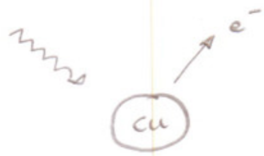
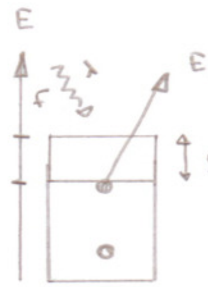
28.13

13. **Review problem.** An isolated copper sphere of radius 5.00 cm, initially uncharged, is illuminated by ultraviolet light of wavelength 200 nm. What charge will the photoelectric effect induce on the sphere? The work function for copper is 4.70 eV.

Lösning:



$\lambda = 200 \cdot 10^{-9}$ m



maximalt värde

$E_{kin} = E_{foton} - \phi$

Cu: $\phi = 4{,}70$ eV

För den första elektronen gäller:

Maximal kinetisk energi

$$E_{max} = hf - \phi$$

$c = \lambda \cdot f$

$\Rightarrow f = \frac{c}{\lambda}$

i Joule: $E_{max} = \frac{hc}{\lambda} - \phi \cdot e$

i eV: $E'_{max} = \frac{hc}{\lambda e} - \phi = \frac{6{,}626 \cdot 10^{-34} \cdot 3 \cdot 10^{8}}{200 \cdot 10^{-9} \cdot 1{,}6 \cdot 10^{-19}} - 4{,}70 =$

$= \underline{\underline{1{,}51 \text{ eV}}}$

När kopparsfären laddas upp pga elektronemission kommer dess potential att ökas successivt. Det innebär att den potentiella energin för de negativt laddade elektronerna att minska successivt. Fältet utanför sfären blir allt starkare (i en given punkt).

Potential $V = \frac{k_e Q}{r}$

Potential ($V_\infty = 0$) vid sfärens yta $V(R) = \frac{k_e Q}{R}$

När $V(R) = 1{,}51$ V kommer emissionen att upphöra. Elektronerna vänder på vägen mot ∞.

$\therefore \frac{k_e Q}{R} = 1{,}51 \quad \Rightarrow \quad Q = \frac{1{,}51 \cdot 5{,}00 \cdot 10^{-2}}{8{,}99 \cdot 10^{9}}$ C $=$

$= \underline{\underline{8{,}41 \cdot 10^{-12} \text{ C}}}$

28.37

37. A free electron has a wave function

$$\psi(x) = Ae^{i(5.00 \times 10^{10}x)}$$

where x is in meters. Find (a) its de Broglie wavelength, (b) its momentum, and (c) its kinetic energy in electron volts.

Lösning:

λ :

$$\psi = A \cdot e^{ikx} \qquad k = \frac{2\pi}{\lambda}$$

här $k = 5{,}00 \cdot 10^{10}$ m

$$\Rightarrow \lambda = \frac{2\pi}{k} = \frac{2\pi}{5{,}00 \cdot 10^{10}} = \underline{\underline{1{,}26 \cdot 10^{-10}\ \text{m}}}$$

---

p :

$$p = \hbar k = \frac{6{,}626 \cdot 10^{-34}}{2\pi} \cdot 5{,}00 \cdot 10^{10} = \underline{\underline{5{,}27 \cdot 10^{-24}}}\ \text{kgm/s}$$

K :

$$K = \frac{1}{2} mv^2 = \frac{1}{2} \frac{(mv)^2}{m} = \frac{1}{2} \frac{p^2}{m} =$$

$$= \frac{1}{2} \frac{(5{,}27 \cdot 10^{-24})^2}{9{,}11 \cdot 10^{-31}}\ \text{J} = 1{,}52 \cdot 10^{-17}\ \text{J} =$$

$$= \frac{1{,}52 \cdot 10^{-17}}{1{,}60 \cdot 10^{-19}}\ \text{eV} = \underline{\underline{95{,}5\ \text{eV}}}$$

Obs!

Delräkningarna i lösn. ovan har gjorts för att illustrera de num. värdena.

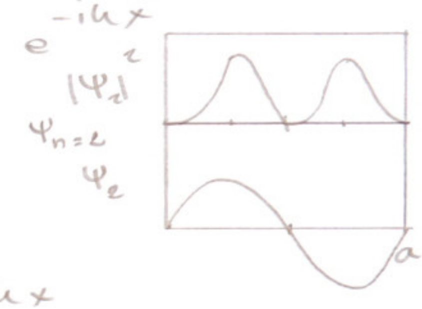
28.41

41. A photon with wavelength λ is absorbed by an electron confined to a box. As a result, the electron moves from state $n = 1$ to $n = 4$. (a) Find the length of the box. (b) What is the wavelength of the photon emitted in the transition of that electron from the state $n = 4$ to the state $n = 2$?

Lösning: Partikel i låda, $V(x) =$ konstant

$\Rightarrow$ Schr. ekv. $-\dfrac{\hbar^2}{2m}\dfrac{\partial^2\psi}{\partial x^2} + 0 = E\psi \Rightarrow -\dfrac{\hbar^2}{2mE}\dfrac{\partial^2\psi}{\partial x^2} + \psi = 0$

$\Rightarrow \psi(x) = A \cdot e^{ikx} + B e^{-ikx}$



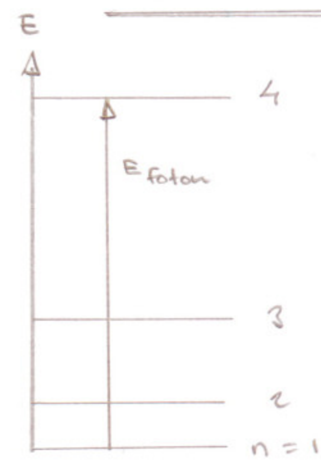
där $k = \dfrac{\sqrt{2mE}}{\hbar} \Rightarrow E = \dfrac{\hbar^2}{2m}k^2$

Randvillkor $\psi(x) = 0 \Rightarrow \psi(x) = C \cdot \sin kx$

$\psi(x=a) \Rightarrow k = n \cdot \dfrac{\pi}{a}$ $n =$ heltal

$\therefore \psi(x) = C \cdot \sin\left[\left(n\dfrac{\pi}{a}\right) \cdot x\right]$

$$E_n = \frac{\hbar^2}{2m}\left(n\frac{\pi}{a}\right)^2 = \frac{h^2\pi^2}{(2\pi)^2\, 2m a^2} n^2 = \underline{\underline{\frac{h^2}{8ma^2} n^2}}$$



$E_{abs} = E_{foton} = E_4 - E_1 =$

$= (4^2 - 1^2)\dfrac{h^2}{8ma^2} = \dfrac{hc}{\lambda}$

$\Rightarrow a = \left(\dfrac{15\lambda}{8mc}\right)^{1/2}$

$E_4 - E_2 = (4^2 - 2^2)\dfrac{h^2}{8ma^2} = \dfrac{hc}{\lambda'}$

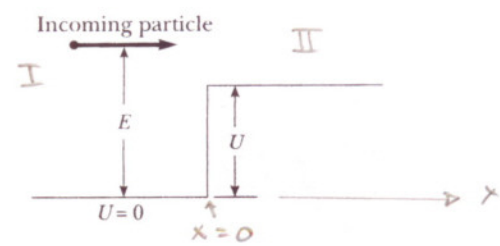
$\Rightarrow 12\dfrac{h^2}{8ma^2} = \dfrac{hc}{\lambda'}$ $\therefore \dfrac{\lambda'}{\lambda} = \dfrac{15}{12} = \underline{\underline{1{,}25}}$

28.62

62. Particles incident from the left are confronted with a step in potential energy shown in Figure P28.62. Located at $x = 0$, the step has a height U. The particles have energy $E > U$. Classically, we would expect all the particles to continue on, although with reduced speed. According to quantum mechanics, a fraction of the particles are reflected at the barrier. (a) Prove that the reflection coefficient R for this case is

$$R = \frac{(k_1 - k_2)^2}{(k_1 + k_2)^2}$$

where $k_1 = 2\pi/\lambda_1$ and $k_2 = 2\pi/\lambda_2$ are the wave numbers for the incident and transmitted particles. Proceed as follows. Show that the wave function $\psi_1 = Ae^{ik_1x} + Be^{-ik_1x}$ satisfies the Schrödinger equation in region 1, for $x < 0$. Here Ae^{ik_1x} represents the incident beam and Be^{-ik_1x} represents the reflected particles. Show that $\psi_2 = Ce^{ik_2x}$ satisfies the Schrödinger equation in region 2, for $x > 0$. Impose the boundary conditions $\psi_1 = \psi_2$ and $d\psi_1/dx = d\psi_2/dx$ at $x = 0$ to find the relationship between B and A. Then evaluate $R = B^2/A^2$. (b) A particle that has kinetic energy $E = 7.00$ eV is incident from a region where the potential energy is zero onto one in which $U = 5.00$ eV. Find its probability of being reflected and its probability of being transmitted.



Lösning:

$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + 0 = E\psi \quad (\mathrm{I})$$

$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + U\psi = E\psi \quad \mathrm{II}$$

$$\mathrm{I}: \quad \psi_{\mathrm{I}} = A\cdot e^{ik_1x} + Be^{-ik_1x} \qquad k_1 = \frac{\sqrt{2mE}}{\hbar}$$

$$\mathrm{II}: \quad \psi_{\mathrm{II}} = C\cdot e^{ik_2x} \qquad k_2 = \frac{\sqrt{2m(E-U)}}{\hbar}$$

$E = 7{,}00$ eV

$U = 5{,}00$ eV

villkor:

$$\psi_{\mathrm{I}}(x=0) = \psi_{\mathrm{II}}(x=0) \Rightarrow A + B = C$$

$$\left(\frac{\partial\psi_{\mathrm{I}}}{\partial x}\right)_{x=0} = \left(\frac{\partial\psi_{\mathrm{II}}}{\partial x}\right)_{x=0} \Rightarrow k_1(A-B) = k_2 C$$

$$\Rightarrow k_1(A-B) = k_2(A+B)$$

$$\Rightarrow (k_1 - k_2)A = (k_1 + k_2)B$$

$$\Rightarrow R = \left(\frac{B}{A}\right)^2 = \left(\frac{k_1 - k_2}{k_1 + k_2}\right)^2 = \left(\frac{1 - \frac{k_2}{k_1}}{1 + \frac{k_2}{k_1}}\right)^2$$

0,535

$$\frac{k_2}{k_1} = \sqrt{\frac{E-U}{E}} = \sqrt{\frac{2}{7}} \quad \Rightarrow R = \left(\frac{1-\sqrt{\frac{2}{7}}}{1+\sqrt{\frac{2}{7}}}\right)^2 = \underline{\underline{0{,}092}}$$

$$T = 1 - R = 1 - 0{,}092 = \underline{\underline{0{,}908}}$$

29.11

11. The ground-state wave function for a hydrogen atom is

$$\psi_{1s}(r) = \frac{1}{\sqrt{\pi a_0^3}} e^{-r/a_0}$$

where r is the radial coordinate of the electron and a_0 is the Bohr radius. (a) Show that the wave function as given is normalized. (b) Find the probability of locating the electron between $r_1 = a_0/2$ and $r_2 = 3a_0/2$.

Lösning:

a) normalisering: $\int_0^\infty |\psi|^2 4\pi r^2 \, dr = 1$

Beräkna: $\int_0^\infty e^{-2r/a_0} \cdot r^2 = \overset{A}{\left[r^2\left(-\frac{a_0}{2}\right)e^{-2r/a_0}\right]_0^\infty} -$

$-\int\left(-\frac{a_0}{2}\right)e^{-2r/a_0}\cdot 2r\,dr = A - \overset{B}{\left[\left(\frac{a_0}{2}\right)^2 e^{-2r/a_0}\cdot 2r\right]_0^\infty} +$

$+\int\left(\frac{a_0}{2}\right)^2 e^{-2r/a_0}\cdot 2\,dr =$

$= A - B - \left[2\left(\frac{a_0}{2}\right)^3 e^{-2r/a_0}\right]_0^\infty = 0 + 0 + \frac{a_0^3}{4}$

$\Rightarrow \frac{1}{\cancel{\pi} a_0^3}\cdot 4\cancel{\pi}\cdot\frac{a_0^3}{4} = 1$ v.s.v.

Sannolikheten att finna elektronen i $\left[\frac{a_0}{2}, 3\frac{a_0}{2}\right]$:

$\int_{a_0/2}^{3a_0/2} e^{-2r/a_0}\cdot r^2 = \left[r^2\left(-\frac{a_0}{2}\right)e^{-2r/a_0}\right]_{\frac{a_0}{2}}^{\frac{3a_0}{2}} - \left[\left(\frac{a_0}{2}\right)^2 e^{-2r/a_0}\cdot 2r\right]_{\frac{a_0}{2}}^{\frac{3a_0}{2}} -$

$-\left[\frac{a_0^3}{4}e^{-2r/a_0}\right]_0^\infty =$

$\boxed{\text{Svar: } 0{,}497}$

$= \frac{9a_0^2}{4}\left(-\frac{a_0}{2}\right)e^{-3a_0/a_0} - \left(\frac{a_0^2}{4}\right)\left(-\frac{a_0}{2}\right)e^{-1} - \left(\frac{a_0}{2}\right)^2 e^{-3}\cdot 3a_0 +$

$+\left(\frac{a_0}{2}\right)^2 e^{-1}\cdot a_0 - \frac{a_0^3}{4}e^{-3} + \frac{a_0^3}{4}e^{-1} = a_0^3\frac{5}{8}e^{-1} - a_0^3\frac{17}{8}e^{-3}$

$\frac{1}{\pi a_0^3}\cdot 4\pi\cdot\left(a_0^3\frac{5}{8}e^{-1} - a_0^3\frac{17}{8}e^{-3}\right) = \frac{20}{8}e^{-1} - \frac{68}{8}e^{-3} = 0{,}9197 - 0{,}4231 =$

89.47

47. *An example of the correspondence principle.* Use Bohr's model of the hydrogen atom to show that when the electron moves from the state n to the state $n-1$, the frequency of the emitted light is

$$f = \left(\frac{2\pi^2 m_e k_e^2 e^4}{h^3 n^2}\right)\frac{2n-1}{(n-1)^2}$$

Show that as $n \to \infty$, this expression varies as $1/n^3$ and reduces to the classical frequency one expects the atom to emit. (*Suggestion:* To calculate the classical frequency, note that the frequency of revolution is $v/2\pi r$, where r is given by Eq. 11.22.)

Lösning:

$$\frac{2n-1}{n^2(n-1)^2} \xrightarrow[n\to\infty]{} \frac{2n}{n^4} = \frac{2}{n^3} \sim \frac{1}{n^3}$$

Klassisk modell for våre atome

$$E \approx \frac{1}{2} m_e v^2 + E_{pot} = \frac{1}{2} m_e v^2 - \frac{e^2}{4\pi\epsilon_0 r}$$

$$\frac{mv^2}{r} = \frac{e^2}{4\pi\epsilon_0 r^2} \Rightarrow \frac{1}{2} mv^2 = \frac{1}{2}\frac{e^2}{4\pi\epsilon_0 r^2}\cdot r = \frac{e^2}{8\pi\epsilon_0 r}$$

Bohrs postulat: $2\pi mvr = nh \Rightarrow r = n\frac{h}{2\pi mv}$

$$\frac{1}{2} mv^2 = \frac{e^2}{8\pi\epsilon_0 r} \Rightarrow (mv)^2 = \frac{e^2}{4\pi\epsilon_0 m\cdot r}$$

samt $v^2 = \frac{e^2}{4\pi\epsilon_0 m^2 r}$

$$r = n\frac{h}{2\pi mv} \Rightarrow r^2 = n^2\frac{h^2}{4\pi^2 (mv)^2} = n^2\frac{m^2 h^2\cdot 4\pi\epsilon_0}{4\pi^2 e^2}\,r$$

$$\Rightarrow \boxed{r = n^2\frac{h^2\epsilon_0}{\pi m e^2}}$$

$$f = \frac{v}{2\pi r} = \frac{e}{\sqrt{4\pi\epsilon_0}\cdot m\, r^{1/2}\cdot 2\pi r} = \frac{1}{2\pi}\frac{e^2}{\sqrt{4\pi\epsilon_0 m}}\frac{1}{r^{3/2}}$$

$$\Rightarrow f = \frac{2\pi^2 m e^4}{4\pi\epsilon_0 h^3}\,\frac{2}{n^3}$$