ N} \cdot \text{s/C} \cdot \text{m})(-\hat{k}) = 3.47 \times 10^{-2} \text{ N}(-\hat{j})$$

(c) We require $\vec{F} = 3.47 \times 10^{-2} \text{ N}(\hat{j}) = q\vec{E} = (-2 \times 10^{-6} \text{ C})\vec{E}$.

So $\vec{E} = -1.73 \times 10^4 \hat{j} \text{ N/C}$

P22.31

We use the Biot-Savart law. For bits of wire along the straight-line sections, $d\vec{s}$ is at 0° or 180° to $\hat{r}$, so $d\vec{s} \times \hat{r} = 0$. Thus, only the curved section of wire contributes to $\vec{B}$ at P . Hence, $d\vec{s}$ is tangent to the arc and $\hat{r}$ is radially inward; so $d\vec{s} \times \hat{r} = |d\vec{s}| \ell \sin 90^\circ = |d\vec{s}| \ell \hat{\phi}$. All points along the curve are the same distance $r = 0.600$ m from the field point, so

$$B = \int \frac{\mu_0 I}{4\pi r^2} |d\vec{s} \times \hat{r}| = \frac{\mu_0 I}{4\pi r^2} \int |d\vec{s}| = \frac{\mu_0 I}{4\pi r^2} s$$

where s is the arc length of the curved wire,

$$s = r\theta = (0.600 \text{ m})(30.0^\circ) \left(\frac{2\pi}{360^\circ} \right) = 0.314 \text{ m}$$

$$\text{Then, } B = (10^{-7} \text{ T}\cdot\text{m/A}) \frac{(3.00 \text{ A})}{(0.600 \text{ m})^2} (0.314 \text{ m}) = 261 \text{ nT into the page}$$

P22.32

Label the wires 1, 2, and 3 as shown in Figure (a) and let the magnetic field created by the currents in these wires be $\vec{B}_1$, $\vec{B}_2$, and $\vec{B}_3$, respectively.

$$(a) \quad \text{At point A: } B_1 = B_2 = \frac{\mu_0 I}{2\pi(a\sqrt{2})} \text{ and } B_3 = \frac{\mu_0 I}{2\pi(3a)}$$

The directions of these fields are shown in Figure (b). Observe that the horizontal components of $\vec{B}_1$ and $\vec{B}_2$ cancel while their vertical components both add to $\vec{B}_3$.

Therefore, the net field at point A is:

$$\begin{aligned} B_A &= B_1 \cos 45.0^\circ + B_2 \cos 45.0^\circ + B_3 \\ &= \frac{\mu_0 I}{2\pi a} \left[\frac{2}{\sqrt{2}} \cos 45.0^\circ + \frac{1}{3} \right] \\ B_A &= \frac{(4\pi \times 10^{-7} \text{ T}\cdot\text{m/A})(2.00 \text{ A})}{2\pi(1.00 \times 10^{-2} \text{ m})} \left[\frac{2}{\sqrt{2}} \cos 45.0^\circ + \frac{1}{3} \right] \\ B_A &= 53.3 \text{ } \mu\text{T toward the bottom of the page} \end{aligned}$$

(b) At point B: $\vec{B}_1$ and $\vec{B}_2$ cancel, leaving

$$\begin{aligned} B_B &= B_3 = \frac{\mu_0 I}{2\pi(2a)} \\ B_B &= \frac{(4\pi \times 10^{-7} \text{ T}\cdot\text{m/A})(2.00 \text{ A})}{2\pi(2)(1.00 \times 10^{-2} \text{ m})} \\ &= 20.0 \text{ } \mu\text{T toward the bottom of the page} \end{aligned}$$

continued on next page

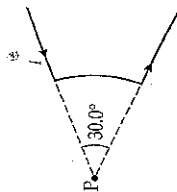


FIG. P22.31



Figure (a)

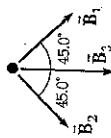


Figure (b)

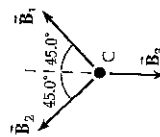


Figure (c)

FIG. P22.32

(c)

At point C: $B_1 = B_2 = \frac{\mu_0 I}{2\pi(a\sqrt{2})}$ and $B_3 = \frac{\mu_0 I}{2\pi a}$ with the directions shown in Figure (c). Again, the horizontal components of $\vec{B}_1$ and $\vec{B}_2$ cancel. The vertical components both oppose $\vec{B}_3$, giving

$$B_C = 2 \left[\frac{\mu_0 I}{2\pi(a\sqrt{2})} \cos 45.0^\circ \right] - \frac{\mu_0 I}{2\pi a} = \frac{\mu_0 I}{2\pi a} \left[\frac{2}{\sqrt{2}} \cos 45.0^\circ - 1 \right] = 0$$

P22.33

(a) Each coil separately produces field given by $B = \frac{N\mu_0 I R^2}{2(R^2 + x^2)^{3/2}}$ at the point halfway between them. Together they produce field

$$2B = \frac{N\mu_0 I R^2}{(R^2 + x^2)^{3/2}} = 4.5 \times 10^{-5} \text{ T} = \frac{50(4\pi \times 10^{-7} \text{ Tm})(0.012 \text{ m})^2}{A[(0.012 \text{ m})^2 + (0.011 \text{ m})^2]^{3/2}} = \frac{9.05 \times 10^{-9} \text{ Tm}^3}{4.31 \times 10^{-6} \text{ m}^3 \text{ A}}$$

$$I = \frac{4.5 \times 10^{-5} \text{ T A}}{2.10 \times 10^{-9} \text{ T}} = 21.5 \text{ mA}$$

$$(b) \quad \Delta V = IR = (0.0215 \text{ A})(210 \Omega) = 4.51 \text{ V}$$

$$(c) \quad \mathcal{P} = (\Delta V)I = (4.51 \text{ V})(0.0215 \text{ A}) = 96.7 \text{ mW}$$

Section 22.8 The Magnetic Force Between Two Parallel Conductors

P22.34 Let both wires carry current in the x direction, the first at $y = 0$ and the second at $y = 10.0$ cm.

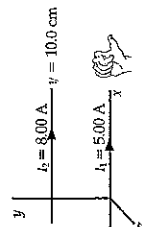


FIG. P22.34(a)

$$(a) \quad \vec{B} = \frac{\mu_0 I}{2\pi r} \hat{k} = \frac{(4\pi \times 10^{-7} \text{ T}\cdot\text{m/A})(5.00 \text{ A})}{2\pi(0.100 \text{ m})} \hat{k}$$

$$\vec{B} = 1.00 \times 10^{-5} \text{ T out of the page}$$

$$(b) \quad \vec{F}_B = I_2 \vec{\ell} \times \vec{B} = (8.00 \text{ A})(1.00 \text{ m}) \hat{i} \times (1.00 \times 10^{-5} \text{ T}) \hat{k} = (8.00 \times 10^{-5} \text{ N})(-\hat{j})$$

$$\vec{F}_B = 8.00 \times 10^{-5} \text{ N toward the first wire}$$

$$(c) \quad \vec{B} = \frac{\mu_0 I}{2\pi r} (-\hat{k}) = \frac{(4\pi \times 10^{-7} \text{ T}\cdot\text{m/A})(8.00 \text{ A})}{2\pi(0.100 \text{ m})} (-\hat{k}) = (1.60 \times 10^{-5} \text{ T})(-\hat{k})$$

$$\vec{B} = 1.60 \times 10^{-5} \text{ T into the page}$$

continued on next page

(d) $\vec{F}_B = I_1 \vec{L} \times \vec{B} = (5.00 \text{ A})[(1.00 \text{ m})\hat{i} \times (1.60 \times 10^{-5} \text{ T})(-\hat{k})] = (8.00 \times 10^{-5} \text{ N})(+\hat{j})$

$\vec{F}_B = 8.00 \times 10^{-5} \text{ N}$ towards the second wire

P22.35

By symmetry, we note that the magnetic forces on the top and bottom segments of the rectangle cancel. The net force on the vertical segments of the rectangle is (using Equation 22.27)

$$\begin{aligned}\vec{F} &= \vec{F}_1 + \vec{F}_2 = \frac{\mu_0 I_1 I_2 \ell}{2\pi} \left(\frac{1}{c+a} - \frac{1}{c} \right) \hat{i} = \frac{\mu_0 I_1 I_2 \ell}{2\pi} \left(\frac{-a}{c(c+a)} \right) \hat{i} \\ \vec{F} &= \frac{(4\pi \times 10^{-7} \text{ N/A}^2)(5.00 \text{ A})(10.0 \text{ A})(0.450 \text{ m})}{2\pi} \left(\frac{-0.150 \text{ m}}{(0.100 \text{ m})(0.250 \text{ m})} \right) \hat{i} \\ \vec{F} &= (-2.70 \times 10^{-5} \text{ N})\hat{i}\end{aligned}$$

or $\vec{F} = 2.70 \times 10^{-5} \text{ N}$ toward the left

*P22.36

Carrying oppositely directed currents, wires 1 and 2 repel each other. If wire 3 were between them, it would have to repel either 1 or 2, so the force on that wire could not be zero. If wire 3 were to the right of wire 2, it would feel a larger force exerted by 2 than that exerted by 1, so the total force on 3 could not be zero. Therefore wire 3 must be to the left of both other wires as shown. It must carry downward current so that it can attract wire 2.

(a) For the equilibrium of wire 3 we have

$$F_{1 \text{ on } 3} = F_{2 \text{ on } 3}$$

$$1.5(20 \text{ cm} + d) = 4d$$

$$d = \frac{30 \text{ cm}}{2.5} = 12.0 \text{ cm to the left of wire 1}$$

(b) For the equilibrium of wire 1,

$$\frac{\mu_0 I_1 (1.5 \text{ A})}{2\pi(12 \text{ cm})} = \frac{\mu_0 (4 \text{ A})(1.5 \text{ A})}{2\pi(20 \text{ cm})}$$

$$I_3 = \frac{12}{20} 4 \text{ A} = 2.40 \text{ A down}$$

We know that wire 2 must be in equilibrium because the forces on it are equal in magnitude to the forces that it exerts on wires 1 and 3, which are equal because they both balance the equal-magnitude forces that 1 exerts on 3 and that 3 exerts on 1.

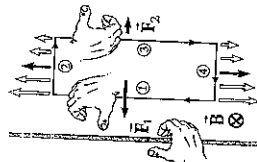


FIG. P22.35

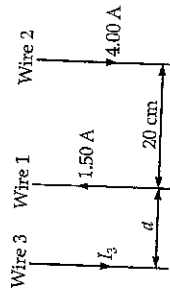


FIG. P22.36

Section 22.9 Ampère's Law

P22.37 Each wire is distant from P by

$$(0.200 \text{ m}) \cos 45.0^\circ = 0.141 \text{ m}$$

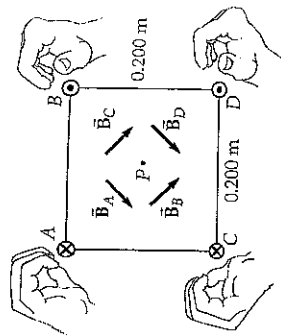
Each wire produces a field at P of equal magnitude:

$$B_A = \frac{\mu_0 I}{2\pi a} = \frac{(2.00 \times 10^{-7} \text{ T} \cdot \text{m/A})(5.00 \text{ A})}{(0.141 \text{ m})} = 7.07 \mu\text{T}$$

Carrying currents into the page, A produces at P a field of $7.07 \mu\text{T}$ to the left and down at -135° , while B creates a field to the right and down at -45° . Carrying currents toward you, C produces a field downward and to the right at -45° , while D's contribution is downward and to the left. The total field is then

$$4(7.07 \mu\text{T}) \sin 45.0^\circ = 20.0 \mu\text{T} \text{ toward the bottom of the page.}$$

FIG. P22.37



P22.38 Let the current I be to the right. It creates a field $B = \frac{\mu_0 I}{2\pi d}$ at the proton's location. And we have a balance between the weight of the proton and the magnetic force

$$mg(-\hat{j}) + qv(-\hat{i}) \times \frac{\mu_0 I}{2\pi d} (\hat{k}) = 0 \text{ at a distance } d \text{ from the wire}$$

$$d = \frac{qv\mu_0 I}{2\pi mg} = \frac{(1.60 \times 10^{-19} \text{ C})(2.30 \times 10^4 \text{ m/s})(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(1.20 \times 10^{-5} \text{ A})}{2\pi(1.67 \times 10^{-27} \text{ kg})(9.80 \text{ m/s}^2)} = 5.40 \text{ cm}$$

P22.39 (a) One wire feels force due to the field of the other ninety-nine.

$$B = \frac{\mu_0 I}{2\pi R} = \frac{(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(99)(2.00 \text{ A})(0.200 \times 10^{-2} \text{ m})}{2\pi(0.500 \times 10^{-2} \text{ m})^2} = 3.17 \times 10^{-3} \text{ T}$$

This field points tangent to a circle of radius 0.200 cm and exerts force $\vec{F} = I\vec{L} \times \vec{B}$ toward the center of the bundle, on the single hundredth wire:

$$\frac{F}{\ell} = IB \sin \theta = (2.00 \text{ A})(3.17 \times 10^{-3} \text{ T}) \sin 90^\circ = 6.34 \text{ mN/m}$$

$$\frac{F_B}{\ell} = 6.34 \times 10^{-3} \text{ N/m inward}$$

(b) $B \propto r$, so B is greatest at the outside of the bundle. Since each wire carries the same current, F is greatest at the outer surface

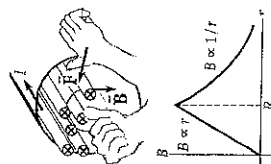


FIG. P22.39

*P22.40 (a) In $B = \frac{\mu_0 I}{2\pi r}$, the field will be one-tenth as large at a ten-times larger distance: 400 cm

(b) $\vec{B} = \frac{\mu_0 I}{2\pi r_1} \hat{k} + \frac{\mu_0 I}{2\pi r_2} (-\hat{k})$ so $B = \frac{4\pi \times 10^{-7} \text{ T} \cdot \text{m}/\text{A}}{2\pi A} \left(\frac{1}{0.3985 \text{ m}} - \frac{1}{0.4015 \text{ m}} \right) = \boxed{7.50 \text{ nT}}$

(c) Call r the distance from cord center to field point and $2d = 3.00 \text{ mm}$ the distance between conductors.

$$B = \frac{\mu_0 I}{2\pi} \left(\frac{1}{r-d} - \frac{1}{r+d} \right) = \frac{\mu_0 I}{2\pi} \frac{2d}{r^2 - d^2}$$

$$7.50 \times 10^{-10} \text{ T} = (2.00 \times 10^{-7} \text{ T} \cdot \text{m}/\text{A})(2.00 \text{ A}) \frac{(3.00 \times 10^{-3} \text{ m})}{r^2 - 2.25 \times 10^{-6} \text{ m}^2} \text{ so } r = \boxed{1.26 \text{ m}}$$

The field of the two-conductor cord is weak to start with and falls off rapidly with distance.

(d) The cable creates zero field at exterior points, since a loop in Ampère's law encloses zero total current. Shall we sell coaxial-cable power cords to people who worry about biological damage from weak magnetic fields?

P22.41 (a) $B_{\text{inner}} = \frac{\mu_0 NI}{2\pi r} = \frac{(4\pi \times 10^{-7} \text{ T} \cdot \text{m}/\text{A})(900)(14.0 \times 10^3 \text{ A})}{2\pi(0.700 \text{ m})} = \boxed{3.60 \text{ T}}$

(b) $B_{\text{outer}} = \frac{\mu_0 NI}{2\pi r} = \frac{(2 \times 10^{-7} \text{ T} \cdot \text{m}/\text{A})(900)(14.0 \times 10^3 \text{ A})}{2\pi(1.30 \text{ m})} = \boxed{1.94 \text{ T}}$

P22.42 We assume the current is vertically upward.

(a) Consider a circle of radius r slightly less than R . It encloses no current so from $\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{\text{inside}}$

$$B(2\pi r) = 0$$

we conclude that the magnetic field is zero.

(b) Now let the r be barely larger than R . Ampère's law becomes $B(2\pi R) = \mu_0 I$,

so $B = \frac{\mu_0 I}{2\pi R}$

The field's direction is $\hat{\phi}$

tangent to the wall of the cylinder in a counterclockwise sense.

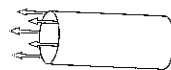


FIG. P22.42(a)

(c) Consider a strip of the wall of width dx and length ℓ . Its width is so small compared to $2\pi R$ that the field at its location would be essentially unchanged if the current in the strip were turned off.

The current it carries is $I_s = \frac{I dx}{2\pi R}$ up.

The force on it is

$$\vec{F} = I_s \vec{\ell} \times \vec{B} = \frac{I dx}{2\pi R} \left(\frac{\mu_0 I}{2\pi R} \right) \text{ up} \times \text{into page} = \frac{\mu_0 I^2 \ell dx}{4\pi^2 R^2} \text{ radially inward.}$$

FIG. P22.42(c)

The pressure on the strip and everywhere on the cylinder is

$$p = \frac{F}{A} = \frac{\mu_0 I^2 \ell dx}{4\pi^2 R^2 \ell dx} = \frac{\mu_0 I^2}{(2\pi R)^2} \text{ inward}$$

The pinch effect makes an effective demonstration when an aluminum can crushes itself as it carries a large current along its length.

P22.43 From $\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I$,
$$I = \frac{2\pi r B}{\mu_0} = \frac{2\pi(1.00 \times 10^{-3})(0.100)}{4\pi \times 10^{-7}} = \boxed{500 \text{ A}}$$

Section 22.10 The Magnetic Field of a Solenoid

*P22.44 The field produced by the solenoid in its interior is given by

$$\vec{B} = \mu_0 n I (-\hat{i}) = (4\pi \times 10^{-7} \text{ T} \cdot \text{m}/\text{A}) \left(\frac{30.0}{10^{-2} \text{ m}} \right) (15.0 \text{ A}) (-\hat{i})$$

$$\vec{B} = -(5.65 \times 10^{-4} \text{ T}) \hat{i}$$

The force exerted on side AB of the square current loop is

$$(\vec{F}_B)_{AB} = \vec{I} \times \vec{B} = (0.200 \text{ A}) [(2.00 \times 10^{-4} \text{ m}) \hat{j}] \times (5.65 \times 10^{-4} \text{ T}) (-\hat{i})$$

$$(\vec{F}_B)_{AB} = (2.26 \times 10^{-4} \text{ N}) \hat{k}$$

Similarly, each side of the square loop experiences a force, lying in the plane of the loop, of

226 μN directed away from the center. From the above

result, it is seen that the net torque exerted on the square loop by the field of the solenoid should be zero. More formally, the magnetic dipole moment of the square loop is given by

$$\vec{\mu} = I \vec{A} = (0.200 \text{ A}) (2.00 \times 10^{-2} \text{ m})^2 (-\hat{i}) = -80.0 \mu\text{A} \cdot \text{m}^2 \hat{i}$$

The torque exerted on the loop is then $\vec{\tau} = \vec{\mu} \times \vec{B} = (-80.0 \mu\text{A} \cdot \text{m}^2 \hat{i}) \times (-5.65 \times 10^{-4} \text{ T}) \hat{i} = \boxed{0}$

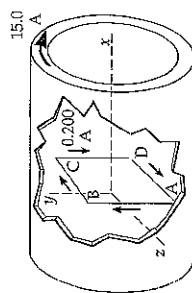
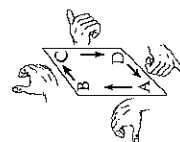


FIG. P22.44



P22.45 $B = \mu_0 \frac{N}{\ell} I$ so $I = \frac{B}{\mu_0 \frac{N}{\ell}} = \frac{(1.00 \times 10^{-4} \text{ T})(0.400 \text{ m})}{(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(1000)} = 31.8 \text{ mA}$

P22.46 Let the axis of the solenoid lie along the y -axis from $y=0$ to $y=\ell$. We will determine the field at $y=a$. This point will be inside the solenoid if $0 < a < \ell$ and outside if $a < 0$ or $a > \ell$. We think of wire and the number of turns per length is $\left(\frac{N}{\ell}\right)$. So the number of turns in each turn of the current in the ring is $I_{\text{ring}} = I \left(\frac{N}{\ell}\right) dy$. Now we use the result of Example 22.6 for the field created by one ring:

$$B_{\text{ring}} = \frac{\mu_0 I_{\text{ring}} R^2}{2(x^2 + R^2)^{3/2}}$$

where x is the name of the distance from the center of the ring, at location y , to the field point $x=a-y$. Each ring creates field in the same direction, along our y -axis, so the whole field of the solenoid is

$$B = \sum_{\text{all rings}} B_{\text{ring}} = \sum \frac{\mu_0 I_{\text{ring}} R^2}{2(x^2 + R^2)^{3/2}} = \int \frac{\mu_0 I (N/\ell) dy R^2}{2((a-y)^2 + R^2)^{3/2}} = \frac{\mu_0 I N R^2}{2\ell} \int_0^\ell \frac{dy}{2((a-y)^2 + R^2)^{3/2}}$$

To perform the integral we change variables to $u=a-y$.

$$B = \frac{\mu_0 I N R^2}{2\ell} \int_a^{a-\ell} \frac{-du}{u^2 \sqrt{u^2 + R^2}}$$

and then use the table of integrals in the appendix:

$$(a) \quad B = \frac{\mu_0 I N R^2}{2\ell} \left[\frac{-u}{R^2 \sqrt{u^2 + R^2}} + \frac{a-\ell}{\sqrt{u^2 + R^2}} \right]_a^{a-\ell}$$

(b) If ℓ is much larger than R and $a=0$,

$$\text{we have } B = \frac{\mu_0 I N}{2\ell} \left[0 - \frac{-\ell}{\sqrt{\ell^2}} \right] = \frac{\mu_0 I N}{2\ell}$$

This is just half the magnitude of the field deep within the solenoid. We would get the same result by substituting $a=\ell$ to describe the other end.

P22.47 The number of turns is $N = \frac{75 \text{ cm}}{0.1 \text{ cm}} = 750$. We assume that the solenoid is long enough to qualify as a long solenoid. Then the field within it (not close to the ends) is $B = \frac{\mu_0 N I}{\ell}$, so

$$I = \frac{B \ell}{\mu_0 N} = \frac{(8 \times 10^{-3} \text{ T})(0.75 \text{ m})}{(4\pi \times 10^{-7} \text{ T} \cdot \text{m})(750)} = 6.37 \text{ A}$$

The resistance of the wire is

$$R = \frac{\rho \ell_{\text{wire}}}{A} = \frac{(1.7 \times 10^{-8} \Omega \cdot \text{m})(2\pi(0.05 \text{ m})/750)}{\pi(0.05 \times 10^{-2} \text{ m})^2} = 5.10 \Omega$$

The power delivered is

$$P = I \Delta V = I^2 R = (6.37 \text{ A})^2 (5.10 \Omega) = 207 \text{ W}$$

The power required would be smaller if wire were wrapped in several layers.

Section 22.11 Magnetism in Matter

P22.48 (a) $I = \frac{e v}{2\pi r}$

$$\mu = I A = \left(\frac{e v}{2\pi r} \right) \pi r^2 = 9.27 \times 10^{-24} \text{ A} \cdot \text{m}^2$$

The Bohr model predicts the correct magnetic moment. However, the "planetary model" is seriously deficient in other regards.

(b) Because the electron is $(-)$, its [conventional] current is clockwise, as seen from above, and μ points downward.



FIG. P22.48

P22.49 (a) Number of unpaired electrons $= \frac{8.00 \times 10^{22} \text{ A} \cdot \text{m}^2}{9.27 \times 10^{-24} \text{ A} \cdot \text{m}^2} = 8.63 \times 10^{45}$

Each iron atom has two unpaired electrons, so the number of iron atoms required is $\frac{1}{2} (8.63 \times 10^{45})$.

(b) Mass $= \frac{(4.31 \times 10^{45} \text{ atoms})(7900 \text{ kg/m}^3)}{(8.50 \times 10^{28} \text{ atoms/m}^3)} = 4.01 \times 10^{20} \text{ kg}$

Section 22.12 Context Connection—The Attractive Model for Magnetic Levitation

P22.50

Let N be the number of charges. For the vehicle we want

$$\Sigma F_y = 0;$$

$$-mg + Nq\epsilon B \sin 90^\circ = 0$$

$$N = \frac{mg}{q\epsilon B} = \frac{5 \times 10^4 \text{ kg}(9.8 \text{ m/s}^2)}{10^{-6} \text{ C}(400 \text{ km/h})(1000/\text{h})(1/3600 \text{ s})(0.1 \text{ N} \cdot \text{s/C} \cdot \text{m})} = \boxed{4 \times 10^{10}}$$

P22.51

The energy per distance is the effective force required to propel the vehicle:

$$\frac{W}{\Delta x} = F = \frac{W/t}{\Delta x/t} = \frac{\mathcal{P}}{v}$$

$$(a) \quad \frac{\mathcal{P}}{v} = \frac{10^2 (10^3 \text{ J/s}) / (3600 \text{ s/h})}{400 \times 10^3 \text{ m/h}} = 900 \text{ N} = (900 \text{ J/m}) \left(\frac{1609 \text{ m}}{1 \text{ mi}} \right) = \boxed{1.4 \times 10^6 \text{ J/mi}}$$

(b) We call 20 mi/gal the fuel economy. Then 1 gal/20 mi is the measure of energy use in which we are interested:

$$\frac{W}{\Delta x} = \frac{1 \text{ gal}}{20 \text{ mi}} \left(\frac{3.786 \text{ L}}{1 \text{ gal}} \right) \left(\frac{10^{-3} \text{ m}^3}{1 \text{ L}} \right) \left(\frac{754 \text{ kg}}{\text{m}^3} \right) \left(\frac{40 \times 10^6 \text{ J}}{\text{kg}} \right) = \boxed{5.7 \times 10^6 \text{ J/mi}}$$

(c) One automobile passenger uses chemical energy at the rate $5.7 \times 10^6 \text{ J/mi}$. One Transrapid passenger uses electric energy at the rate

$$\frac{1.4 \times 10^6 \text{ J/mi}}{100} = 1.4 \times 10^4 \text{ J/mi}, \quad \boxed{\text{smaller by 400 times}}$$

If we suppose that electric energy must be generated by burning a fossil fuel with limited efficiency, then a fair comparison is between the output energies propelling the vehicles, namely $0.20(5.7 \times 10^6 \text{ J/mi}) = 1.1 \times 10^6 \text{ J/mi}$ for the car and $1.4 \times 10^4 \text{ J/mi}$ for the maglev vehicle. The latter is 80 times smaller.

The dependence of American society on gasoline is a dangerous and destructive addition that we cannot continue in the long run.

Additional Problems

P22.52

(a) The element $d\vec{s}$ is a distance r from P . The direction of the field at P due to this element is out of the page, since $d\vec{s} \times \hat{r}$ is out of the page. In fact, all elements give contributions directly out of the page at P . Therefore we have only to determine the magnitude of the field.

Since $d\vec{s} = \hat{i}dx$ in this case,

$$|d\vec{s} \times \hat{r}| = dx \sin \theta.$$

we see that

Using this in the Biot-Savart law

$$\text{we get} \quad dB = \frac{\mu_0 I}{4\pi} \frac{dx \sin \theta}{r^2}. \quad (1)$$

In order to integrate this expression, we must relate the variables θ , x , and r . One approach is to express x and r in terms of θ . From the geometry in Figure P22.52a and some simple differentiation, we obtain the following relationships:

$$r = \frac{a}{\sin \theta} = a \csc \theta. \quad (2)$$

Since $\tan \theta = -\frac{a}{x}$ from the right triangle in Figure 22.52a, we have $x = -a \cot \theta$, so

$$dx = a \csc^2 \theta d\theta. \quad (3)$$

Substitution of (2) and (3) into (1) gives

$$dB = \frac{\mu_0 I}{4\pi a} \sin \theta d\theta. \quad (4)$$

Thus, we have reduced the expression to one involving only the variable θ . We can now obtain the total field at P by integrating (4) over all elements that subtend angles ranging from θ_1 to θ_2 , as defined in Figure 22.52. This gives

$$B = \frac{\mu_0 I}{4\pi a} \int_{\theta_1}^{\theta_2} \sin \theta d\theta = \frac{\mu_0 I}{4\pi a} (\cos \theta_1 - \cos \theta_2).$$

We can apply this result to find the magnetic field of any straight wire—if we know the geometry and the hence the angles θ_1 and θ_2 .

(b) Consider the point P a distance a from the wire. Then for an infinite wire,

$$\theta_1 \rightarrow 0$$

and $\theta_2 \rightarrow \pi$ radians

$$\text{and} \quad B = \frac{\mu_0 I}{2\pi a}$$

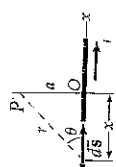


FIG. P22.52(a)

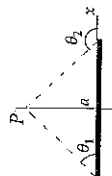


FIG. P22.52(b)

P22.53 $J_s = \frac{I}{\ell}$

From Ampère's Law,

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I$$

$$B(2\ell) = \mu_0 J_s \ell \quad \therefore \quad B = \frac{\mu_0 J_s}{2}$$

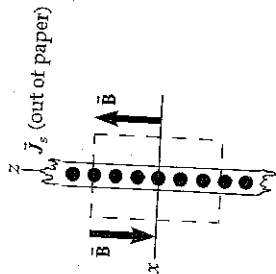


FIG. P22.53

- P22.54 (a) The boundary between a region of strong magnetic field and a region of zero field cannot be perfectly sharp, but we ignore the thickness of the transition zone. In the field the electron moves on an arc of a circle:

$$\sum \vec{r} = ma$$

$$|q|vB \sin 90^\circ = \frac{mv^2}{r}$$

$$\frac{v}{r} = \omega = \frac{|q|B}{m} = \frac{(1.60 \times 10^{-19} \text{ C})(10^{-3} \text{ N} \cdot \text{s/C} \cdot \text{m})}{(9.11 \times 10^{-31} \text{ kg})} = 1.76 \times 10^6 \text{ rad/s}$$

The time for one half revolution is,

$$\Delta t = \frac{\Delta \theta}{\omega} = \frac{\pi \text{ rad}}{1.76 \times 10^6 \text{ rad/s}} = 1.79 \times 10^{-6} \text{ s}$$

- (b) The maximum depth of penetration is the radius of the path.

$$\text{Then } v = \omega r = (1.76 \times 10^6 \text{ s}^{-1})(0.02 \text{ m}) = 3.51 \times 10^6 \text{ m/s}$$

and

$$K = \frac{1}{2}mv^2 = \frac{1}{2}(9.11 \times 10^{-31} \text{ kg})(3.51 \times 10^6 \text{ m/s})^2 = 5.62 \times 10^{-18} \text{ J} = \frac{5.62 \times 10^{-18} \text{ J}}{1.60 \times 10^{-19} \text{ C}} = 35.1 \text{ eV}$$

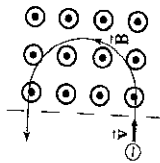


FIG. P22.54(a)

- P22.55 (a) Define vector $\vec{h}$ to have the downward direction of the current, and vector $\vec{L}$ to be along the pipe into the page as shown. The electric current experiences a magnetic force

$$I(\vec{h} \times \vec{B}) \text{ in the direction of } \vec{L}.$$

- (b) The sodium, consisting of ions and electrons, flows along the pipe transporting no net charge. But inside the section of length L , electrons drift upward to constitute downward electric current $J \times (\text{area}) = J L w$.

$$\text{The current then feels a magnetic force } I(\vec{h} \times \vec{B}) = J L w h B \sin 90^\circ.$$

This force along the pipe axis will make the fluid move, exerting pressure

$$\frac{F}{\text{area}} = \frac{J L w h B}{L w} = \frac{J L B}{L}.$$

- (c) Charge moves within the fluid inside the length L , but charge does not accumulate: the fluid is not charged after it leaves the pump. It is not current-carrying and it is not magnetized.

$$\sum F_y = 0: \quad +n - mg = 0$$

$$\sum F_x = 0: \quad -\mu_k n + IB L \sin 90.0^\circ = 0$$

$$B = \frac{\mu_k mg}{I L} = \frac{0.100(0.200 \text{ kg})(9.80 \text{ m/s}^2)}{(10.0 \text{ A})(0.500 \text{ m})} = 39.2 \text{ mT}$$

- P22.57 (a) The net force is the Lorentz force given by

$$\vec{F} = q\vec{E} + q\vec{v} \times \vec{B} = q(\vec{E} + \vec{v} \times \vec{B})$$

$$\vec{F} = (3.20 \times 10^{-19} \text{ C})[(4\hat{i} - 1\hat{j} - 2\hat{k}) + (2\hat{i} + 3\hat{j} - 1\hat{k}) \times (2\hat{i} + 4\hat{j} + 1\hat{k})] \text{ N}$$

Carrying out the indicated operations, we find:

$$\vec{F} = (3.52\hat{i} - 1.60\hat{j}) \times 10^{-18} \text{ N}$$

$$(b) \quad \theta = \cos^{-1} \left(\frac{F_x}{F} \right) = \cos^{-1} \left(\frac{3.52}{\sqrt{(3.52)^2 + (1.60)^2}} \right) = 24.4^\circ$$

- P22.58 The magnetic force on each proton, $\vec{F}_B = q\vec{v} \times \vec{B} = qvB \sin 90^\circ$ downward perpendicular to velocity, causes centripetal acceleration, guiding it into a circular path of radius r , with

$$qvB = \frac{mv^2}{r}$$

$$\text{and} \quad r = \frac{mv}{qB}$$

We compute this radius by first finding the proton's speed:

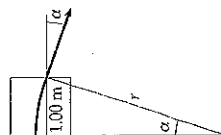


FIG. P22.58

continued on next page

$$K = \frac{1}{2}mv^2$$

$$v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2(5.00 \times 10^6 \text{ eV})(1.60 \times 10^{-19} \text{ J/eV})}{1.67 \times 10^{-27} \text{ kg}}} = 3.10 \times 10^7 \text{ m/s.}$$

$$\text{Now, } r = \frac{mv}{qB} = \frac{(1.67 \times 10^{-27} \text{ kg})(3.10 \times 10^7 \text{ m/s})}{(1.60 \times 10^{-19} \text{ C})(0.0500 \text{ N/C} \cdot \text{m})} = 6.46 \text{ m.}$$

(b) From the figure, observe that

$$\sin \alpha = \frac{100 \text{ m}}{r} = \frac{1 \text{ m}}{6.46 \text{ m}}$$

$$\alpha = 8.90^\circ$$

(a) The magnitude of the proton momentum stays constant, and its final y component is

$$-(1.67 \times 10^{-27} \text{ kg})(3.10 \times 10^7 \text{ m/s}) \sin 8.90^\circ = -8.00 \times 10^{-21} \text{ kg} \cdot \text{m/s}$$

P22.59

Suppose the input power is

$$120 \text{ W} = (120 \text{ V})I;$$

$$I \sim 1 \text{ A} = 10^0 \text{ A.}$$

Suppose

$$\omega = 2000 \text{ rev/min} \left(\frac{1 \text{ min}}{60 \text{ s}} \right) \left(\frac{2\pi \text{ rad}}{1 \text{ rev}} \right) \sim 200 \text{ rad/s}$$

and the output power is

$$20 \text{ W} = \tau\omega = \tau(200 \text{ rad/s}) \quad \tau \sim 10^{-1} \text{ N} \cdot \text{m.}$$

Suppose the area is about

$$(3 \text{ cm}) \times (4 \text{ cm}), \quad \text{or} \quad A \sim 10^{-3} \text{ m}^2$$

Suppose that the field is

$$B \sim 10^{-1} \text{ T.}$$

Then, the number of turns in the coil may be found from $\tau \approx NIAB$:

$$0.1 \text{ N} \cdot \text{m} \sim N(1 \text{ C/s})(10^{-3} \text{ m}^2)(10^{-1} \text{ N/C} \cdot \text{m})$$

giving

$$N \sim 10^3.$$

P22.60

$$\sum \vec{F} = m\vec{a} \text{ or } qvB \sin 90.0^\circ = \frac{mv^2}{r}$$

$\therefore$ the angular frequency for each ion is $\frac{v}{r} = \omega = \frac{qB}{m}$ and

$$\Delta\omega = \omega_{12} - \omega_{14} = qB \left(\frac{1}{m_{12}} - \frac{1}{m_{14}} \right) = \frac{(1.60 \times 10^{-19} \text{ C})(2.40 \text{ T})}{(1.66 \times 10^{-27} \text{ kg/u})} \left(\frac{1}{12.0 \text{ u}} - \frac{1}{14.0 \text{ u}} \right)$$

$$\Delta\omega = \omega_{12} - \omega_{14} = 2.75 \times 10^6 \text{ s}^{-1} = 2.75 \text{ Mrad/s}$$

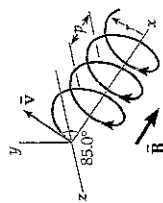


FIG. P22.61

Let v_x and $v_\perp$ be the components of the velocity of the positron parallel to and perpendicular to the direction of the magnetic field.

(a) The pitch of trajectory is the distance moved along x by the positron during each period, T .

$$p = v_x T = (v \cos 85.0^\circ) \left(\frac{2\pi m}{Bq} \right)$$

$$p = \frac{(5.00 \times 10^6)(\cos 85.0^\circ)(2\pi)(9.11 \times 10^{-31})}{(0.150)(1.60 \times 10^{-19})} = 1.04 \times 10^{-4} \text{ m}$$

(b) From Equation 22.3,

$$r = \frac{mv_\perp}{Bq} = \frac{mv \sin 85.0^\circ}{Bq}$$

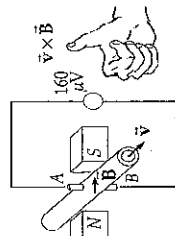
$$r = \frac{(9.11 \times 10^{-31})(5.00 \times 10^6)(\sin 85.0^\circ)}{(0.150)(1.60 \times 10^{-19})} = 1.89 \times 10^{-4} \text{ m}$$

(a) The magnetic force acting on ions in the blood stream will deflect positive charges toward point A and negative charges toward point B. This separation of charges produces an electric field directed from A toward B. At equilibrium, the electric force caused by this field must balance the magnetic force, so

$$qvB = qE = q \left(\frac{\Delta V}{d} \right)$$

$$\text{or } v = \frac{\Delta V}{Bd} = \frac{(160 \times 10^{-6} \text{ V})}{(0.0400 \text{ T})(3.00 \times 10^{-3} \text{ m})} = 1.33 \text{ m/s.}$$

FIG. P22.62



(b) **No.** Negative ions moving in the direction of v would be deflected toward point B, giving A a higher potential than B. Positive ions moving in the direction of v would be deflected toward A, again giving A a higher potential than B. Therefore, the sign of the potential difference does not depend on whether the ions in the blood are positively or negatively charged.

P22.63

Consider a longitudinal filament of the strip of width dr as shown in the sketch. The contribution to the field at point P due to the current di in the element dr is

$$dB = \frac{\mu_0 di}{2\pi r}$$

$$\text{where } di = i \left(\frac{dr}{w} \right)$$

$$\vec{B} = \int_{b+w}^{b+w} \frac{\mu_0 i dr}{2\pi wr} \hat{k} = \frac{\mu_0 i}{2\pi w} \ln \left(1 + \frac{w}{b} \right) \hat{k}.$$

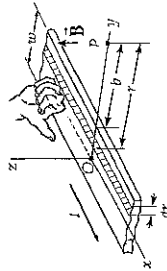


FIG. P22.63

P22.64 $B = \frac{\mu_0 I R^2}{2(R^2 + R^2)^{3/2}} = \frac{\mu_0 I}{2^{5/2} R}$
 so $I = 2.01 \times 10^9 \text{ A}$ toward the west

P22.65 On the axis of a current loop, the magnetic field is given by

$$B = \frac{\mu_0 I R^2}{2(x^2 + R^2)^{3/2}}$$

where in this case $I = \frac{q}{2\pi(\omega)}$

when $x = \frac{R}{2}$

Therefore,

$$B = \frac{\mu_0 \omega R^2 q}{4\pi(x^2 + R^2)^{3/2}}$$

then

$$B = \frac{\mu_0 \omega R^2 q}{4\pi(\frac{5}{4}R^2)^{3/2}} = \frac{\mu_0 \omega q}{2.5\sqrt{5}\pi R}$$

P22.66 (a) Use Equation 22.33 twice: $B_x = \frac{\mu_0 I R^2}{2(x^2 + R^2)^{3/2}}$

If each coil has N turns, the field is just N times larger.

$$B = B_{x1} + B_{x2} = \frac{N\mu_0 I R^2}{2} \left[\frac{1}{(x^2 + R^2)^{3/2}} + \frac{1}{(R-x)^2 + R^2)^{3/2}} \right]$$

$$B = \frac{N\mu_0 I R^2}{2} \left[\frac{1}{(x^2 + R^2)^{3/2}} + \frac{1}{(2R^2 + x^2 - 2xR)^{3/2}} \right]$$

(b) $\frac{dB}{dx} = \frac{N\mu_0 I R^2}{2} \left[-\frac{3}{2}(2x)(x^2 + R^2)^{-5/2} - \frac{3}{2}(2R^2 + x^2 - 2xR)^{-5/2}(2x - 2R) \right]$

Substituting $x = \frac{R}{2}$ and canceling terms, $\frac{dB}{dx} = 0$.

$$\frac{d^2B}{dx^2} = \frac{-3N\mu_0 I R^2}{2} \left[(x^4 + R^2)^{-5/2} - 5x^2(x^2 + R^2)^{-7/2} + (2R^2 + x^2 - 2xR)^{-5/2} - 5(x-R)^2(2R^2 + x^2 - 2xR)^{-7/2} \right]$$

Again substituting $x = \frac{R}{2}$ and canceling terms, $\frac{d^2B}{dx^2} = 0$.

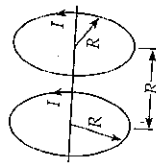


FIG. P22.66

P22.67 Model the two wires as straight parallel wires (!)

(a) $F_B = \frac{\mu_0 I^2 \ell}{2\pi a}$ (Equation 22.27)

$$F_B = \frac{(4\pi \times 10^{-7})(140)^2(2\pi)(0.100)}{2\pi(1.00 \times 10^{-3})}$$

$$= 2.46 \text{ N upward}$$

(b) $a_{\text{loop}} = \frac{2.46 \text{ N} - m_{\text{loop}}g}{m_{\text{loop}}} = \frac{107 \text{ m/s}^2}{\text{upward}}$

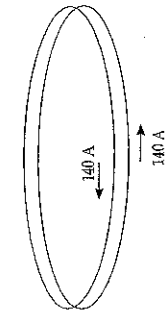


FIG. P22.67

P22.68 (a) $B = \frac{\mu_0 I}{2\pi r} = \frac{(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(24.0 \text{ A})}{2\pi(0.0175 \text{ m})} = 2.74 \times 10^{-4} \text{ T}$

(b) At point C, conductor AB produces a field $\frac{1}{2}(2.74 \times 10^{-4} \text{ T})(-\hat{j})$, conductor DE produces a field of $\frac{1}{2}(2.74 \times 10^{-4} \text{ T})(-\hat{j})$, and BD produces no field, and AE produces negligible field.

The total field at C is $2.74 \times 10^{-4} \text{ T}(-\hat{j})$.

(c) $\vec{F}_B = I\vec{L} \times \vec{B} = (24.0 \text{ A})(0.0350 \text{ m}\hat{k}) \times [5(2.74 \times 10^{-4} \text{ T})(-\hat{j})] = (1.15 \times 10^{-3} \text{ N})\hat{i}$

(d) $\vec{a} = \frac{\sum \vec{F}}{m} = \frac{(1.15 \times 10^{-3} \text{ N})\hat{i}}{3.0 \times 10^{-3} \text{ kg}} = (0.384 \text{ m/s}^2)\hat{i}$

(e) The bar is already so far from AE that it moves through nearly constant magnetic field. The force acting on the bar is constant, and therefore the bar's acceleration is constant.

(f) $v_f^2 = v_i^2 + 2ax = 0 + 2(0.384 \text{ m/s}^2)(1.30 \text{ m})$, so $\vec{v}_f = (0.999 \text{ m/s})\hat{i}$

ANSWERS TO EVEN PROBLEMS

P22.2 (a) west; (b) no deflection; (c) up; (d) down

P22.10 244 kV/m

P22.4 (a) 7.90 pN; (b) 0

P22.12 (a) $7.66 \times 10^7 \text{ rad/s}$; (b) 26.8 Mm/s;

(c) 3.76 MeV; (d) $3.13 \times 10^3 \text{ rev}$; (e) 257 μs

P22.6 2.34 aN

P22.14 (a) see the solution; (b) $n = \frac{IB}{q\Delta V_H}$

P22.8 (a) 5.00 cm; (b) $8.78 \times 10^6 \text{ m/s}$