$$"Q" = mc\Delta T = \left(6\times10^{-15}\ \text{kg}\right)\left(\frac{300\ \text{J}}{\text{kg}\cdot{}^\circ\text{C}}\right)\left(600\ \text{K} - 300\ \text{K}\right)\left(\frac{1^\circ\text{C}}{1\ \text{K}}\right)$$

$$"Q" \approx 6\times10^{-10}\ \text{J}$$

The pit moves past in a time interval of $\Delta t = \dfrac{2\times10^{-6}\ \text{m}}{1\ \text{m/s}} = 2\times10^{-6}\ \text{s}$.

So, the intensity is $I = \dfrac{\mathcal{P}}{A} = \dfrac{"Q"}{A\Delta t}$ $\quad I \approx \dfrac{6\times10^{-10}\ \text{J}}{\left[\pi\left(10^{-6}\ \text{m}\right)^2\right]\left(2\times10^{-6}\ \text{s}\right)} = 9\times10^{7}\ \text{W/m}^2$

$$I\ \boxed{\sim10^{8}\ \text{W/m}^2}$$

ANSWERS TO EVEN CONTEXT 8 CONCLUSION PROBLEMS

CC8.2 74.2 grooves/mm

CC8.4 48 059

Chapter 28 Quantum Physics

CHAPTER OUTLINE

ANSWERS TO QUESTIONS

Q28.1 Planck made two new assumptions: (1) molecular energy is quantized and (2) molecules emit or absorb energy in discrete irreducible packets. These assumptions contradict the classical idea of energy as continuously divisible. They also imply that an atom must have a definite structure—it cannot just be a soup of electrons orbiting the nucleus.

Q28.2 (c) UV light has the highest frequency of the three, and hence each photon delivers more energy to a skin cell. This explains why you can become sunburned on a cloudy day: clouds block visible light and infrared, but not much ultraviolet. You usually do not become sunburned through window glass, even though you can see the visible light from the Sun coming through the window, because the glass absorbs much of the ultraviolet and reemits it as infrared.

Q28.3 No. The second metal may have a larger work function than the first, in which case the incident photons may not have enough energy to eject photoelectrons.

Q28.4 The Compton effect describes the *scattering* of photons from electrons, while the photoelectric effect predicts the ejection of electrons due to the *absorption* of photons by a material.

Q28.5 Wave theory predicts that the photoelectric effect should occur at any frequency, provided the light intensity is high enough. However, as seen in the photoelectric experiments, the light must have a sufficiently high frequency for the effect to occur.

Q28.6 A few photons would only give a few dots of exposure, apparently randomly scattered.

Q28.7 The x-ray photon transfers some of its energy to the electron. Thus, its frequency must decrease.

Q28.8 Light has both classical-wave and classical-particle characteristics. In single- and double-slit experiments light behaves like a wave. In the photoelectric effect light behaves like a particle. Light may be characterized as an electromagnetic wave with a particular wavelength or frequency, yet at the same time light may be characterized as a stream of photons, each carrying a discrete energy, hf. Since light displays *both* wave and particle characteristics, perhaps it would be fair to call light a "wavicle". It is customary to call a photon a quantum particle, different from a classical particle.

Q28.9 An electron has both classical-wave and classical-particle characteristics. In single- and double-slit diffraction and interference experiments, electrons behave like classical waves. An electron has mass and charge. It carries kinetic energy and momentum in parcels of definite size, as classical particles do. At the same time it has a particular wavelength and frequency. Since an electron displays characteristics of both classical waves and classical particles, it is neither a classical wave nor a classical particle. It is customary to call it a quantum particle, but another invented term, such as "wavicle", could serve equally well.

Q28.10 The discovery of electron diffraction by Davisson and Germer was a fundamental advance in our understanding of the motion of material particles. Newton's laws fail to properly describe the motion of an object with small mass. It moves as a wave, not as a classical particle. Proceeding from this recognition, the development of quantum mechanics made possible describing the motion of electrons in atoms; understanding molecular structure and the behavior of matter at the atomic scale, including electronics, photonics, and engineered materials; accounting for the motion of nucleons in nuclei; and studying elementary particles.

Q28.11 Any object of macroscopic size—including a grain of dust—has an undetectably small wavelength and does not exhibit quantum behavior.

Q28.12 If we set $\frac{p^2}{2m} = q\Delta V$, which is the same for both particles, then we see that the electron has the smaller momentum and therefore the longer wavelength $\left(\lambda = \frac{h}{p}\right)$.

Q28.13 The *intensity* of electron waves in some small region of space determines the *probability* that an electron will be found in that region.

Q28.14 (a) The slot is blacker than any black material or pigment. Any radiation going in through the hole will be absorbed by the walls or the contents of the box, perhaps after several reflections. Essentially none of that energy will come out through the hole again. Figure 28.1 in the text shows this effect if you imagine the beam getting weaker at each reflection.

(b) The open slots between the glowing tubes are brightest. When you look into a slot, you receive direct radiation emitted by the wall on the far side of a cavity enclosed by the fixture; and you also receive radiation that was emitted by other sections of the cavity wall and has bounced around a few or many times before escaping through the slot. In Figure 28.1 in the text, reverse all of the arrowheads and imagine the beam getting stronger at each reflection. Then the figure shows the extra efficiency of a cavity radiator. Here is the conclusion of Kirchhoff's thermodynamic argument: ... energy radiated. A poor reflector—a good absorber—avoids rising in temperature by being an efficient emitter. Its emissivity is equal to its absorptivity: $e = a$. The slot in the box in part (a) of the question is a black body with reflectivity zero and absorptivity 1, so it must also be the most efficient possible radiator, to avoid rising in temperature above its surroundings in thermal equilibrium. Its emissivity in Stefan's law is $100\% = 1$, higher than perhaps 0.9 for black paper, 0.1 for light-colored paint, or 0.04 for shiny metal. Only in this way can the material objects underneath these different surfaces maintain equal temperatures after they come to thermal equilibrium and continue to exchange energy by electromagnetic radiation. By considering one blackbody facing another, Kirchhoff proved logically that the material forming the walls of the cavity made no difference to the radiation. By thinking about inserting color filters between two cavity radiators, he proved that the spectral distribution of blackbody radiation must be a universal function of wavelength, the same for all materials and depending only on the temperature. Blackbody radiation is a fundamental connection between the matter and the energy that physicists had previously studied separately.

Q28.15 The motion of the quantum particle does not consist of moving through successive points. The particle has no definite position. It can sometimes be found on one side of a node and sometimes on the other side, but never at the node itself. There is no contradiction here, for the quantum particle is moving as a wave. It is not a classical particle. In particular, the particle does not speed up to infinite speed to cross the node.

SOLUTIONS TO PROBLEMS

Section 28.1 Blackbody Radiation and Planck's Theory

***P28.1** This is an example of Stefan's law.

At the lower temperature, $\mathcal{P}_1 = \sigma A e T_1^4$.

At the higher, $\mathcal{P}_2 = \sigma A e T_2^4$.

Then $\frac{\mathcal{P}_2}{\mathcal{P}_1} = \left(\frac{T_2}{T_1}\right)^4 = \left(\frac{273+38.3}{273+37}\right)^4 = (1.004\,19)^4 = 1.016\,9$.

The percentage increase in power is $(1.016\,9 - 1) = 0.016\,9 = \boxed{1.69\%}$, four times larger than the percentage increase in temperature.

P28.2 (a) $\mathcal{P} = eA\sigma T^4$, so

$$T = \left(\frac{\mathcal{P}}{eA\sigma}\right)^{1/4} = \left[\frac{3.85\times10^{26}\ \text{W}}{1\left[4\pi\left(6.96\times10^8\ \text{m}\right)^2\right]\left(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4\right)}\right]^{1/4} = \boxed{5.78\times10^3\ \text{K}}$$

(b) $$\lambda_{\max} = \frac{2.898\times10^{-3}\ \text{m}\cdot\text{K}}{T} = \frac{2.898\times10^{-3}\ \text{m}\cdot\text{K}}{5.78\times10^3\ \text{K}} = 5.01\times10^{-7}\ \text{m} = \boxed{501\ \text{nm}}$$

***P28.3** The peak radiation occurs at approximately 560 nm wavelength. From Wien's displacement law,

$$T = \frac{0.289\,8\times10^{-2}\ \text{m}\cdot\text{K}}{\lambda_{\max}} = \frac{0.289\,8\times10^{-2}\ \text{m}\cdot\text{K}}{560\times10^{-9}\ \text{m}} \approx \boxed{5\,200\ \text{K}}.$$

Clearly, a firefly is not at this temperature, so $\boxed{\text{this is not blackbody radiation}}$

P28.4 (a) $$E = hf = \left(6.626\times10^{-34}\ \text{J}\cdot\text{s}\right)\left(620\times10^{12}\ \text{s}^{-1}\right)\left(\frac{1.00\ \text{eV}}{1.60\times10^{-19}\ \text{J}}\right) = \boxed{2.57\ \text{eV}}$$

(b) $$E = hf = \left(6.626\times10^{-34}\ \text{J}\cdot\text{s}\right)\left(3.10\times10^{9}\ \text{s}^{-1}\right)\left(\frac{1.00\ \text{eV}}{1.60\times10^{-19}\ \text{J}}\right) = \boxed{1.28\times10^{-5}\ \text{eV}}$$

(c) $$E = hf = \left(6.626\times10^{-34}\ \text{J}\cdot\text{s}\right)\left(46.0\times10^{6}\ \text{s}^{-1}\right)\left(\frac{1.00\ \text{eV}}{1.60\times10^{-19}\ \text{J}}\right) = \boxed{1.91\times10^{-7}\ \text{eV}}$$

continued on next page

(d) $\lambda = \frac{c}{f} = \frac{3.00\times10^8 \text{ m/s}}{620\times10^{12} \text{ Hz}} = 4.84\times10^{-7} \text{ m} = \boxed{484 \text{ nm, visible light (blue)}}$

$\lambda = \frac{c}{f} = \frac{3.00\times10^8 \text{ m/s}}{3.10\times10^9 \text{ Hz}} = 9.68\times10^{-2} \text{ m} = \boxed{9.68 \text{ cm, radio wave}}$

$\lambda = \frac{c}{f} = \frac{3.00\times10^8 \text{ m/s}}{46.0\times10^6 \text{ Hz}} = \boxed{6.52 \text{ m, radio wave}}$

P28.5 Each photon has an energy $E = hf = (6.626\times10^{-34})(99.7\times10^6) = 6.61\times10^{-26}$ J.

This implies that there are $\frac{150\times10^3 \text{ J/s}}{6.61\times10^{-26} \text{ J/photon}} = \boxed{2.27\times10^{30} \text{ photons/s}}$.

P28.6 Energy of a single 500-nm photon:

$$E_\gamma = hf = \frac{hc}{\lambda} = \frac{(6.626\times10^{-34} \text{ J}\cdot\text{s})(3.00\times10^8 \text{ m/s})}{(500\times10^{-9} \text{ m})} = 3.98\times10^{-19} \text{ J}.$$

The energy entering the eye each second

$$E = \mathcal{P}\Delta t = IA\Delta t = (4.00\times10^{-11} \text{ W/m}^2)\left[\frac{\pi}{4}(8.50\times10^{-3} \text{ m})^2\right](1.00 \text{ s}) = 2.27\times10^{-15} \text{ J}.$$

The number of photons required to yield this energy

$$n = \frac{E}{E_\gamma} = \frac{2.27\times10^{-15} \text{ J}}{3.98\times10^{-19} \text{ J/photon}} = \boxed{5.71\times10^3 \text{ photons}}$$

P28.7 We take $\theta = 0.030\,0$ radians. Then the pendulum's total energy is

$E = mgh = mg(L - L\cos\theta)$

$E = (1.00 \text{ kg})(9.80 \text{ m/s}^2)(1.00 - 0.999\,5) = 4.41\times10^{-3}$ J

The frequency of oscillation is $f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{g}{L}} = 0.498$ Hz.

The energy is quantized, $E = nhf$.

Therefore, $n = \frac{E}{hf} = \frac{4.41\times10^{-3} \text{ J}}{(6.626\times10^{-34} \text{ J}\cdot\text{s})(0.498 \text{ s}^{-1})}$

$= \boxed{1.34\times10^{31}}$

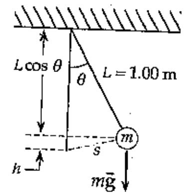

FIG. P28.7

***P28.8** (a) $V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(0.02 \text{ m})^3 = 3.35\times10^{-5} \text{ m}^3$

$m = \rho V = 7.86\times10^3 \text{ kg/m}^3 \; 3.35\times10^{-5} \text{ m}^3 = \boxed{0.263 \text{ kg}}$

continued on next page

(b) $A = 4\pi r^2 = 4\pi(0.02 \text{ m})^2 = 5.03\times10^{-3} \text{ m}^2$

$\mathcal{P} = \sigma AeT^4 = 5.67\times10^{-8} \text{ W/m}^2\text{K}^4 \; 5.03\times10^{-3} \text{ m}^2 \; 0.86(293 \text{ K})^4 = \boxed{1.81 \text{ W}}$

(c) It emits but does not absorb radiation, so its temperature must drop according to

$$Q = mc\Delta T = mc(T_f - T_i) \qquad \frac{dQ}{dt} = mc\frac{dT_f}{dt}$$

$$\frac{dT_f}{dt} = \frac{dQ/dt}{mc} = \frac{-\mathcal{P}}{mc} = \frac{-1.81 \text{ J/s}}{0.263 \text{ kg } 448 \text{ J/kg}\cdot\text{C}^\circ} = \boxed{-0.015\,3\ ^\circ\text{C/s}} = -0.919\ ^\circ\text{C/min}$$

(d) $\lambda_{max}T = 2.898\times10^{-3} \text{ m}\cdot\text{K}$

$\lambda_{max} = \frac{2.898\times10^{-3} \text{ m}\cdot\text{K}}{293 \text{ K}} = \boxed{9.89\times10^{-6} \text{ m}}$ infrared

(e) $E = hf = \frac{hc}{\lambda} = \frac{6.63\times10^{-34} \text{ Js } 3\times10^8 \text{ m/s}}{9.89\times10^{-6} \text{ m}} = \boxed{2.01\times10^{-20} \text{ J}}$

(f) The energy output each second is carried by photons according to

$$\mathcal{P} = \left(\frac{N}{\Delta t}\right)E$$

$$\frac{N}{\Delta t} = \frac{\mathcal{P}}{E} = \frac{1.81 \text{ J/s}}{2.01\times10^{-20} \text{ J/photon}} = \boxed{8.98\times10^{19} \text{ photon/s}}$$

Matter is coupled to radiation, quite strongly, in terms of photon numbers.

Section 28.2 The Photoelectric Effect

P28.9 (a) $\lambda_c = \frac{hc}{\phi} = \frac{(6.626\times10^{-34} \text{ J}\cdot\text{s})(3.00\times10^8 \text{ m/s})}{(4.20 \text{ eV})(1.60\times10^{-19} \text{ J/eV})} = \boxed{296 \text{ nm}}$

$f_c = \frac{c}{\lambda_c} = \frac{3.00\times10^8 \text{ m/s}}{296\times10^{-9} \text{ m}} = \boxed{1.01\times10^{15} \text{ Hz}}$

(b) $\frac{hc}{\lambda} = \phi + e\Delta V_S$: $\frac{(6.626\times10^{-34})(3.00\times10^8)}{180\times10^{-9}} = (4.20 \text{ eV})(1.60\times10^{-19} \text{ J/eV}) + (1.60\times10^{-19})\Delta V_S$

Therefore, $\boxed{\Delta V_S = 2.71 \text{ V}}$

P28.10 $K_{max} = \frac{1}{2}mv_{max}^2 = \frac{1}{2}(9.11\times10^{-31})(4.60\times10^5)^2 = 9.64\times10^{-20}\text{ J} = 0.602\text{ eV}$

(a) $\phi = E - K_{max} = \frac{1\,240\text{ eV}\cdot\text{nm}}{625\text{ nm}} - 0.602\text{ nm} = \boxed{1.38\text{ eV}}$

(b) $f_c = \frac{\phi}{h} = \frac{1.38\text{ eV}}{6.626\times10^{-34}\text{ J}\cdot\text{s}}\left(\frac{1.60\times10^{-19}\text{ J}}{1\text{ eV}}\right) = \boxed{3.34\times10^{14}\text{ Hz}}$

P28.11 (a) $e\Delta V_S = \frac{hc}{\lambda} - \phi \to \phi = \frac{1\,240\text{ nm}\cdot\text{eV}}{546.1\text{ nm}} - 0.376\text{ eV} = \boxed{1.90\text{ eV}}$

(b) $e\Delta V_S = \frac{hc}{\lambda} - \phi = \frac{1\,240\text{ nm}\cdot\text{eV}}{587.5\text{ nm}} - 1.90\text{ eV} \to \boxed{\Delta V_S = 0.216\text{ V}}$

P28.12 The energy needed is $E = 1.00\text{ eV} = 1.60\times10^{-19}\text{ J}$.

The energy absorbed in time interval Δt is $E = \mathcal{P}\Delta t = IA\Delta t$

so $\Delta t = \frac{E}{IA} = \frac{1.60\times10^{-19}\text{ J}}{(500\text{ J/s}\cdot\text{m}^2)\left[\pi(2.82\times10^{-15}\text{ m})^2\right]} = 1.28\times10^7\text{ s} = \boxed{148\text{ days}}$.

The gross failure of the classical theory of the photoelectric effect contrasts with the success of quantum mechanics.

P28.13 Ultraviolet photons will be absorbed to knock electrons out of the sphere with maximum kinetic energy $K_{max} = hf - \phi$,

or $K_{max} = \frac{(6.626\times10^{-34}\text{ J}\cdot\text{s})(3.00\times10^8\text{ m/s})}{200\times10^{-9}\text{ m}}\left(\frac{1.00\text{ eV}}{1.60\times10^{-19}\text{ J}}\right) - 4.70\text{ eV} = 1.51\text{ eV}$.

The sphere is left with positive charge and so with positive potential relative to $V = 0$ at $r = \infty$. As its potential approaches 1.51 V, no further electrons will be able to escape, but will fall back onto the sphere. Its charge is then given by

$V = \frac{k_e Q}{r}$ or $Q = \frac{rV}{k_e} = \frac{(5.00\times10^{-2}\text{ m})(1.51\text{ N}\cdot\text{m/C})}{8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2} = \boxed{8.41\times10^{-12}\text{ C}}$.

Section 28.3 The Compton Effect

P28.14 $E = \frac{hc}{\lambda} = \frac{(6.626\times10^{-34}\text{ J}\cdot\text{s})(3.00\times10^8\text{ m/s})}{700\times10^{-9}\text{ m}} = 2.84\times10^{-19}\text{ J} = \boxed{1.78\text{ eV}}$

$p = \frac{h}{\lambda} = \frac{6.626\times10^{-34}\text{ J}\cdot\text{s}}{700\times10^{-9}\text{ m}} = \boxed{9.47\times10^{-28}\text{ kg}\cdot\text{m/s}}$

P28.15 (a) $\Delta\lambda = \frac{h}{m_e c}(1-\cos\theta)$: $\Delta\lambda = \frac{6.626\times10^{-34}}{(9.11\times10^{-31})(3.00\times10^8)}(1-\cos 37.0°) = \boxed{4.88\times10^{-13}\text{ m}}$

(b) $E_0 = \frac{hc}{\lambda_0}$ $\quad (300\times10^3\text{ eV})(1.60\times10^{-19}\text{ J/eV}) = \frac{(6.626\times10^{-34})(3.00\times10^8\text{ m/s})}{\lambda_0}$

$\lambda_0 = 4.14\times10^{-12}\text{ m}$

and $\lambda' = \lambda_0 + \Delta\lambda = 4.63\times10^{-12}\text{ m}$

$E' = \frac{hc}{\lambda'} = \frac{(6.626\times10^{-34}\text{ J}\cdot\text{s})(3.00\times10^8\text{ m/s})}{4.63\times10^{-12}\text{ m}} = 4.30\times10^{-14}\text{ J} = \boxed{268\text{ keV}}$

(c) $K_e = E_0 - E' = 300\text{ keV} - 268.5\text{ keV} = \boxed{31.5\text{ keV}}$

P28.16 This is Compton scattering through 180°:

FIG. P28.16

$E_0 = \frac{hc}{\lambda_0} = \frac{(6.626\times10^{-34}\text{ J}\cdot\text{s})(3.00\times10^8\text{ m/s})}{(0.110\times10^{-9}\text{ m})(1.60\times10^{-19}\text{ J/eV})} = 11.3\text{ keV}$

$\Delta\lambda = \frac{h}{m_e c}(1-\cos\theta) = (2.43\times10^{-12}\text{ m})(1-\cos 180°) = 4.86\times10^{-12}\text{ m}$

$\lambda' = \lambda_0 + \Delta\lambda = 0.115\text{ nm}$ so $E' = \frac{hc}{\lambda'} = 10.8\text{ keV}$.

By conservation of momentum for the photon-electron system, $\frac{h}{\lambda_0}\hat{\mathrm{i}} = \frac{h}{\lambda'}(-\hat{\mathrm{i}}) + p_e\hat{\mathrm{i}}$

and $p_e = h\left(\frac{1}{\lambda_0} + \frac{1}{\lambda'}\right)$

$p_e = (6.626\times10^{-34}\text{ J}\cdot\text{s})\left(\frac{(3.00\times10^8\text{ m/s})/c}{1.60\times10^{-19}\text{ J/eV}}\right)\left(\frac{1}{0.110\times10^{-9}\text{ m}} + \frac{1}{0.115\times10^{-9}\text{ m}}\right) = \boxed{\frac{22.1\text{ keV}}{c}}$

By conservation of system energy, $11.3\text{ keV} = 10.8\text{ keV} + K_e$

so that $\boxed{K_e = 478\text{ eV}}$.

Check: $E^2 = p^2c^2 + m_e^2c^4$ or $(m_ec^2 + K_e)^2 = (pc)^2 + (m_ec^2)^2$

$(511\text{ keV} + 0.478\text{ keV})^2 = (22.1\text{ keV})^2 + (511\text{ keV})^2$

$2.62\times10^{11} = 2.62\times10^{11}$

P28.17 With $K_e = E'$, $K_e = E_0 - E'$ gives $E' = E_0 - E'$

$$E' = \frac{E_0}{2} \text{ and } \lambda' = \frac{hc}{E'} \qquad \lambda' = \frac{hc}{E_0/2} = 2\frac{hc}{E_0} = 2\lambda_0$$

$$\lambda' = \lambda_0 + \lambda_C(1-\cos\theta) \qquad 2\lambda_0 = \lambda_0 + \lambda_C(1-\cos\theta)$$

$$1-\cos\theta = \frac{\lambda_0}{\lambda_C} = \frac{0.001\,60}{0.002\,43} \qquad \theta = \boxed{70.0°}$$

P28.18 (a) $K = \frac{1}{2}m_e v^2$. $K = \frac{1}{2}(9.11\times10^{-31}\text{ kg})(1.40\times10^6\text{ m/s})^2 = 8.93\times10^{-19}\text{ J} = 5.58\text{ eV}$

$$E_0 = \frac{hc}{\lambda_0} = \frac{1\,240\text{ eV}\cdot\text{nm}}{0.800\text{ nm}} = 1\,550\text{ eV}$$

$E' = E_0 - K$ and $\lambda' = \frac{hc}{E'} = \frac{1\,240\text{ eV}\cdot\text{nm}}{1\,550\text{ eV} - 5.58\text{ eV}} = 0.803\text{ nm}$

$$\Delta\lambda = \lambda' - \lambda_0 = 0.002\,88\text{ nm} = \boxed{2.88\text{ pm}}$$

(b) $\Delta\lambda = \lambda_C(1-\cos\theta)$: $\cos\theta = 1 - \frac{\Delta\lambda}{\lambda_C} = 1 - \frac{0.002\,88\text{ nm}}{0.002\,43\text{ nm}} = -0.189$,

so $\boxed{\theta = 101°}$

Section 28.4 Photons and Electromagnetic Waves

*P28.19 With photon energy $10.0\text{ eV} = hf$

$$f = \frac{10.0(1.6\times10^{-19}\text{ J})}{6.63\times10^{-34}\text{ J}\cdot\text{s}} = 2.41\times10^{15}\text{ Hz}.$$

Any electromagnetic wave with frequency higher than 2.41×10^{15} Hz counts as ionizing radiation. This includes far ultraviolet light, x-rays, and gamma rays.

Section 28.5 The Wave Properties of Particles

P28.20 $\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{6.626\times10^{-34}\text{ J}\cdot\text{s}}{(1.67\times10^{-27}\text{ kg})(1.00\times10^6\text{ m/s})} = \boxed{3.97\times10^{-13}\text{ m}}$

P28.21 (a) Electron: $\lambda = \frac{h}{p}$ and $K = \frac{1}{2}m_e v^2 = \frac{m_e^2 v^2}{2m_e} = \frac{p^2}{2m_e}$ so $p = \sqrt{2m_e K}$

and $\lambda = \frac{h}{\sqrt{2m_e K}} = \frac{6.626\times10^{-34}\text{ J}\cdot\text{s}}{\sqrt{2(9.11\times10^{-31}\text{ kg})(3.00)(1.60\times10^{-19}\text{ J})}}$

$$\lambda = 7.09\times10^{-10}\text{ m} = \boxed{0.709\text{ nm}}.$$

(b) Photon: $\lambda = \frac{c}{f}$ and $E = hf$ so $f = \frac{E}{h}$

and $\lambda = \frac{hc}{E} = \frac{(6.626\times10^{-34}\text{ J}\cdot\text{s})(3.00\times10^8\text{ m/s})}{3(1.60\times10^{-19}\text{ J})} = 4.14\times10^{-7}\text{ m} = \boxed{414\text{ nm}}$.

P28.22 From the condition for Bragg reflection,

$$m\lambda = 2d\sin\theta = 2d\cos\left(\frac{\phi}{2}\right).$$

But $d = a\sin\left(\frac{\phi}{2}\right)$

where a is the lattice spacing.

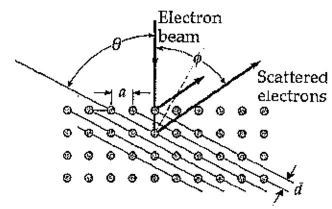

FIG. P28.22

Thus, with $m = 1$, $\lambda = 2a\sin\left(\frac{\phi}{2}\right)\cos\left(\frac{\phi}{2}\right) = a\sin\phi$

$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2m_e K}}$ $\qquad \lambda = \frac{6.626\times10^{-34}\text{ J}\cdot\text{s}}{\sqrt{2(9.11\times10^{-31}\text{ kg})(54.0\times1.60\times10^{-19}\text{ J})}} = 1.67\times10^{-10}\text{ m}.$

Therefore, the lattice spacing is $a = \frac{\lambda}{\sin\phi} = \frac{1.67\times10^{-10}\text{ m}}{\sin 50.0°} = 2.18\times10^{-10} = \boxed{0.218\text{ nm}}$.

P28.23 (a) $\lambda \sim 10^{-14}$ m or less. $\qquad p = \frac{h}{\lambda} \sim \frac{6.6\times10^{-34}\text{ J}\cdot\text{s}}{10^{-14}\text{ m}} \doteq 10^{-19}\text{ kg}\cdot\text{m/s}$ or more.

The energy of the electron is $E = \sqrt{p^2c^2 + m_e^2c^4} \sim \sqrt{(10^{-19})^2(3\times10^8)^2 + (9\times10^{-31})^2(3\times10^8)^4}$

or $E \sim 10^{-11}\text{ J} \sim 10^8$ eV or more,

so that $K = E - m_e c^2 \sim 10^8\text{ eV} - (0.5\times10^6\text{ eV})\boxed{\sim 10^8\text{ eV}}$ or more.

(b) The electric potential energy of the electron-nucleus system would be

$$U_e = \frac{k_e q_1 q_2}{r} \sim \frac{(9\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(10^{-19}\text{ C})(-e)}{10^{-14}\text{ m}} \sim -10^5\text{ eV}.$$

With its $K + U_e >> 0$, $\boxed{\text{the electron would immediately escape the nucleus}}$.

P28.24 (a) The wavelength of the student is $\lambda = \frac{h}{p} = \frac{h}{mv}$. If w is the width of the diffracting aperture,

then we need $w \le 10.0\lambda = 10.0\left(\frac{h}{mv}\right)$

so that $v \le 10.0\frac{h}{mw} = 10.0\left(\frac{6.626\times10^{-34}\ \text{J}\cdot\text{s}}{(80.0\ \text{kg})(0.750\ \text{m})}\right) = \boxed{1.10\times10^{-34}\ \text{m/s}}$.

(b) Using $\Delta t = \frac{d}{v}$ we get: $\Delta t \ge \frac{0.150\ \text{m}}{1.10\times10^{-34}\ \text{m/s}} = \boxed{1.36\times10^{33}\ \text{s}}$.

(c) $\boxed{\text{No}}$. The minimum time to pass through the door is over 10^{15} times the age of the Universe.

P28.25 $\lambda = \frac{h}{p}$ $\quad p = \frac{h}{\lambda} = \frac{6.626\times10^{-34}\ \text{J}\cdot\text{s}}{1.00\times10^{-11}\ \text{m}} = 6.63\times10^{-23}\ \text{kg}\cdot\text{m/s}$

(a) electrons: $K_e = \frac{p^2}{2m_e} = \frac{(6.63\times10^{-23})^2}{2(9.11\times10^{-31})}\ \text{J} = 15.1\ \text{keV}$

The relativistic answer is more precisely correct:

$$K_e = \sqrt{p^2c^2 + m_e^2c^4} - m_ec^2 = \boxed{14.9\ \text{keV}}.$$

(b) photons: $E_\gamma = pc = (6.63\times10^{-23})(3.00\times10^8) = \boxed{124\ \text{keV}}$

Section 28.6 The Quantum Particle

***P28.26** $E = K = \frac{1}{2}mu^2 = hf$ and $\lambda = \frac{h}{mu}$

$$v_{\text{phase}} = f\lambda = \frac{mu^2}{2h}\frac{h}{mu} = \boxed{\frac{u}{2} = v_{\text{phase}}}.$$

This is different from the speed u at which the particle transports mass, energy, and momentum.

P28.27 As a bonus, we begin by proving that the phase speed $v_p = \frac{\omega}{k}$ is not the speed of the particle.

$$v_p = \frac{\omega}{k} = \frac{\sqrt{p^2c^2 + m^2c^4}\,\hbar}{\hbar\gamma mv} = \frac{\sqrt{\gamma^2m^2v^2c^2 + m^2c^4}}{\sqrt{\gamma^2m^2v^2}} = c\sqrt{1 + \frac{c^2}{\gamma^2v^2}} = c\sqrt{1 + \frac{c^2}{v^2}\left(1 - \frac{v^2}{c^2}\right)} = c\sqrt{1 + \frac{c^2}{v^2} - 1} = \frac{c^2}{v}$$

In fact, the phase speed is larger than the speed of light. A point of constant phase in the wave function carries no mass, no energy, and no information.

Now for the group speed:

continued on next page

$$v_g = \frac{d\omega}{dk} = \frac{d\hbar\omega}{d\hbar k} = \frac{dE}{dp} = \frac{d}{dp}\sqrt{m^2c^4 + p^2c^2}$$

$$v_g = \frac{1}{2}\left(m^2c^4 + p^2c^2\right)^{-1/2}\left(0 + 2pc^2\right) = \sqrt{\frac{p^2c^4}{p^2c^2 + m^2c^4}}$$

$$v_g = c\sqrt{\frac{\gamma^2m^2v^2}{\gamma^2m^2v^2 + m^2c^2}} = c\sqrt{\frac{v^2/(1 - v^2/c^2)}{v^2/(1 - v^2/c^2) + c^2}} = c\sqrt{\frac{v^2/(1 - v^2/c^2)}{(v^2 + c^2 - v^2)/(1 - v^2/c^2)}} = v$$

It is this speed at which mass, energy, and momentum are transported.

Section 28.7 The Double-Slit Experiment Revisited

P28.28 Consider the first bright band away from the center:

$d\sin\theta = m\lambda$ $\quad (6.00\times10^{-8}\ \text{m})\sin\left(\tan^{-1}\left[\frac{0.400}{200}\right]\right) = (1)\lambda = 1.20\times10^{-10}\ \text{m}$

$\lambda = \frac{h}{m_ev}$ so $\quad m_ev = \frac{h}{\lambda}$

and $\quad K = \frac{1}{2}m_ev^2 = \frac{m_e^2v^2}{2m_e} = \frac{h^2}{2m_e\lambda^2} = e\Delta V$

$\Delta V = \frac{h^2}{2em_e\lambda^2}$ $\quad \Delta V = \frac{(6.626\times10^{-34}\ \text{J}\cdot\text{s})^2}{2(1.60\times10^{-19}\ \text{C})(9.11\times10^{-31}\ \text{kg})(1.20\times10^{-10}\ \text{m})^2} = \boxed{105\ \text{V}}$.

P28.29 (a) $\lambda = \frac{h}{mv} = \frac{6.626\times10^{-34}\ \text{J}\cdot\text{s}}{(1.67\times10^{-27}\ \text{kg})(0.400\ \text{m/s})} = \boxed{9.92\times10^{-7}\ \text{m}}$

(b) For destructive interference in a multiple-slit experiment, $d\sin\theta = \left(m + \frac{1}{2}\right)\lambda$, with $m = 0$ for the first minimum.

Then, $\quad \theta = \sin^{-1}\left(\frac{\lambda}{2d}\right) = 0.028\,4°$

so $\frac{y}{L} = \tan\theta$ $\quad y = L\tan\theta = (10.0\ \text{m})(\tan 0.028\,4°) = \boxed{4.96\ \text{mm}}$

(c) We cannot say the neutron passed through one slit. We can only say it passed through the slits.

*P28.30 We find the speed of each electron from energy conservation in the firing process:

$$0 = K_f + U_f = \frac{1}{2}mv^2 - eV$$

$$v = \sqrt{\frac{2eV}{m}} = \sqrt{\frac{2\times 1.6\times 10^{-19}\ \text{C}(45\ \text{V})}{9.11\times 10^{-31}\ \text{kg}}} = 3.98\times 10^{6}\ \text{m/s}$$

The time of flight is $\Delta t = \frac{\Delta x}{v} = \frac{0.28\ \text{m}}{3.98\times 10^{6}\ \text{m/s}} = 7.04\times 10^{-8}\ \text{s}$. The current when electrons are 28 cm apart is $I = \frac{q}{t} = \frac{e}{\Delta t} = \frac{1.6\times 10^{-19}\ \text{C}}{7.04\times 10^{-8}\ \text{s}} = \boxed{2.27\times 10^{-12}\ \text{A}}$.

Section 28.8 The Uncertainty Principle

P28.31 For the electron, $\Delta p = m_e \Delta v = (9.11\times 10^{-31}\ \text{kg})(500\ \text{m/s})(1.00\times 10^{-4}) = 4.56\times 10^{-32}\ \text{kg}\cdot\text{m/s}$

$$\Delta x = \frac{h}{4\pi \Delta p} = \frac{6.626\times 10^{-34}\ \text{J}\cdot\text{s}}{4\pi(4.56\times 10^{-32}\ \text{kg}\cdot\text{m/s})} = \boxed{1.16\ \text{mm}}.$$

For the bullet, $\Delta p = m\Delta v = (0.020\,0\ \text{kg})(500\ \text{m/s})(1.00\times 10^{-4}) = 1.00\times 10^{-3}\ \text{kg}\cdot\text{m/s}$

$$\Delta x = \frac{h}{4\pi \Delta p} = \boxed{5.28\times 10^{-32}\ \text{m}}.$$

P28.32 (a) $\Delta p \Delta x = m\Delta v \Delta x \ge \frac{\hbar}{2}$ so $\Delta v \ge \frac{h}{4\pi m \Delta x} = \frac{2\pi\ \text{J}\cdot\text{s}}{4\pi(2.00\ \text{kg})(1.00\ \text{m})} = \boxed{0.250\ \text{m/s}}$.

(b) The duck might move by $(0.25\ \text{m/s})(5\ \text{s}) = 1.25\ \text{m}$. With original position uncertainty of 1.00 m, we can think of Δx growing to $1.00\ \text{m} + 1.25\ \text{m} = \boxed{2.25\ \text{m}}$.

P28.33 $\frac{\Delta y}{x} = \frac{\Delta p_y}{p_x}$ and $d\Delta p_y \ge \frac{h}{4\pi}$

Eliminate Δp_y and solve for x.

$$x = 4\pi p_x(\Delta y)\frac{d}{h} \qquad x = 4\pi(1.00\times 10^{-3}\ \text{kg})(100\ \text{m/s})(1.00\times 10^{-2}\ \text{m})\frac{(2.00\times 10^{-3}\ \text{m})}{(6.626\times 10^{-34}\ \text{J}\cdot\text{s})}$$

The answer, $x = \boxed{3.79\times 10^{28}\ \text{m}}$, is 190 times greater than the diameter of the observable Universe!

P28.34 From the uncertainty principle $\Delta E \Delta t \ge \frac{\hbar}{2}$

or $\Delta(mc^2)\Delta t = \frac{\hbar}{2}$

Therefore,
$$\frac{\Delta m}{m} = \frac{h}{4\pi c^2(\Delta t)m} = \frac{h}{4\pi(\Delta t)E_R}$$

$$\frac{\Delta m}{m} = \frac{6.626\times 10^{-34}\ \text{J}\cdot\text{s}}{4\pi(8.70\times 10^{-17}\ \text{s})(135\ \text{MeV})}\left(\frac{1\ \text{MeV}}{1.60\times 10^{-13}\ \text{J}}\right) = \boxed{2.81\times 10^{-8}}$$

P28.35 (a) At the top of the ladder, the woman holds a pellet inside a small region Δx_i. Thus, the uncertainty principle requires her to release it with typical horizontal momentum $\Delta p_x = m\Delta v_x = \frac{\hbar}{2\Delta x_i}$. It falls to the floor in a travel time given by $H = 0 + \frac{1}{2}gt^2$ as $t = \sqrt{\frac{2H}{g}}$, so the total width of the impact points is

$$\Delta x_f = \Delta x_i + (\Delta v_x)t = \Delta x_i + \left(\frac{\hbar}{2m\Delta x_i}\right)\sqrt{\frac{2H}{g}} = \Delta x_i + \frac{A}{\Delta x_i}$$

where
$$A = \frac{\hbar}{2m}\sqrt{\frac{2H}{g}}$$

To minimize Δx_f, we require $\frac{d(\Delta x_f)}{d(\Delta x_i)} = 0$ or $1 - \frac{A}{\Delta x_i^2} = 0$

so $\Delta x_i = \sqrt{A}$.

The minimum width of the impact points is

$$(\Delta x_f)_{\min} = \left(\Delta x_i + \frac{A}{\Delta x_i}\right)\bigg|_{\Delta x_i = \sqrt{A}} = 2\sqrt{A} = \boxed{\sqrt{\frac{2\hbar}{m}}\left(\frac{2H}{g}\right)^{1/4}}.$$

(b)
$$(\Delta x_f)_{\min} = \left[\frac{2(1.054\,6\times 10^{-34}\ \text{J}\cdot\text{s})}{5.00\times 10^{-4}\ \text{kg}}\right]^{1/2}\left[\frac{2(2.00\ \text{m})}{9.80\ \text{m/s}^2}\right]^{1/4} = \boxed{5.19\times 10^{-16}\ \text{m}}$$

Section 28.9 An Interpretation of Quantum Mechanics

P28.36 Probability
$$P = \int_{-a}^{a}|\psi(x)|^2 = \int_{-a}^{a}\frac{a}{\pi(x^2+a^2)}dx = \left(\frac{a}{\pi}\right)\left(\frac{1}{a}\right)\tan^{-1}\left(\frac{x}{a}\right)\bigg|_{-a}^{a}$$

$$P = \frac{1}{\pi}\left[\tan^{-1}1 - \tan^{-1}(-1)\right] = \frac{1}{\pi}\left[\frac{\pi}{4} - \left(-\frac{\pi}{4}\right)\right] = \boxed{\frac{1}{2}}$$

P28.37 (a) $\psi(x) = Ae^{i(5.00\times10^{10}x)} = A\cos(5\times10^{10}x) + Ai\sin(5\times10^{10}x) = A\cos(kx) + Ai\sin(kx)$ goes through a full cycle when x changes by λ and when kx changes by 2π. Then $k\lambda = 2\pi$ where $k = 5.00\times10^{10}\ \text{m}^{-1} = \frac{2\pi}{\lambda}$. Then $\lambda = \frac{2\pi\,\text{m}}{(5.00\times10^{10})} = \boxed{1.26\times10^{-10}\ \text{m}}$.

(b) $p = \frac{h}{\lambda} = \frac{6.626\times10^{-34}\ \text{J}\cdot\text{s}}{1.26\times10^{-10}\ \text{m}} = \boxed{5.27\times10^{-24}\ \text{kg}\cdot\text{m/s}}$

(c) $m_e = 9.11\times10^{-31}\ \text{kg}$

$$K = \frac{m_e^2v^2}{2m_e} = \frac{p^2}{2m} = \frac{(5.27\times10^{-24}\ \text{kg}\cdot\text{m/s})^2}{(2\times9.11\times10^{-31}\ \text{kg})} = 1.52\times10^{-17}\ \text{J} = \frac{1.52\times10^{-17}\ \text{J}}{1.60\times10^{-19}\ \text{J/eV}} = \boxed{95.5\ \text{eV}}$$

Section 28.10 A Particle in a Box

P28.38 For an electron wave to "fit" into an infinitely deep potential well, an integral number of half-wavelengths must equal the width of the well.

$$\frac{n\lambda}{2} = 1.00\times10^{-9}\ \text{m} \qquad \text{so} \qquad \lambda = \frac{2.00\times10^{-9}}{n} = \frac{h}{p}$$

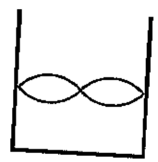

FIG. P28.38

(a) Since $K = \frac{p^2}{2m_e} = \frac{(h^2/\lambda^2)}{2m_e} = \frac{h^2}{2m_e}\frac{n^2}{(2\times10^{-9})^2} = (0.377n^2)\ \text{eV}$

For $K \approx 6\ \text{eV}$ $\quad \boxed{n=4}$

(b) With $n = 4$, $\quad \boxed{K = 6.03\ \text{eV}}$

P28.39 (a) We can draw a diagram that parallels our treatment of standing mechanical waves. In each state, we measure the distance d from one node to another (N to N), and base our solution upon that:

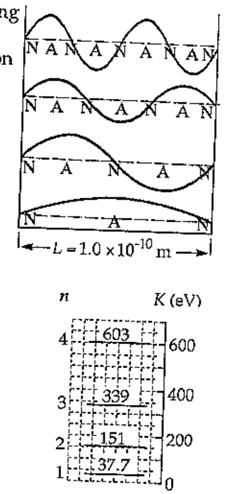

FIG. P28.39

Since $d_{\text{N to N}} = \frac{\lambda}{2}$ and $\lambda = \frac{h}{p}$

$$p = \frac{h}{\lambda} = \frac{h}{2d}$$

Next, $K = \frac{p^2}{2m_e} = \frac{h^2}{8m_ed^2} = \frac{1}{d^2}\left[\frac{(6.626\times10^{-34}\ \text{J}\cdot\text{s})^2}{8(9.11\times10^{-31}\ \text{kg})}\right]$.

Evaluating, $K = \frac{6.02\times10^{-38}\ \text{J}\cdot\text{m}^2}{d^2}$ $\qquad K = \frac{3.77\times10^{-19}\ \text{eV}\cdot\text{m}^2}{d^2}$

In state 1, $d = 1.00\times10^{-10}\ \text{m}$ $\qquad K_1 = 37.7\ \text{eV}$.

In state 2, $d = 5.00\times10^{-11}\ \text{m}$ $\qquad K_2 = 151\ \text{eV}$.

In state 3, $d = 3.33\times10^{-11}\ \text{m}$ $\qquad K_3 = 339\ \text{eV}$.

In state 4, $d = 2.50\times10^{-11}\ \text{m}$ $\qquad K_4 = 603\ \text{eV}$.

(b) When the electron falls from state 2 to state 1, it puts out energy

$$E = 151\ \text{eV} - 37.7\ \text{eV} = 113\ \text{eV} = hf = \frac{hc}{\lambda}$$

into emitting a photon of wavelength

$$\lambda = \frac{hc}{E} = \frac{(6.626\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^8\ \text{m/s})}{(113\ \text{eV})(1.60\times10^{-19}\ \text{J/eV})} = 11.0\ \text{nm}.$$

The wavelengths of the other spectral lines we find similarly:

Transition	$4\to3$	$4\to2$	$4\to1$	$3\to2$	$3\to1$	$2\to1$
E(eV)	264	452	565	188	302	113
λ(nm)	4.71	2.75	2.20	6.60	4.12	11.0

P28.40 The confined proton can be described in the same way as a standing wave on a string. At level 1, the node-to-node distance of the standing wave is 1.00×10^{-14} m, so the wavelength is twice this distance: $\frac{h}{p}=2.00\times10^{-14}$ m.

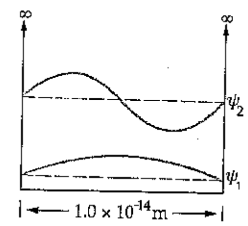

FIG. P28.40

The proton's kinetic energy is

$$K=\frac{1}{2}mv^2=\frac{p^2}{2m}=\frac{h^2}{2m\lambda^2}=\frac{\left(6.626\times10^{-34}\text{ J}\cdot\text{s}\right)^2}{2\left(1.67\times10^{-27}\text{ kg}\right)\left(2.00\times10^{-14}\text{ m}\right)^2}$$

$$=\frac{3.29\times10^{-13}\text{ J}}{1.60\times10^{-19}\text{ J/eV}}=2.05\text{ MeV}$$

In the first excited state, level 2, the node-to-node distance is half as long as in state 1. The momentum is two times larger and the energy is four times larger: $K=8.22$ MeV.

The proton has mass, has charge, moves slowly compared to light in a standing wave state, and stays inside the nucleus. When it falls from level 2 to level 1, its energy change is

$2.05\text{ MeV}-8.22\text{ MeV}=-6.16\text{ MeV}$.

Therefore, we know that a photon (a traveling wave with no mass and no charge) is emitted at the speed of light, and that it has an energy of $\boxed{+6.16\text{ MeV}}$.

Its frequency is $$f=\frac{E}{h}=\frac{\left(6.16\times10^{6}\text{ eV}\right)\left(1.60\times10^{-19}\text{ J/eV}\right)}{6.626\times10^{-34}\text{ J}\cdot\text{s}}=1.49\times10^{21}\text{ Hz}.$$

And its wavelength is $$\lambda=\frac{c}{f}=\frac{3.00\times10^{8}\text{ m/s}}{1.49\times10^{21}\text{ s}^{-1}}=\boxed{2.02\times10^{-13}\text{ m}}.$$

This is a gamma ray, according to the electromagnetic spectrum chart in Chapter 24.

*P28.41 (a) The energies of the confined electron are $E_n=\frac{h^2}{8m_eL^2}n^2$. Its energy gain in the quantum jump from state 1 to state 4 is $\frac{h^2}{8m_eL^2}\left(4^2-1^2\right)$ and this is the photon energy:

$\frac{h^2 15}{8m_eL^2}=hf=\frac{hc}{\lambda}$. Then $8m_ecL^2=15h\lambda$ and $\boxed{L=\left(\frac{15h\lambda}{8m_ec}\right)^{1/2}}$.

(b) Let λ' represent the wavelength of the photon emitted: $\frac{hc}{\lambda'}=\frac{h^2}{8m_eL^2}4^2-\frac{h^2}{8m_eL^2}2^2=\frac{12h^2}{8m_eL^2}$.

Then $\frac{hc}{\lambda}\frac{\lambda'}{hc}=\frac{h^2 15\left(8m_eL^2\right)}{8m_eL^2 12h^2}=\frac{5}{4}$ and $\boxed{\lambda'=1.25\lambda}$.

Section 28.11 The Quantum Particle Under Boundary Conditions

Section 28.12 The Schrödinger Equation

P28.42 $\psi(x)=A\cos kx+B\sin kx$ $\qquad\frac{\partial\psi}{\partial x}=-kA\sin kx+kB\cos kx$

$\frac{\partial^2\psi}{\partial x^2}=-k^2A\cos kx-k^2B\sin kx$ $\qquad-\frac{2m}{\hbar}(E-U)\psi=-\frac{2mE}{\hbar^2}(A\cos kx+B\sin kx)$

Therefore the Schrödinger equation is satisfied if

$\frac{\partial^2\psi}{\partial x^2}=\left(-\frac{2m}{\hbar^2}\right)(E-U)\psi$ or $\qquad-k^2(A\cos kx+B\sin kx)=\left(-\frac{2mE}{\hbar^2}\right)(A\cos kx+B\sin kx)$.

This is true as an identity (functional equality) for all x if $\boxed{E=\frac{\hbar^2k^2}{2m}}$.

P28.43 We have $\psi=Ae^{i(kx-\omega t)}$ $\qquad\frac{\partial\psi}{\partial x}=ik\psi$ $\qquad$ and $\frac{\partial^2\psi}{\partial x^2}=-k^2\psi$.

Schrödinger's equation: $\qquad\frac{\partial^2\psi}{\partial x^2}=-k^2\psi=-\frac{2m}{\hbar^2}(E-U)\psi$.

Since $\qquad k^2=\frac{(2\pi)^2}{\lambda^2}=\frac{(2\pi p)^2}{h^2}=\frac{p^2}{\hbar^2}$ $\qquad$ and $\qquad E-U=\frac{p^2}{2m}$

$\boxed{\text{Thus this equation balances.}}$

P28.44 (a) With $\psi(x)=Ae^{ikx}$

$\frac{d}{dx}Ae^{ikx}=Aikxe^{ikx}$ and $\frac{d^2\psi}{dx^2}=-Ak^2e^{ikx}$

Then $-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2}=+\frac{\hbar^2k^2}{2m}Ae^{ikx}=\frac{h^2}{4\pi^2}\frac{4\pi^2}{\lambda^2 2m}\psi=\frac{p^2}{2m}\psi=\frac{m^2v^2}{2m}\psi=\frac{1}{2}mv^2\psi=K\psi$.

(b) With $\psi(x)=\sqrt{\frac{2}{L}}\sin\left(\frac{n\pi x}{L}\right)=A\sin kx$, $\frac{d\psi}{dx}=Ak\cos kx$ and $\frac{d^2\psi}{dx^2}=-Ak^2\sin kx$

Then $-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2}=+\frac{\hbar^2}{2m}Ak^2\sin kx=\frac{h^2 4\pi^2}{4\pi^2\lambda^2 2m}\psi=\frac{p^2}{2m}\psi=K\psi$.

P28.45 (a) $\langle x\rangle=\int_0^L x\frac{2}{L}\sin^2\left(\frac{2\pi x}{L}\right)dx=\frac{2}{L}\int_0^L x\left(\frac{1}{2}-\frac{1}{2}\cos\frac{4\pi x}{L}\right)dx$

$\langle x\rangle=\frac{1}{L}\frac{x^2}{2}\Big|_0^L-\frac{1}{L}\frac{L^2}{16\pi^2}\left[\frac{4\pi x}{L}\sin\frac{4\pi x}{L}+\cos\frac{4\pi x}{L}\right]_0^L=\boxed{\frac{L}{2}}$

continued on next page

(b) $$\text{Probability} = \int_{0.490L}^{0.510L} \frac{2}{L}\sin^2\left(\frac{2\pi x}{L}\right)dx = \left[\frac{1}{L}x - \frac{1}{L}\frac{L}{4\pi}\sin\frac{4\pi x}{L}\right]_{0.490L}^{0.510L}$$

$$\text{Probability} = 0.020 - \frac{1}{4\pi}(\sin 2.04\pi - \sin 1.96\pi) = \boxed{5.26\times10^{-5}}$$

(c) $$\text{Probability}\left[\frac{x}{L} - \frac{1}{4\pi}\sin\frac{4\pi x}{L}\right]_{0.240L}^{0.260L} = \boxed{3.99\times10^{-2}}$$

(d) In the $n = 2$ graph in Figure 28.23 (b), it is more probable to find the particle either near $x = \frac{L}{4}$ or $x = \frac{3L}{4}$ than at the center, where the probability density is zero.

Nevertheless, the symmetry of the distribution means that the average position is $\frac{L}{2}$.

P28.46 Normalization requires

$$\int_{\text{all space}} |\psi|^2 dx = 1 \quad \text{or} \quad \int_0^L A^2\sin^2\left(\frac{n\pi x}{L}\right)dx = 1$$

$$\int_0^L A^2\sin^2\left(\frac{n\pi x}{L}\right)dx = A^2\left(\frac{L}{2}\right) = 1 \quad \text{or} \quad \boxed{A = \sqrt{\frac{2}{L}}}.$$

P28.47 The desired probability is $P = \int_0^{L/4} |\psi|^2 dx = \frac{2}{L}\int_0^{L/4}\sin^2\left(\frac{2\pi x}{L}\right)dx$

where $\sin^2\theta = \frac{1-\cos 2\theta}{2}$

Thus, $P = \left(\frac{x}{L} - \frac{1}{4\pi}\sin\frac{4\pi x}{L}\right)\Big|_0^{L/4} = \left(\frac{1}{4} - 0 - 0 + 0\right) = \boxed{0.250}$.

P28.48 (a) $\psi(x) = A\left(1 - \frac{x^2}{L^2}\right)$ $\qquad \frac{d\psi}{dx} = -\frac{2Ax}{L^2}$ $\qquad \frac{d^2\psi}{dx^2} = -\frac{2A}{L^2}$

Schrödinger's equation $\qquad \frac{d^2\psi}{dx^2} = -\frac{2m}{\hbar^2}(E-U)\psi$

becomes $$-\frac{2A}{L^2} = -\frac{2m}{\hbar^2}EA\left(1-\frac{x^2}{L^2}\right) + \frac{2m}{\hbar^2}\frac{\left(-\hbar^2x^2\right)A\left(1-x^2/L^2\right)}{mL^2\left(L^2-x^2\right)}$$

$$-\frac{1}{L^2} = -\frac{mE}{\hbar^2} + \frac{mEx^2}{\hbar^2L^2} - \frac{x^2}{L^4}.$$

This will be true for all x if both $\qquad \frac{1}{L^2} = \frac{mE}{\hbar^2}$

and $\qquad \frac{mE}{\hbar^2L^2} - \frac{1}{L^4} = 0$

both these conditions are satisfied for a particle of energy $\boxed{E = \frac{\hbar^2}{L^2m}}$.

continued on next page

(b) For normalization, $$1 = \int_{-L}^{L} A^2\left(1-\frac{x^2}{L^2}\right)^2 dx = A^2\int_{-L}^{L}\left(1-\frac{2x^2}{L^2}+\frac{x^4}{L^4}\right)dx$$

$$1 = A^2\left[x - \frac{2x^3}{3L^2} + \frac{x^5}{5L^4}\right]_{-L}^{L} = A^2\left[L - \frac{2}{3}L + \frac{L}{5} + L - \frac{2}{3}L + \frac{L}{5}\right] = A^2\left(\frac{16L}{15}\right) \qquad \boxed{A = \sqrt{\frac{15}{16L}}}.$$

(c) $$P = \int_{-L/3}^{L/3}\psi^2 dx = \frac{15}{16L}\int_{-L/3}^{L/3}\left(1-\frac{2x^2}{L^2}+\frac{x^4}{L^4}\right)dx = \frac{15}{16L}\left[x - \frac{2x^3}{3L^2} + \frac{x^5}{5L^5}\right]_{-L/3}^{L/3} = \frac{30}{16L}\left[\frac{L}{3} - \frac{2L}{81} + \frac{L}{1\,215}\right]$$

$$P = \frac{47}{81} = \boxed{0.580}$$

Section 28.13 Tunneling Through a Potential Energy Barrier

P28.49 $T = e^{-2CL}$ where $C = \frac{\sqrt{2m(U-E)}}{\hbar}$

$$2CL = \frac{2\sqrt{2\left(9.11\times10^{-31}\right)\left(8.00\times10^{-19}\right)}}{1.055\times10^{-34}}\left(2.00\times10^{-10}\right) = 4.58$$

(a) $T = e^{-4.58} = \boxed{0.010\,3}$, a 1% chance of transmission.

(b) $R = 1 - T = \boxed{0.990}$, a 99% chance of reflection.

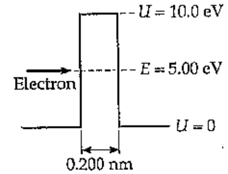

FIG. P28.49

P28.50 $$C = \frac{\sqrt{2\left(9.11\times10^{-31}\right)(5.00-4.50)\left(1.60\times10^{-19}\right)}\ \text{kg}\cdot\text{m/s}}{1.055\times10^{-34}\ \text{J}\cdot\text{s}} = 3.62\times10^{9}\ \text{m}^{-1}$$

$$T = e^{-2CL} = \exp\left[-2\left(3.62\times10^{9}\ \text{m}^{-1}\right)\left(950\times10^{-12}\ \text{m}\right)\right] = \exp(-6.88)$$

$T = \boxed{1.03\times10^{-3}}$

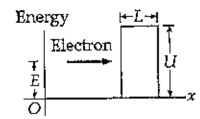

FIG. P28.50

***P28.51** The original tunneling probability is $T = e^{-2CL}$ where

$$C = \frac{\left(2m(U-E)\right)^{1/2}}{\hbar} = \frac{2\pi\left(2\times9.11\times10^{-31}\ \text{kg}(20-12)1.6\times10^{-19}\ \text{J}\right)^{1/2}}{6.626\times10^{-34}\ \text{J}\cdot\text{s}} = 1.448\,1\times10^{10}\ \text{m}^{-1}$$

The photon energy is $hf = \frac{hc}{\lambda} = \frac{1\,240\ \text{eV}\cdot\text{nm}}{546\ \text{nm}} = 2.27\ \text{eV}$, to make the electron's new kinetic energy $12 + 2.27 = 14.27\ \text{eV}$ and its decay coefficient inside the barrier

$$C' = \frac{2\pi\left(2\times9.11\times10^{-31}\ \text{kg}(20-14.27)1.6\times10^{-19}\ \text{J}\right)^{1/2}}{6.626\times10^{-34}\ \text{J}\cdot\text{s}} = 1.225\,5\times10^{10}\ \text{m}^{-1}$$

Now the factor of increase in transmission probability is

$$\frac{e^{-2C'L}}{e^{-2CL}} = e^{2L(C-C')} = e^{2\times10^{-9}\ \text{m}\times0.223\times10^{10}\ \text{m}^{-1}} = e^{4.45} = \boxed{85.9}.$$

Section 28.14 Context Connection—The Cosmic Temperature

P28.52 The radiation wavelength of $\lambda' = 500$ nm that is observed by observers on Earth is not the true wavelength, λ, emitted by the star because of the Doppler effect. The true wavelength is related to the observed wavelength using:

$$\frac{c}{\lambda'} = \frac{c}{\lambda}\sqrt{\frac{1-(v/c)}{1+(v/c)}} \qquad \lambda = \lambda'\sqrt{\frac{1-(v/c)}{1+(v/c)}} = (500\text{ nm})\sqrt{\frac{1-(0.280)}{1+(0.280)}} = 375\text{ nm}.$$

The temperature of the star is given by $\lambda_{max}T = 2.898\times10^{-3}\text{ m}\cdot\text{K}$:

$$T = \frac{2.898\times10^{-3}\text{ m}\cdot\text{K}}{\lambda_{max}} \qquad T = \frac{2.898\times10^{-3}\text{ m}\cdot\text{K}}{375\times10^{-9}} = \boxed{7.73\times10^3\text{ K}}.$$

P28.53 (a) Wien's law: $\lambda_{max}T = 2.898\times10^{-3}\text{ m}\cdot\text{K}$.

Thus, $\lambda_{max} = \dfrac{2.898\times10^{-3}\text{ m}\cdot\text{K}}{T} = \dfrac{2.898\times10^{-3}\text{ m}\cdot\text{K}}{2.73\text{ K}} = 1.06\times10^{-3}\text{ m} = \boxed{1.06\text{ mm}}$

(b) This is a $\boxed{\text{microwave}}$.

P28.54 We suppose that the fireball of the Big Bang is a black body.

$$I = e\sigma T^4 = (1)(5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)(2.73\text{ K})^4 = \boxed{3.15\times10^{-6}\text{ W/m}^2}$$

As a bonus, we can find the current power of direct radiation from the Big Bang in the section of the universe observable to us. If it is fifteen billion years old, the fireball is a perfect sphere of radius fifteen billion light years, centered at the point halfway between your eyes:

$$\mathcal{P} = IA = I(4\pi r^2) = (3.15\times10^{-6}\text{ W/m}^2)(4\pi)(15\times10^9\text{ ly})^2\left(\frac{3\times10^8\text{ m/s}}{1\text{ ly/yr}}\right)^2(3.156\times10^7\text{ s/yr})^2$$

$$\mathcal{P} = 7.98\times10^{47}\text{ W}.$$

Additional Problems

***P28.55** The condition on electric power delivered to the filament is $\mathcal{P} = I\Delta V = \dfrac{(\Delta V)^2}{R} = \dfrac{(\Delta V)^2 A}{\rho\ell} = \dfrac{(\Delta V)^2\pi r^2}{\rho\ell}$

so $r = \left(\dfrac{\mathcal{P}\rho\ell}{\pi(\Delta V)^2}\right)^{1/2}$. Here $\mathcal{P} = 75$ W, $\rho = 7.13\times10^{-7}\,\Omega\cdot\text{m}$, and $\Delta V = 120$ V. As the filament radiates in steady state, it must emit all of this power through its lateral surface area $\mathcal{P} = \sigma eAT^4 = \sigma e2\pi r\ell T^4$. We combine the conditions by substitution:

continued on next page

$$\mathcal{P} = \sigma e2\pi\left(\frac{\mathcal{P}\rho\ell}{\pi(\Delta V)^2}\right)^{1/2}\ell T^4$$

$$(\Delta V)\mathcal{P}^{1/2} = \sigma e2\pi^{1/2}\rho^{1/2}\ell^{3/2}T^4$$

$$\ell = \left(\frac{(\Delta V)\mathcal{P}^{1/2}}{\sigma e2\pi^{1/2}\rho^{1/2}T^4}\right)^{2/3} = \left(\frac{120\text{ V}(75\text{ W})^{1/2}\text{ m}^2\text{K}^4}{5.67\times10^{-8}\text{ W }0.450(2)\pi^{1/2}(7.13\times10^{-7}\,\Omega\cdot\text{m})^{1/2}(2\,900\text{ K})^4}\right)^{2/3}$$

$$= (0.192\text{ m}^{3/2})^{2/3} = \boxed{0.333\text{ m} = \ell}$$

and $r = \left(\dfrac{\mathcal{P}\rho\ell}{\pi(\Delta V)^2}\right)^{1/2} = \left(\dfrac{75\text{ W }7.13\times10^{-7}\,\Omega\cdot\text{m }0.333\text{ m}}{\pi(120\text{ V})^2}\right)^{1/2} = \boxed{r = 1.98\times10^{-5}\text{ m}}.$

P28.56 $\Delta V_S = \left(\dfrac{h}{e}\right)f - \dfrac{\phi}{e}$

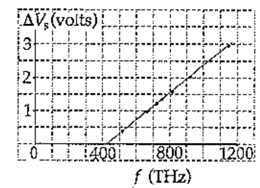

FIG. P28.56

From two points on the graph $\quad 0 = \left(\dfrac{h}{e}\right)(4.1\times10^{14}\text{ Hz}) - \dfrac{\phi}{e}$

and $\quad 3.3\text{ V} = \left(\dfrac{h}{e}\right)(12\times10^{14}\text{ Hz}) - \dfrac{\phi}{e}.$

Combining these two expressions we find:

(a) $\phi = \boxed{1.7\text{ eV}}$

(b) $\dfrac{h}{e} = \boxed{4.2\times10^{-15}\text{ V}\cdot\text{s}}$

(c) At the cutoff wavelength $\dfrac{hc}{\lambda_c} = \phi = \left(\dfrac{h}{e}\right)\dfrac{ec}{\lambda_c}$

$$\lambda_c = (4.2\times10^{-15}\text{ V}\cdot\text{s})(1.6\times10^{-19}\text{ C})\frac{(3\times10^8\text{ m/s})}{(1.7\text{ eV})(1.6\times10^{-19}\text{ J/eV})} = \boxed{730\text{ nm}}$$

P28.57 We want an Einstein plot of K_{max} versus $f = \frac{c}{\lambda}$

λ, nm	f, 10^{14} Hz	K_{max}, eV
588	5.10	0.67
505	5.94	0.98
445	6.74	1.35
399	7.52	1.63

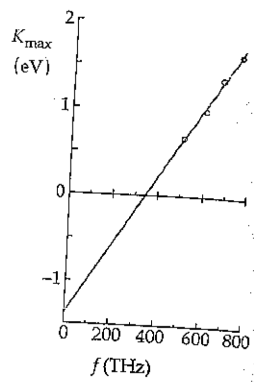

FIG. P28.57

(a) slope $= \frac{0.402 \text{ eV}}{10^{14} \text{ Hz}} \pm 8\%$

(b) $e\Delta V_S = hf - \phi$

$$h = (0.402)\left(\frac{1.60\times10^{-19} \text{ J}\cdot\text{s}}{10^{14}}\right) = \boxed{6.4\times10^{-34} \text{ J}\cdot\text{s} \pm 8\%}$$

(c) $K_{max} = 0$

at $f \approx 344\times10^{12}$ Hz

$$\phi = hf = 2.32\times10^{-19} \text{ J} = \boxed{1.4 \text{ eV}}$$

P28.58 From the path the electrons follow in the magnetic field, the maximum kinetic energy is seen to be:

$$K_{max} = \frac{e^2B^2R^2}{2m_e}$$

From the photoelectric equation, $K_{max} = hf - \phi = \frac{hc}{\lambda} - \phi$.

Thus, the work function is $\phi = \frac{hc}{\lambda} - K_{max} = \boxed{\frac{hc}{\lambda} - \frac{e^2B^2R^2}{2m_e}}$.

P28.59 (a) $mgy_i = \frac{1}{2}mv_f^2$

$$v_f = \sqrt{2gy_i} = \sqrt{2(9.80 \text{ m/s}^2)(50.0 \text{ m})} = 31.3 \text{ m/s}$$

$$\lambda = \frac{h}{mv} = \frac{6.626\times10^{-34} \text{ J}\cdot\text{s}}{(75.0 \text{ kg})(31.3 \text{ m/s})} = \boxed{2.82\times10^{-37} \text{ m}} \quad \text{(not observable)}$$

(b) $\Delta E\Delta t \geq \frac{\hbar}{2}$

$$\text{so } \Delta E \geq \frac{6.626\times10^{-34} \text{ J}\cdot\text{s}}{4\pi(5.00\times10^{-3} \text{ s})} = \boxed{1.06\times10^{-32} \text{ J}}$$

(c) $$\frac{\Delta E}{E} = \frac{1.06\times10^{-32} \text{ J}}{(75.0 \text{ kg})(9.80 \text{ m/s}^2)(50.0 \text{ m})} = \boxed{2.87\times10^{-35}\%}$$

P28.60 (a) $\lambda = 2L = \boxed{2.00\times10^{-10} \text{ m}}$

(b) $$p = \frac{h}{\lambda} = \frac{6.626\times10^{-34} \text{ J}\cdot\text{s}}{2.00\times10^{-10} \text{ m}} = \boxed{3.31\times10^{-24} \text{ kg}\cdot\text{m/s}}$$

(c) $$E = \frac{p^2}{2m} = \boxed{0.172 \text{ eV}}$$

P28.61 (a) $\boxed{\text{See the figure.}}$ (b) $\boxed{\text{See the figure.}}$

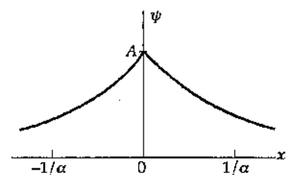

FIG. P28.61(a)

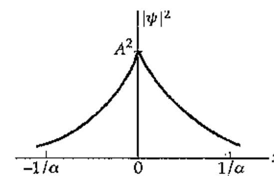

FIG. P28.61(b)

(c) ψ is continuous and $\psi \to 0$ as $x \to \pm\infty$. The function can be normalized. It describes a particle bound near $x = 0$, by a very deep, very narrow well of potential energy.

(d) Since ψ is symmetric,

$$\int_{-\infty}^{\infty} |\psi|^2 dx = 2\int_0^{\infty} |\psi|^2 dx = 1$$

or $$2A^2 \int_0^{\infty} e^{-2\alpha x} dx = \left(\frac{2A^2}{-2\alpha}\right)\left(e^{-\infty} - e^0\right) = 1.$$

This gives $\boxed{A = \sqrt{\alpha}}$.

(e) $$P_{(-1/2\alpha)\to(1/2\alpha)} = 2\left(\sqrt{\alpha}\right)^2 \int_{x=0}^{1/2\alpha} e^{-2\alpha x} dx = \left(\frac{2\alpha}{-2\alpha}\right)\left(e^{-2\alpha/2\alpha} - 1\right) = \left(1 - e^{-1}\right) = \boxed{0.632}$$

P28.62 (a) Use Schrödinger's equation

$$\frac{\partial^2\psi}{\partial x^2} = -\frac{2m}{\hbar^2}(E - U)\psi$$

with solutions

$$\psi_1 = Ae^{ik_1x} + Be^{-ik_1x} \quad \text{[region I]}$$

$$\psi_2 = Ce^{ik_2x} \quad \text{[region II]}.$$

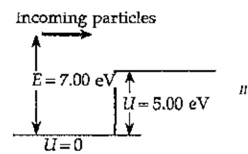

FIG. P28.62(a)

continued on next page

Where $k_1 = \frac{\sqrt{2mE}}{\hbar}$

and $k_2 = \frac{\sqrt{2m(E-U)}}{\hbar}$

Then, matching functions and derivatives at $x=0$

$(\psi_1)_0 = (\psi_2)_0$ gives $A+B=C$

and $\left(\frac{d\psi_1}{dx}\right)_0 = \left(\frac{d\psi_2}{dx}\right)_0$ gives $k_1(A-B)=k_2C$.

Then $B = \frac{1-k_2/k_1}{1+k_2/k_1}A$

and $C = \frac{2}{1+k_2/k_1}A$.

Incident wave Ae^{ikx} reflects Be^{-ikx}, with probability $R = \frac{B^2}{A^2} = \frac{(1-k_2/k_1)^2}{(1+k_2/k_1)^2} = \boxed{\frac{(k_1-k_2)^2}{(k_1+k_2)^2}}$.

(b) With $E = 7.00$ eV

and $U = 5.00$ eV

$$\frac{k_2}{k_1} = \sqrt{\frac{E-U}{E}} = \sqrt{\frac{2.00}{7.00}} = 0.535.$$

The reflection probability is $R = \frac{(1-0.535)^2}{(1+0.535)^2} = \boxed{0.092\,0}$.

The probability of transmission is $T = 1 - R = \boxed{0.908}$.

P28.63 $\langle x^2 \rangle = \int_{-\infty}^{\infty} x^2 |\psi|^2 dx$

For a one-dimensional box of width L, $\psi_n = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right)$.

Thus, $\langle x^2 \rangle = \frac{2}{L}\int_0^L x^2 \sin^2\left(\frac{n\pi x}{L}\right)dx = \boxed{\frac{L^2}{3} - \frac{L^2}{2n^2\pi^2}}$ (from integral tables).

***P28.64** (a) The requirement that $\frac{n\lambda}{2} = L$ so $p = \frac{h}{\lambda} = \frac{nh}{2L}$ is still valid.

$$E = \sqrt{(pc)^2 + (mc^2)^2} \Rightarrow E_n = \sqrt{\left(\frac{nhc}{2L}\right)^2 + (mc^2)^2}$$

$$K_n = E_n - mc^2 = \boxed{\sqrt{\left(\frac{nhc}{2L}\right)^2 + (mc^2)^2} - mc^2}$$

(b) Taking $L = 1.00\times10^{-12}$ m, $m = 9.11\times10^{-31}$ kg, and $n = 1$, we find $K_1 = \boxed{4.69\times10^{-14}\text{ J}}$

Nonrelativistic, $E_1 = \frac{h^2}{8mL^2} = \frac{(6.626\times10^{-34}\text{ J}\cdot\text{s})^2}{8(9.11\times10^{-31}\text{ kg})(1.00\times10^{-12}\text{ m})^2} = 6.02\times10^{-14}$ J.

Comparing this to K_1, we see that this value is too large by $\boxed{28.6\%}$.

P28.65 For a particle with wave function

$\psi(x) = \sqrt{\frac{2}{a}}e^{-x/a}$ for $x>0$

and 0 for $x<0$.

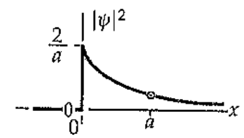

FIG. P28.65

(a) $|\psi(x)|^2 = 0,\ x<0$ and $|\psi^2(x)| = \frac{2}{a}e^{-2x/a},\ x>0$

(b) $\text{Prob}(x<0) = \int_{-\infty}^{0}|\psi(x)|^2 dx = \int_{-\infty}^{0}(0)dx = \boxed{0}$

(c) Normalization $\int_{-\infty}^{\infty}|\psi(x)|^2 dx = \int_{-\infty}^{0}|\psi|^2 dx + \int_0^{\infty}|\psi|^2 dx = 1$

$$\int_{-\infty}^{0} 0dx + \int_0^{\infty}\left(\frac{2}{a}\right)e^{-2x/a}dx = 0 - e^{-2x/a}\Big|_0^{\infty} = -\left(e^{-\infty} - 1\right) = 1$$

$$\text{Prob}(0<x<a) = \int_0^a |\psi|^2 dx = \int_0^a \left(\frac{2}{a}\right)e^{-2x/a}dx = -e^{-2x/a}\Big|_0^a = 1 - e^{-2} = \boxed{0.865}$$

This wave function would require the potential energy to be $+\infty$ for $x<0$ and $-\infty$ for $x=0$. This potential energy function cannot be realistic in detail.