ANSWERS TO EVEN PROBLEMS

P28.2 (a) 5.78×10^{3} K; (b) 501 nm

P28.4 (a) 2.57 eV; (b) 12.8 μeV; (c) 191 neV; (d) 484 nm visible, 9.68 cm and 6.52 m radio waves

P28.6 5.71×10^{3} photons

P28.8 (a) 0.263 kg; (b) 1.81 W; (c) $-0.015\,3$ °C/s $= -0.919$ °C/min; (d) 9.89 μm; (e) 2.01×10^{-20} J; (f) 8.98×10^{19} photon/s

P28.10 (a) 1.38 eV; (b) 334 THz

P28.12 148 days, absurdly large

P28.14 1.78 eV, 9.47×10^{-28} kg·m/s

P28.16 $\frac{22.1\text{ keV}}{c}$, 478 eV

P28.18 (a) 2.88 pm; (b) 101°

P28.20 397 fm

P28.22 0.218 nm

P28.24 (a) 1.10×10^{-34} m/s; (b) 1.36×10^{33} s; (c) no, the time is over 10^{15} times the age of the universe

P28.26 $v_{\text{phase}} = \frac{u}{2}$

P28.28 105 V

P28.30 2.27 pA

P28.32 (a) 0.250 m/s; (b) 2.25 m

P28.34 2.81×10^{-8}

P28.36 $\frac{1}{2}$

P28.38 (a) $n = 4$; (b) 6.03 eV

P28.40 6.16 MeV, 202 fm, a gamma ray

P28.42 see the solution, $E = \frac{\hbar^2 k^2}{2m}$

P28.44 see the solution

P28.46 see the solution

P28.48 (a) $E = \frac{\hbar^2}{mL^2}$,

(b) requiring $\int_{-L}^{L} A^2\left(1-\frac{x^2}{L^2}\right)^2 dx = 1$ gives $A = \left(\frac{15}{16L}\right)^{1/2}$,

(c) $\frac{47}{81} = 0.580$

P28.50 1.03×10^{-3}

P28.52 7.73×10^{3} K

P28.54 3.15 μW/m^2

P28.56 (a) 1.7 eV; (b) 4.2 fV·s; (c) 730 nm

P28.58 $\frac{hc}{\lambda} - \frac{e^2B^2R^2}{2m_e}$

P28.60 (a) 2.00×10^{-10} m; (b) 3.31×10^{-24} kg·m/s; (c) 0.172 eV

P28.62 (a) see the solution; (b) $R = 0.092\,0$, $T = 0.908$

P28.64 (a) $\sqrt{\left(\frac{nhc}{2L}\right)^2 + m^2c^4} - mc^2$; (b) 46.9 fJ, 28.6%

Chapter 29
Atomic Physics

CHAPTER OUTLINE

ANSWERS TO QUESTIONS

Q29.1 Bohr modeled the electron as moving in a perfect circle, with zero uncertainty in its radial coordinate. Then its radial velocity is always zero with zero uncertainty. Bohr's theory violates the uncertainty principle by making the uncertainty product $\Delta r\Delta p_r$ be zero, less than the minimum allowable $\frac{\hbar}{2}$.

Q29.2 Fundamentally, three quantum numbers describe an orbital wave function because we live in three-dimensional space. They arise mathematically from boundary conditions on the wave function, expressed as a product of a function of r, a function of θ, and a function of ϕ.

Q29.3 Bohr's theory pictures the electron as moving in a flat circle like a classical particle described by $\sum F = ma$. Schrödinger's theory pictures the electron as a cloud of probability amplitude in the three-dimensional space around the hydrogen nucleus, with its motion described by a wave equation. In the Bohr model, the ground-state angular momentum is $1\hbar$; in the Schrödinger model the ground-state angular momentum is zero. Both models predict that the electron's energy is limited to discrete energy levels, given by $\frac{-13.606\text{ eV}}{n^2}$ with $n = 1, 2, 3$.

Q29.4 The term *electron cloud* refers to the unpredictable location of an electron around an atomic nucleus. It is a cloud of probability amplitude. An electron in an s subshell has a spherically symmetric probability distribution. Electrons in p, d, and f subshells have directionality to their distribution. The shape of these electron clouds influences how atoms form molecules and chemical compounds.

Q29.5 The direction of the magnetic moment due to an orbiting charge is given by the right hand rule, but assumes a *positive* charge. Since the electron is negatively charged, its magnetic moment is in the opposite direction to its angular momentum.

Q29.6 The deflecting force on an atom with a magnetic moment is proportional to the *gradient* of the magnetic field. Thus, atoms with oppositely directed magnetic moments would be deflected in *opposite* directions in an inhomogeneous magnetic field.

Q29.7 Practically speaking, no. Ions have a net charge and the magnetic force $q(\vec{\mathbf{v}}\times\vec{\mathbf{B}})$ would deflect the beam, making it difficult to separate the atoms with different orientations of magnetic moments.

Q29.8 The Stern-Gerlach experiment with hydrogen atoms shows that the component of an electron's spin angular momentum along an applied magnetic field can have only one of two allowed values. So does electron spin resonance on atoms with one unpaired electron.

Q29.9 If the exclusion principle were not valid, the elements and their chemical behavior would be grossly different because every electron would end up in the lowest energy level of the atom. All matter would be nearly alike in its chemistry and composition, since the shell structures of all elements would be identical. Most materials would have a much higher density. The spectra of atoms and molecules would be very simple, and there would be very little color in the world.

Q29.10 An atom is a quantum system described by a wave function. The electric force of attraction to the nucleus imposes a constraint on the electrons. The physical constraint implies mathematical boundary conditions on the wave functions, with consequent quantization so that only certain wave functions are allowed to exist. The Schrödinger equation assigns a definite energy to each allowed wave function. Each wave function is spread out in space, describing an electron with no definite position. If you like analogies, think of a classical standing wave on a string fixed at both ends. Its position is spread out to fill the whole string, but its frequency is one of a certain set of quantized values.

Q29.11 The three elements have similar electronic configurations. Each has filled inner shells plus one electron in an *s* orbital. Their single outer electrons largely determine their chemical interactions with other atoms.

Q29.12 In a neutral helium atom, one electron can be modeled as moving in an electric field created by the nucleus and the other electron. According to Gauss's law, if the electron is above the ground state it moves in the electric field of a net charge of $+2e-1e=+1e$. We say the nuclear charge is *screened* by the inner electron. The electron in a He^+ ion moves in the field of the unscreened nuclear charge of 2 protons. Then the potential energy function for the electron is about double that of one electron in the neutral atom.

Q29.13 At low density, the gas consists of essentially separate atoms. As the density increases, the atoms interact with each other. This has the effect of giving different atoms levels at slightly different energies, at any one instant. The collection of atoms can then emit photons in lines or bands, narrower or wider, depending on the density.

Q29.14 Each of the electrons must have at least one quantum number different from the quantum numbers of each of the other electrons. They can differ (in m_s) by being spin-up or spin-down. They can also differ (in ℓ) in angular momentum and in the general shape of the wave function. Those electrons with $\ell=1$ can differ (in m_ℓ) in orientation of angular momentum—look at Figure Q29.14.

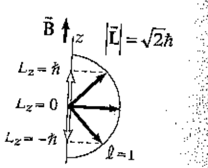

FIG. Q29.14

Q29.15 The Mosely graph shows that the reciprocal square root of the wavelength of K_α characteristic x-rays is a linear function of atomic number. Then measuring this wavelength for a new chemical element reveals its location on the graph, including its atomic number.

Q29.16 (a) The terms "I define" and "this part of the universe" seem vague, in contrast to the precision of the rest of the statement. But the statement is true in the sense of being experimentally verifiable. The way to test the orientation of the magnetic moment of an electron is to apply a magnetic field to it. When that is done for any electron, it has precisely a 50% chance of being either spin-up or spin-down. Its spin magnetic moment vector must make one of two allowed angles with the applied magnetic field. They are given by $\cos\theta=\frac{S_z}{S}=\frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}}$ and $\cos\theta=\frac{-\frac{1}{2}}{\frac{\sqrt{3}}{2}}$. You can calculate as many digits of the two angles allowed by "space quantization" as you wish.

(b) This statement may be true. There is no reason to suppose that an ant can comprehend the cosmos, and no reason to suppose that a human can comprehend all of it. Our experience with macroscopic objects does not prepare us to understand quantum particles. On the other hand, what seems strange to us now may be the common knowledge of tomorrow. Looking back at the past 150 years of physics, great strides in understanding the Universe—from the quantum to the galactic scale—have been made. Think of trying to explain the photoelectric effect using Newtonian mechanics. What seems strange sometimes just has an underlying structure that has not yet been described fully. On the other hand still, it has been demonstrated that a "hidden-variable" theory, that would model quantum uncertainty as caused by some determinate but fluctuating quantity, cannot agree with experiment.

Q29.17 People commonly say that science is difficult to learn. Many other branches of knowledge are constructed by humans. Learning them can require getting your mind to follow the path of someone else's mind. A well-developed science does not mirror human patterns of thought, but the way nature works. Thus we agree with the view stated in the problem. A scientific discovery can be like a garbled communication in the sense that it does not explain itself. It can seem fragmentary and in need of context. It can seem unfriendly in the sense that it has no regard for ease of human comprehension. Evolution has adapted the human mind for catching rabbits and outwitting bison and leopards, but not necessarily for understanding atoms. Education in science gives the student opportunities for understanding a wide variety of ideas; for evaluating new information; and for recognizing that extraordinary evidence must be adduced for extraordinary claims.

SOLUTIONS TO PROBLEMS

Section 29.1 Early Structural Models of the Atom

P29.1 (a) For a classical atom, the centripetal acceleration is

$$a=\frac{v^2}{r}=\frac{1}{4\pi\epsilon_0}\frac{e^2}{r^2m_e}$$

$$E=-\frac{e^2}{4\pi\epsilon_0 r}+\frac{m_e v^2}{2}=-\frac{e^2}{8\pi\epsilon_0 r}$$

so $$\frac{dE}{dt}=\frac{e^2}{8\pi\epsilon_0 r^2}\frac{dr}{dt}=\frac{-1}{6\pi\epsilon_0}\frac{e^2a^2}{c^3}=\frac{-e^2}{6\pi\epsilon_0 c^3}\left(\frac{e^2}{4\pi\epsilon_0 r^2 m_e}\right)^2.$$

Therefore, $$\frac{dr}{dt}=-\frac{e^4}{12\pi^2\epsilon_0^2 r^2 m_e^2 c^3}.$$

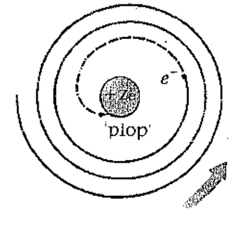

FIG. P29.1

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(b) $$-\int_{2.00\times10^{-10}\text{ m}}^{0} 12\pi^2 \epsilon_0^2 r^2 m_e^2 c^3\, dr = e^4 \int_0^T dt \qquad \frac{12\pi^2 \epsilon_0^2 m_e^2 c^3}{e^4}\frac{r^3}{3}\bigg|_0^{2.00\times10^{-10}} = T = \boxed{8.46\times10^{-10}\text{ s}}$$

Since atoms last a lot longer than 0.8 ns, the classical laws (fortunately!) do not hold for systems of atomic size.

P29.2 (a) The point of closest approach is found when

$$E = K + U = 0 + \frac{k_e q_\alpha q_{Au}}{r}$$

or $$r_{min} = \frac{k_e(2e)(79e)}{E}$$

$$r_{min} = \frac{(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(158)(1.60\times10^{-19}\text{ C})^2}{(4.00\text{ MeV})(1.60\times10^{-13}\text{ J/MeV})} = \boxed{5.68\times10^{-14}\text{ m}}.$$

(b) The maximum force exerted on the alpha particle is

$$F_{max} = \frac{k_e q_\alpha q_{Au}}{r_{min}^2} = \frac{(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(158)(1.60\times10^{-19}\text{ C})^2}{(5.68\times10^{-14}\text{ m})^2} = \boxed{11.3\text{ N}}$$ away from the nucleus.

***P29.3** (a) The speed of the moon in its orbit is $v = \frac{2\pi r}{T} = \frac{2\pi(3.84\times10^8\text{ m})}{2.36\times10^6\text{ s}} = 1.02\times10^3\text{ m/s}$.

So, $$L = mvr = (7.36\times10^{22}\text{ kg})(1.02\times10^3\text{ m/s})(3.84\times10^8\text{ m}) = \boxed{2.89\times10^{34}\text{ kg}\cdot\text{m}^2/\text{s}}$$

(b) We have $L = n\hbar$

or $$n = \frac{L}{\hbar} = \frac{2.89\times10^{34}\text{ kg}\cdot\text{m}^2/\text{s}}{1.055\times10^{-34}\text{ J}\cdot\text{s}} = \boxed{2.74\times10^{68}}.$$

(c) We have $n\hbar = L = mvr = m\left(\frac{GM_e}{r}\right)^{1/2} r$,

so $$r = \frac{\hbar^2}{m^2 GM_e} n^2 = Rn^2 \text{ and } \frac{\Delta r}{r} = \frac{(n+1)^2 R - n^2 R}{n^2 R} = \frac{2n+1}{n^2}$$

which is approximately equal to $\frac{2}{n} = \boxed{7.30\times10^{-69}}$.

***P29.4** (a) The fifth excited state must lie above the second excited state by the photon energy

$$E_{52} = hf = \frac{hc}{\lambda} = \frac{6.63\times10^{-34}\text{ J}\cdot\text{s}\; 3\times10^8\text{ m/s}}{520\times10^{-9}\text{ m}} = 3.82\times10^{-19}\text{ J}.$$

The sixth excited state exceeds the second in energy by

$$E_{62} = \frac{6.63\times10^{-34}\text{ J}\cdot\text{s}\; 3\times10^8\text{ m/s}}{410\times10^{-9}\text{ m}} = 4.85\times10^{-19}\text{ J}.$$

Then the sixth excited state is above the fifth by $(4.85-3.82)10^{-19}\text{ J} = 1.03\times10^{-19}\text{ J}$. In the 6 to 5 transition the atom emits a photon with the infrared wave length

$$\lambda = \frac{hc}{E_{62}} = \frac{6.63\times10^{-34}\text{ J}\cdot\text{s}\; 3\times10^8\text{ m/s}}{1.03\times10^{-19}\text{ J}} = \boxed{1.94\times10^{-6}\text{ m}}.$$

(b) The same steps solve the symbolic problem.

$$E_{CA} = E_{CB} + E_{BA}$$
$$\frac{hc}{\lambda_{CA}} = \frac{hc}{\lambda_{CB}} + \frac{hc}{\lambda_{BA}}$$
$$\frac{1}{\lambda_{CB}} = \frac{1}{\lambda_{CA}} - \frac{1}{\lambda_{BA}}$$
$$\boxed{\lambda_{CB} = \left(\frac{1}{\lambda_{CA}} - \frac{1}{\lambda_{BA}}\right)^{-1}}$$

Section 29.2 The Hydrogen Atom Revisited

P29.5 (a) Longest wavelength implies lowest frequency and smallest energy:

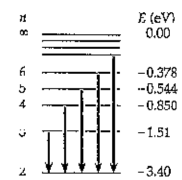
the atom falls from $n = 3$

to $n = 2$

losing energy $-\frac{13.6\text{ eV}}{3^2} + \frac{13.6\text{ eV}}{2^2} = \boxed{1.89\text{ eV}}$

The photon frequency is $f = \frac{\Delta E}{h}$

and its wavelength is



Balmer Series

FIG. P29.5

$$\lambda = \frac{c}{f} = \frac{hc}{\Delta E} = \frac{(6.626\times10^{-34}\text{ J}\cdot\text{s})(3.00\times10^8\text{ m/s})}{(1.89\text{ eV})}\left(\frac{\text{eV}}{1.60\times10^{-19}\text{ J}}\right)$$
$$\lambda = \boxed{656\text{ nm}}$$

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(b) The biggest energy loss is for an atom to fall from an ionized configuration,

$n=\infty$

to the $n=2$ state.

It loses energy $-\dfrac{13.6\text{ eV}}{\infty}+\dfrac{13.6\text{ eV}}{2^2}=\boxed{3.40\text{ eV}}$

to emit light of wavelength $\lambda=\dfrac{hc}{\Delta E}=\dfrac{(6.626\times10^{-34}\text{ J}\cdot\text{s})(3.00\times10^8\text{ m/s})}{(3.40\text{ eV})(1.60\times10^{-19}\text{ J/eV})}=\boxed{365\text{ nm}}$.

P29.6 (a) The photon has energy 2.28 eV.

And $\dfrac{13.6\text{ eV}}{2^2}=3.40$ eV is required to ionize a hydrogen atom from state $n=2$. So while the photon cannot ionize a hydrogen atom pre-excited to $n=2$, it can ionize a hydrogen atom in the $n=\boxed{3}$ state, with energy $-\dfrac{13.6\text{ eV}}{3^2}=-1.51$ eV.

(b) The electron thus freed can have kinetic energy $K_e=2.28\text{ eV}-1.51\text{ eV}=0.769\text{ eV}=\dfrac{1}{2}m_e v^2$

Therefore, $v=\sqrt{\dfrac{2(0.769)(1.60\times10^{-19})\text{ J}}{9.11\times10^{-31}\text{ kg}}}=\boxed{520\text{ km/s}}$.

P29.7 The reduced mass of positronium is **less** than hydrogen, so the photon energy will be **less** for positronium than for hydrogen. This means that the wavelength of the emitted photon will be **longer** than 656.3 nm. On the other hand, helium has about the same reduced mass but more charge than hydrogen, so its transition energy will be **larger**, corresponding to a wavelength **shorter** than 656.3 nm.

All the factors in the given equation are constant for this problem except for the reduced mass and the nuclear charge. Therefore, the wavelength corresponding to the energy difference for the transition can be found simply from the ratio of mass and charge variables.

For hydrogen, $\mu=\dfrac{m_p m_e}{m_p+m_e}\approx m_e$. The photon energy is $\Delta E=E_3-E_2$.

Its wavelength is $\lambda=656.3$ nm, where $\lambda=\dfrac{c}{f}=\dfrac{hc}{\Delta E}$

(a) For positronium, $\mu=\dfrac{m_e m_e}{m_e+m_e}=\dfrac{m_e}{2}$

so the energy of each level is one half as large as in hydrogen, which we could call "protonium". The photon energy is inversely proportional to its wavelength, so for positronium,

$\lambda_{32}=2(656.3\text{ nm})=\boxed{1.31\ \mu\text{m}}$ (in the infrared region).

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(b) For He^+, $\mu\approx m_e$, $q_1=e$, and $q_2=2e$,

so the transition energy is $2^2=4$ times larger than hydrogen.

Then, $\lambda_{32}=\left(\dfrac{656}{4}\right)\text{nm}=\boxed{164\text{ nm}}$ (in the ultraviolet region).

P29.8 (a) For a particular transition from n_i to n_f, $\Delta E_H=-\dfrac{\mu_H k_e^2 e^4}{2\hbar^2}\left(\dfrac{1}{n_f^2}-\dfrac{1}{n_i^2}\right)=\dfrac{hc}{\lambda_H}$ and

$\Delta E_D=-\dfrac{\mu_D k_e^2 e^4}{2\hbar^2}\left(\dfrac{1}{n_f^2}-\dfrac{1}{n_i^2}\right)=\dfrac{hc}{\lambda_D}$ where $\mu_H=\dfrac{m_e m_p}{m_e+m_p}$ and $\mu_D=\dfrac{m_e m_D}{m_e+m_D}$. By division,

$\dfrac{\Delta E_H}{\Delta E_D}=\dfrac{\mu_H}{\mu_D}=\dfrac{\lambda_D}{\lambda_H}$ or $\lambda_D=\left(\dfrac{\mu_H}{\mu_D}\right)\lambda_H$. Then, $\boxed{\lambda_H-\lambda_D=\left(1-\dfrac{\mu_H}{\mu_D}\right)\lambda_H}$

(b) $\dfrac{\mu_H}{\mu_D}=\left(\dfrac{m_e m_p}{m_e+m_p}\right)\left(\dfrac{m_e+m_D}{m_e m_D}\right)=\dfrac{(1.007\ 276\text{ u})(0.000\ 549\text{ u}+2.013\ 553\text{ u})}{(0.000\ 549\text{ u}+1.007\ 276\text{ u})(2.013\ 553\text{ u})}=0.999\ 728$

$\lambda_H-\lambda_D=(1-0.999\ 728)(656.3\text{ nm})=\boxed{0.179\text{ nm}}$

***P29.9** (a) $\Delta x\Delta p\ge\dfrac{\hbar}{2}$ so if $\Delta x=r$, $\Delta p\ge\boxed{\dfrac{\hbar}{2r}}$.

(b) Arbitrarily choosing $\Delta p\approx\dfrac{\hbar}{r}$, we find $K=\dfrac{p^2}{2m_e}\approx\dfrac{(\Delta p)^2}{2m_e}=\boxed{\dfrac{\hbar^2}{2m_e r^2}}$

$U=\dfrac{-k_e e^2}{r}$, so $E=K+U\approx\boxed{\dfrac{\hbar^2}{2m_e r^2}-\dfrac{k_e e^2}{r}}$

(c) To minimize E,

$\dfrac{dE}{dr}=-\dfrac{\hbar^2}{m_e r^3}+\dfrac{k_e e^2}{r^2}=0\to r=\boxed{\dfrac{\hbar^2}{m_e k_e e^2}=a_0}$ (the Bohr radius).

Then, $E=\dfrac{\hbar^2}{2m_e}\left(\dfrac{m_e k_e e^2}{\hbar^2}\right)^2-k_e e^2\left(\dfrac{m_e k_e e^2}{\hbar^2}\right)=-\dfrac{m_e k_e^2 e^4}{2\hbar^2}=\boxed{-13.6\text{ eV}}$.

Section 29.3 The Wave Functions for Hydrogen

P29.10 $\psi_{1s}(r)=\frac{1}{\sqrt{\pi a_0^3}}e^{-r/a_0}$ (Eq. 29.3)

$P_{1s}(r)=\frac{4r^2}{a_0^3}e^{-2r/a_0}$ (Eq. 29.7)

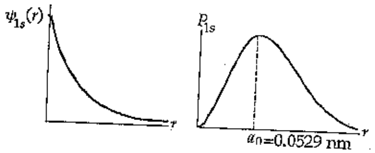

FIG. P29.10

P29.11 (a) $\int|\psi|^2 dV=4\pi\int_0^\infty|\psi|^2 r^2 dr=4\pi\left(\frac{1}{\pi a_0^3}\right)\int_0^\infty r^2 e^{-2r/a_0}dr$

Using integral tables, $\int|\psi|^2 dV=-\frac{2}{a_0^2}\left[e^{-2r/a_0}\left(r^2+a_0 r+\frac{a_0^2}{2}\right)\right]_0^\infty=\left(-\frac{2}{a_0^2}\right)\left(-\frac{a_0^2}{2}\right)=\boxed{1}$

so the wave function as given is normalized.

(b) $P_{a_0/2\to 3a_0/2}=4\pi\int_{a_0/2}^{3a_0/2}|\psi|^2 r^2 dr=4\pi\left(\frac{1}{\pi a_0^3}\right)\int_{a_0/2}^{3a_0/2}r^2 e^{-2r/a_0}dr$

Again, using integral tables,

$$P_{a_0/2\to 3a_0/2}=-\frac{2}{a_0^2}\left[e^{-2r/a_0}\left(r^2+a_0 r+\frac{a_0^2}{2}\right)\right]_{a_0/2}^{3a_0/2}=-\frac{2}{a_0^2}\left[e^{-3}\left(\frac{17a_0^2}{4}\right)-e^{-1}\left(\frac{5a_0^2}{4}\right)\right]=\boxed{0.497}.$$

P29.12 $\psi=\frac{1}{\sqrt{3}}\frac{1}{(2a_0)^{3/2}}\frac{r}{a_0}e^{-r/2a_0}$

so $P_r=4\pi r^2|\psi^2|=4\pi r^2\frac{r^2}{24a_0^5}e^{-r/a_0}$

Set $\frac{dP}{dr}=\frac{4\pi}{24a_0^5}\left[4r^3 e^{-r/a_0}+r^4\left(-\frac{1}{a_0}\right)e^{-r/a_0}\right]=0.$

Solving for r, this is a maximum at $\boxed{r=4a_0}$

P29.13 $\psi=\frac{1}{\sqrt{\pi a_0^3}}e^{-r/a_0}$ $\quad\frac{2}{r}\frac{d\psi}{dr}=\frac{-2}{r\sqrt{\pi a_0^5}}e^{-r/a_0}=-\frac{2}{ra_0}\psi$

$\frac{d^2\psi}{dr^2}=\frac{1}{\sqrt{\pi a_0^7}}e^{-r/a_0}=\frac{1}{a_0^2}\psi$ $\quad-\frac{\hbar^2}{2m_e}\left(\frac{1}{a_0^2}-\frac{2}{ra_0}\right)\psi-\frac{e^2}{4\pi\in_0 r}\psi=E\psi$

But $a_0=\frac{\hbar^2(4\pi\in_0)}{m_e e^2}$

so $-\frac{e^2}{8\pi\in_0 a_0}=E$

or $\boxed{E=-\frac{k_e e^2}{2a_0}}$

This is true, so the Schrödinger equation is satisfied.

P29.14 The hydrogen ground-state radial probability density is

$P(r)=4\pi r^2|\psi_{1s}|^2=\frac{4r^2}{a_0^3}\exp\left(-\frac{2r}{a_0}\right).$

The number of observations at $2a_0$ is, by proportion

$$N=1\,000\frac{P(2a_0)}{P\left(\frac{a_0}{2}\right)}=1\,000\frac{(2a_0)^2}{\left(\frac{a_0}{2}\right)^2}\frac{e^{-4a_0/a_0}}{e^{-a_0/a_0}}=1\,000(16)e^{-3}=\boxed{797\text{ times}}.$$

Section 29.4 Physical Interpretation of the Quantum Numbers

Note: Problems 22.20 and 22.17 in Chapter 22 can be assigned with Section 29.4.

P29.15 (a) In the $3d$ subshell, $n=3$ and $\ell=2$,

we have

n	3	3	3	3	3	3	3	3	3	3
ℓ	2	2	2	2	2	2	2	2	2	2
m_ℓ	+2	+2	+1	+1	0	0	−1	−1	−2	−2
m_s	+1/2	−1/2	+1/2	−1/2	+1/2	−1/2	+1/2	−1/2	+1/2	−1/2

(A total of 10 states)

(b) In the $3p$ subshell, $n=3$ and $\ell=1$,

we have

n	3	3	3	3	3	3
ℓ	1	1	1	1	1	1
m_ℓ	+1	+1	+0	+0	−1	−1
m_s	+1/2	−1/2	+1/2	−1/2	+1/2	−1/2

(A total of 6 states)

P29.16 (a) For the d state, $\ell = 2$, $L = \boxed{\sqrt{6}\hbar} = 2.58 \times 10^{-34}$ J·s.

(b) For the f state, $\ell = 3$, $L = \sqrt{\ell(\ell+1)}\hbar = \boxed{\sqrt{12}\hbar} = 3.65 \times 10^{-34}$ J·s.

P29.17 $L = \sqrt{\ell(\ell+1)}\hbar$:

$$4.714 \times 10^{-34} = \sqrt{\ell(\ell+1)}\left(\frac{6.626 \times 10^{-34}}{2\pi}\right)$$

$$\ell(\ell+1) = \frac{\left(4.714 \times 10^{-34}\right)^2 (2\pi)^2}{\left(6.626 \times 10^{-34}\right)^2} = 1.998 \times 10^1 \approx 20 = 4(4+1)$$

so $\boxed{\ell = 4}$.

P29.18 The 5th excited state has $n = 6$, energy $\dfrac{-13.6 \text{ eV}}{36} = -0.378$ eV.

The atom loses this much energy: $\dfrac{hc}{\lambda} = \dfrac{\left(6.626 \times 10^{-34} \text{ J·s}\right)\left(3.00 \times 10^8 \text{ m/s}\right)}{\left(1\,090 \times 10^{-9} \text{ m}\right)\left(1.60 \times 10^{-19} \text{ J/eV}\right)} = 1.14$ eV

to end up with energy $-0.378 \text{ eV} - 1.14 \text{ eV} = -1.52$ eV

which is the energy in state 3: $-\dfrac{13.6 \text{ eV}}{3^3} \approx -1.51$ eV.

While $n = 3$, ℓ can be as large as 2, giving angular momentum $\sqrt{\ell(\ell+1)}\hbar = \boxed{\sqrt{6}\hbar}$.

P29.19 (a) $n = 1$: For $n = 1$, $\ell = 0$, $m_\ell = 0$, $m_s = \pm\dfrac{1}{2}$

n	ℓ	m_ℓ	m_s
1	0	0	−1/2
1	0	0	+1/2

Yields 2 sets; $2n^2 = 2(1)^2 = \boxed{2}$

(b) $n = 2$: For $n = 2$.

we have

n	ℓ	m_ℓ	m_s
2	0	0	±1/2
2	1	−1	±1/2
2	1	0	±1/2
2	1	1	±1/2

yields 8 sets; $2n^2 = 2(2)^2 = \boxed{8}$

continued on next page

Note that the number is twice the number of m_ℓ values. Also, for each ℓ there are $(2\ell+1)$ different m_ℓ values. Finally, ℓ can take on values ranging from 0 to $n-1$.

So the general expression is number $= \sum_0^{n-1} 2(2\ell+1)$.

The series is an arithmetic progression: $2 + 6 + 10 + 14 \ldots$

the sum of which is number $= \dfrac{n}{2}\left[2a + (n-1)d\right]$

where $a = 2$, $d = 4$: number $= \dfrac{n}{2}\left[4 + (n-1)4\right] = 2n^2$

(c) $n = 3$: $2(1) + 2(3) + 2(5) = 2 + 6 + 10 = 18$ $\quad 2n^2 = 2(3)^2 = \boxed{18}$

(d) $n = 4$: $2(1) + 2(3) + 2(5) + 2(7) = 32$ $\quad 2n^2 = 2(4)^2 = \boxed{32}$

(e) $n = 5$: $32 + 2(9) = 32 + 18 = 50$ $\quad 2n^2 = 2(5)^2 = \boxed{50}$

P29.20 For a $3d$ state, $n = 3$ and $\ell = 2$.

Therefore, $L = \sqrt{\ell(\ell+1)}\hbar = \boxed{\sqrt{6}\hbar} = 2.58 \times 10^{-34}$ J·s

m_ℓ can have the values $-2, -1, 0, 1$, and 2

so $\boxed{L_z \text{ can have the values } -2\hbar,\ -\hbar,\ 0,\ \hbar \text{ and } 2\hbar}$.

Using the relation $\cos\theta = \dfrac{L_z}{L}$

we find the possible values of θ $\boxed{145°,\ 114°,\ 90.0°,\ 65.9°, \text{ and } 35.3°}$.

P29.21 (a) Density of a proton: $\rho = \dfrac{m}{V} = \dfrac{1.67 \times 10^{-27} \text{ kg}}{(4/3)\pi\left(1.00 \times 10^{-15} \text{ m}\right)^3} = \boxed{3.99 \times 10^{17} \text{ kg/m}^3}$.

(b) Size of model electron: $r = \left(\dfrac{3m}{4\pi\rho}\right)^{1/3} = \left(\dfrac{3\left(9.11 \times 10^{-31} \text{ kg}\right)}{4\pi\left(3.99 \times 10^{17} \text{ kg/m}^3\right)}\right)^{1/3} = \boxed{8.17 \times 10^{-17} \text{ m}}$

continued on next page

(c) Moment of inertia: $I=\frac{2}{5}mr^2=\frac{2}{5}\left(9.11\times10^{-31}\text{ kg}\right)\left(8.17\times10^{-17}\text{ m}\right)^2=2.43\times10^{-63}\text{ kg}\cdot\text{m}^2$

$$L_z=I\omega=\frac{\hbar}{2}=\frac{Iv}{r}$$

Therefore, $v=\frac{\hbar r}{2I}=\frac{\left(6.626\times10^{-34}\text{ J}\cdot\text{s}\right)\left(8.17\times10^{-17}\text{ m}\right)}{2\pi\left(2\times2.43\times10^{-63}\text{ kg}\cdot\text{m}^2\right)}=\boxed{1.77\times10^{12}\text{ m/s}}$.

(d) This is $\boxed{5.91\times10^3 \text{ times larger}}$ than the speed of light.

P29.22 In the N shell, $n=4$. For $n=4$, ℓ can take on values of 0, 1, 2, and 3. For each value of ℓ, m_ℓ can be $-\ell$ to ℓ in integral steps. Thus, the maximum value for m_ℓ is 3. Since $L_z=m_\ell\hbar$, the maximum value for L_z is $L_z=\boxed{3\hbar}$.

P29.23 The 3*d* subshell has $\ell=2$, and $n=3$. Also, we have $s=1$.

Therefore, we can have $\boxed{n=3,\ \ell=2;\ m_\ell=-2,\ -1,\ 0,\ 1,\ 2;\ s=1;\ \text{and } m_s=-1,\ 0,\ 1}$

leading to the following table:

n	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3
ℓ	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
m_ℓ	–2	–2	–2	–1	–1	–1	0	0	0	1	1	1	2	2	2
s	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
m_s	–1	0	1	–1	0	1	–1	0	1	–1	0	1	–1	0	1

Section 29.5 The Exclusion Principle and the Periodic Table

P29.24 (a) $\boxed{1s^2 2s^2 2p^4}$

(b) For the 1*s* electrons, $n=1,\ \ell=0,\ m_\ell=0,\quad m_s=+\frac{1}{2}$ and $-\frac{1}{2}$.

For the two 2*s* electrons, $n=2,\ \ell=0,\ m_\ell=0,\quad m_s=+\frac{1}{2}$ and $-\frac{1}{2}$

For the four 2*p* electrons, $n=2;\ \ell=1;\ m_\ell=-1,0,\text{ or }1;$ and $m_s=+\frac{1}{2}$ or $-\frac{1}{2}$

P29.25 The $\boxed{4s \text{ subshell fills first}}$, for potassium and calcium, before the 3*d* subshell starts to fill for scandium through zinc. Thus, we would first suppose that $[\text{Ar}]3d^4 4s^2$ would have lower energy than $[\text{Ar}]3d^5 4s^1$. But the latter has more unpaired spins, six instead of four, and Hund's rule suggests that this could give the latter configuration lower energy. In fact it must, for $[\text{Ar}]3d^5 4s^1$ is the ground state for chromium.

P29.26

Electronic configuration:			Sodium to Argon
$[1s^2 2s^2 2p^6]$	$+3s^1$	→	Na^{11}
	$+3s^2$	→	Mg^{12}
	$+3s^2 3p^1$	→	Al^{13}
	$+3s^2 3p^2$	→	Si^{14}
	$+3s^2 3p^3$	→	P^{15}
	$+3s^2 3p^4$	→	S^{16}
	$+3s^2 3p^5$	→	Cl^{17}
	$+3s^2 3p^6$	→	Ar^{18}
$[1s^2 2s^2 2p^6 3s^2 3p^6]4s^1$		→	K^{19}

P29.27 In the table of electronic configurations in the text, or on a periodic table, we look for the element whose last electron is in a 3*p* state and which has three electrons outside a closed shell. Its electron configuration then ends in $3s^2 3p^1$. The element is $\boxed{\text{aluminum}}$.

P29.28 (a) For electron one and also for electron two, $n=3$ and $\ell=1$. The possible states are listed here in columns giving the other quantum numbers:

electron one	m_ℓ	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0
	m_s	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
electron two	m_ℓ	1	0	0	–1	–1	1	0	0	–1	–1	1	1	0	–1	–1
	m_s	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$

electron one	m_ℓ	0	0	0	0	0	–1	–1	–1	–1	–1	–1	–1	–1	–1	–1
	m_s	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$
electron two	m_ℓ	1	1	0	–1	–1	1	1	0	0	–1	1	1	0	0	–1
	m_s	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$

There are thirty allowed states, since electron one can have any of three possible values for m_ℓ for both spin up and spin down, amounting to six states, and the second electron can have any of the other five states.

(b) Were it not for the exclusion principle, there would be $\boxed{36}$ possible states, six for each electron independently.

P29.29 (a)

$n+\ell$	1	2	3	4	5	6	7
subshell	$1s$	$2s$	$2p, 3s$	$3p, 4s$	$3d, 4p, 5s$	$4d, 5p, 6s$	$4f, 5d, 6p, 7s$

(b) $Z=15$:

Filled subshells: $1s, 2s, 2p, 3s$ (12 electrons)

Valence subshell: 3 electrons in $3p$ subshell

Prediction: Valence $=+5$ or -3

Element is phosphorus, Valence $=+5$ or -3 (Prediction correct)

$Z=47$:

Filled subshells: $1s, 2s, 2p, 3s, 3p, 4s, 3d, 4p, 5s$ (38 electrons)

Outer subshell: 9 electrons in $4d$ subshell

Prediction: Valence $=-1$

Element is silver, (Prediction fails) Valence is $+1$

$Z=86$:

Filled subshells: $1s, 2s, 2p, 3s, 3p, 4s, 3d, 4p, 5s, 4d, 5p, 6s, 4f, 5d, 6p$ (86 electrons)

Prediction Outer subshell is full: inert gas

Element is radon, inert (Prediction correct)

P29.30 Listing subshells in the order of filling, we have for element 110,

$1s^2 2s^2 2p^6 3s^2 3p^6 4s^2 3d^{10} 4p^6 5s^2 4d^{10} 5p^6 6s^2 4f^{14} 5d^{10} 6p^6 7s^2 5f^{14} 6d^8$.

In order of increasing principal quantum number, this is

$\boxed{1s^2 2s^2 2p^6 3s^2 3p^6 3d^{10} 4s^2 4p^6 4d^{10} 4f^{14} 5s^2 5p^6 5d^{10} 5f^{14} 6s^2 6p^6 6d^8 7s^2}$.

P29.31 In the ground state of sodium, the outermost electron is in an s state. This state is spherically symmetric, so it generates no magnetic field by orbital motion, and has the same energy no matter whether the electron is spin-up or spin-down. The energies of the states $3p\uparrow$ and $3p\downarrow$ above $3s$ are $hf_1=\frac{hc}{\lambda}$ and $hf_2=\frac{hc}{\lambda_2}$

The energy difference is

$$2\mu_B B = hc\left(\frac{1}{\lambda_1}-\frac{1}{\lambda_2}\right)$$

continued on next page

so $$B=\frac{hc}{2\mu_B}\left(\frac{1}{\lambda_1}-\frac{1}{\lambda_2}\right)=\frac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})(3\times10^8\ \text{m/s})}{2(9.27\times10^{-24}\ \text{J/T})}\left(\frac{1}{588.995\times10^{-9}\ \text{m}}-\frac{1}{589.592\times10^{-9}\ \text{m}}\right)$$

$B=\boxed{18.4\ \text{T}}$.

Section 29.6 More on Atomic Spectra: Visible and X-Ray

P29.32 (a) $\boxed{n=3,\ \ell=0,\ m_\ell=0}$

$\boxed{n=3,\ \ell=1,\ m_\ell=-1,0,1}$

For $\boxed{n=3,\ \ell=2,\ m_\ell=-2,-1,0,1,2}$

(b) ψ_{300} corresponds to $E_{300}=-\frac{Z^2E_0}{n^2}=-\frac{2^2(13.6)}{3^2}=\boxed{-6.05\ \text{eV}}$.

ψ_{31-1}, ψ_{310}, ψ_{311} have the same energy since n is the same.

ψ_{32-2}, ψ_{32-1}, ψ_{320}, ψ_{321}, ψ_{322} have the same energy since n is the same.

All states are degenerate.

P29.33 $E=\frac{hc}{\lambda}=e\Delta V$ $\quad \frac{(6.626\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^8\ \text{m/s})}{(10.0\times10^{-9}\ \text{m})}=(1.60\times10^{-19})\Delta V$

$\Delta V=\boxed{124\ \text{V}}$

P29.34 Some electrons can give all their kinetic energy $K_e=e\Delta V$ to the creation of a single photon of x-radiation, with

$hf=\frac{hc}{\lambda}=e\Delta V$

$$\lambda=\frac{hc}{e\Delta V}=\frac{(6.6261\times10^{-34}\ \text{J}\cdot\text{s})(2.9979\times10^8\ \text{m/s})}{(1.6022\times10^{-19}\ \text{C})\Delta V}=\boxed{\frac{1240\ \text{nm}\cdot\text{V}}{\Delta V}}$$

P29.35 Following Example 29.8 $\quad E_\gamma=\frac{3}{4}(42-1)^2(13.6\ \text{eV})=1.71\times10^4\ \text{eV}=2.74\times10^{-15}\ \text{J}$

$f=4.14\times10^{18}\ \text{Hz}$

and $\quad \lambda=\boxed{0.0725\ \text{nm}}$.

P29.36 $E = \frac{hc}{\lambda} = \frac{1\,240\text{ eV}\cdot\text{nm}}{\lambda} = \frac{1.240\text{ keV}\cdot\text{nm}}{\lambda}$

For $\lambda_1 = 0.018\,5\text{ nm}$, $E = 67.03\text{ keV}$

$\lambda_2 = 0.020\,9\text{ nm}$, $E = 59.3\text{ keV}$

$\lambda_3 = 0.021\,5\text{ nm}$, $E = 57.7\text{ keV}$

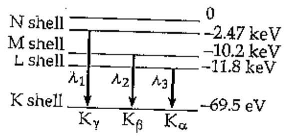

FIG. P29.36

The ionization energy for the K shell is 69.5 keV, so the ionization energies for the other shells are:

$\boxed{\text{L shell} = 11.8\text{ keV}}$ $\boxed{\text{M shell} = 10.2\text{ keV}}$ $\boxed{\text{N shell} = 2.47\text{ keV}}$.

P29.37 The K_β x-rays are emitted when there is a vacancy in the ($n=1$) K shell and an electron from the ($n=3$) M shell falls down to fill it. Then this electron is shielded by nine electrons originally and by one in its final state.

$$\frac{hc}{\lambda} = -\frac{13.6(Z-9)^2}{3^2}\text{ eV} + \frac{13.6(Z-1)^2}{1^2}\text{ eV}$$

$$\frac{(6.626\times10^{-34}\text{ J}\cdot\text{s})(3.00\times10^8\text{ m/s})}{(0.152\times10^{-9}\text{ m})(1.60\times10^{-19}\text{ J/eV})} = (13.6\text{ eV})\left(-\frac{Z^2}{9} + \frac{18Z}{9} - \frac{81}{9} + Z^2 - 2Z + 1\right)$$

$$8.17\times10^3\text{ eV} = (13.6\text{ eV})\left(\frac{8Z^2}{9} - 8\right)$$

so $601 = \frac{8Z^2}{9} - 8$

and $Z = 26$ $\boxed{\text{Iron}}$

Section 29.7 Context Connection—Atoms in Space

P29.38 (a) The configuration we may model as $\boxed{\text{SN}}$ $\boxed{\text{NS}}$ has higher energy than $\boxed{\text{SN}}$ $\boxed{\text{SN}}$. The higher energy state has antiparallel magnetic moments, so it has $\boxed{\text{parallel spins}}$ of the oppositely charged particles.

(b) $E = \frac{hc}{\lambda} = 9.42\times10^{-25}\text{ J} = \boxed{5.89\ \mu\text{eV}}$

(c) $\Delta E\Delta t \approx \frac{\hbar}{2}$ so $\Delta E \approx \frac{1.055\times10^{-34}\text{ J}\cdot\text{s}}{2(10^7\text{ yr})(3.16\times10^7\text{ s/yr})}\left(\frac{1.00\text{ eV}}{1.60\times10^{-19}\text{ J}}\right) = \boxed{1.04\times10^{-30}\text{ eV}}$

P29.39 Section 24.3 says $f_{\text{observer}} = f_{\text{source}}\sqrt{\frac{1+v_a/c}{1-v_a/c}}$.

The velocity of approach, v_a, is the negative of the velocity of mutual recession: $v_a = -v$.

Then, $\frac{c}{\lambda'} = \frac{c}{\lambda}\sqrt{\frac{1-v/c}{1+v/c}}$ and $\boxed{\lambda' = \lambda\sqrt{\frac{1+v/c}{1-v/c}}}$.

***P29.40** (a) The energy of the ground state is: $E_1 = -\frac{hc}{\lambda_{\text{series limit}}} = -\frac{1\,240\text{ eV}\cdot\text{nm}}{152.0\text{ nm}} = \boxed{-8.16\text{ eV}}$.

From the wavelength of the Lyman α line: $E_2 - E_1 = \frac{hc}{\lambda} = \frac{1\,240\text{ nm}\cdot\text{eV}}{202.6\text{ nm}} = 6.12\text{ eV}$

$E_2 = E_1 + 6.12\text{ eV} = \boxed{-2.04\text{ eV}}$.

The wavelength of the Lyman β line gives: $E_3 - E_1 = \frac{1\,240\text{ nm}\cdot\text{eV}}{170.9\text{ nm}} = 7.26\text{ eV}$

so $E_3 = \boxed{-0.902\text{ eV}}$.

Next, using the Lyman γ line gives: $E_4 - E_1 = \frac{1\,240\text{ nm}\cdot\text{eV}}{162.1\text{ nm}} = 7.65\text{ eV}$

and $E_4 = \boxed{-0.508\text{ eV}}$.

From the Lyman δ line, $E_5 - E_1 = \frac{1\,240\text{ nm}\cdot\text{eV}}{158.3\text{ nm}} = 7.83\text{ eV}$

so $E_5 = \boxed{-0.325\text{ eV}}$

(b) For the Balmer series, $\frac{hc}{\lambda} = E_i - E_2$, or $\lambda = \frac{1\,240\text{ nm}\cdot\text{eV}}{E_i - E_2}$

For the α line, $E_i = E_3$ and so $\lambda_\alpha = \frac{1\,240\text{ nm}\cdot\text{eV}}{(-0.902\text{ eV})-(-2.04\text{ eV})} = \boxed{1\,090\text{ nm}}$

Similarly, the wavelengths of the β line, γ line, and the short wavelength limit are found to be: $\boxed{811\text{ nm}}$, $\boxed{724\text{ nm}}$, and $\boxed{609\text{ nm}}$.

(c) Computing 60.0% of the wavelengths of the spectral lines shown on the energy-level diagram gives:

$0.600(202.6\text{ nm}) = \boxed{122\text{ nm}}$, $0.600(170.9\text{ nm}) = \boxed{103\text{ nm}}$, $0.600(162.1\text{ nm}) = \boxed{97.3\text{ nm}}$, $0.600(158.3\text{ nm}) = \boxed{95.0\text{ nm}}$, and $0.600(152.0\text{ nm}) = \boxed{91.2\text{ nm}}$

These are seen to be the wavelengths of the α, β, γ, and δ lines as well as the short wavelength limit for the Lyman series in Hydrogen.

continued on next page

(d) The observed wavelengths could be the result of Doppler shift when the source moves away from the Earth. The required speed of the source is found from

$$\frac{f'}{f}=\frac{\lambda}{\lambda'}=\sqrt{\frac{1-(v/c)}{1+(v/c)}}=0.600 \qquad \text{yielding} \qquad \boxed{v=0.471c}.$$

P29.41 (a) $\lambda'=\lambda\sqrt{\frac{1+v/c}{1-v/c}}$ $\qquad 510\text{ nm}=434\text{ nm}\sqrt{\frac{1+v/c}{1-v/c}}$

$$1.18^2=\frac{1+v/c}{1-v/c}=1.381$$

$$1+\frac{v}{c}=1.381-1.381\frac{v}{c} \qquad 2.38\frac{v}{c}=0.381$$

$$\frac{v}{c}=0.160 \quad \text{or} \quad v=\boxed{0.160c}=4.80\times10^{7}\text{ m/s}$$

(b) $v=HR$: $\qquad R=\frac{v}{H}=\frac{4.80\times10^{7}\text{ m/s}}{17\times10^{-3}\text{ m/s}\cdot\text{ly}}=\boxed{2.82\times10^{9}\text{ ly}}$

Additional Problems

***P29.42** (a) The energy emitted by the atom is $\Delta E=E_4-E_2=-13.6\text{ eV}\left(\frac{1}{4^2}-\frac{1}{2^2}\right)=2.55\text{ eV}$. The wavelength of the photon produced is then

$$\lambda=\frac{hc}{E_\gamma}=\frac{hc}{\Delta E}=\frac{(6.626\times10^{-34}\text{ J}\cdot\text{s})(3.00\times10^{8}\text{ m/s})}{(2.55\text{ eV})(1.60\times10^{-19}\text{ J/eV})}=4.87\times10^{-7}\text{ m}=\boxed{487\text{ nm}}$$

(b) Since momentum must be conserved, the photon and the atom go in opposite directions with equal magnitude momenta. Thus, $p=m_{\text{atom}}v=\frac{h}{\lambda}$ or

$$v=\frac{h}{m_{\text{atom}}\lambda}=\frac{6.626\times10^{-34}\text{ J}\cdot\text{s}}{(1.67\times10^{-27}\text{ kg})(4.87\times10^{-7}\text{ m})}=\boxed{0.814\text{ m/s}}.$$

P29.43 (a) $$\Delta E=\frac{e\hbar B}{m_e}=\frac{1.60\times10^{-19}\text{ C}(6.63\times10^{-34}\text{ J}\cdot\text{s})(5.26\text{ T})}{2\pi(9.11\times10^{-31}\text{ kg})}\left(\frac{\text{N}\cdot\text{s}}{\text{T}\cdot\text{C}\cdot\text{m}}\right)\left(\frac{\text{kg}\cdot\text{m}}{\text{N}\cdot\text{s}^2}\right)=9.75\times10^{-23}\text{ J}$$

$$=\boxed{609\ \mu\text{eV}}$$

(b) $k_BT=(1.38\times10^{-23}\text{ J/K})(80\times10^{-3}\text{ K})=1.10\times10^{-24}\text{ J}=\boxed{6.90\ \mu\text{eV}}$

continued on next page

(c) $$f=\frac{\Delta E}{h}=\frac{9.75\times10^{-23}\text{ J}}{6.63\times10^{-34}\text{ J}\cdot\text{s}}=\boxed{1.47\times10^{11}\text{ Hz}}$$

$$\lambda=\frac{c}{f}=\frac{3\times10^{8}\text{ m/s}}{1.47\times10^{11}\text{ Hz}}=\boxed{2.04\times10^{-3}\text{ m}}$$

P29.44 (a) The energy difference between these two states is equal to the energy that is absorbed.

Thus, $\qquad E=E_2-E_1=\frac{(-13.6\text{ eV})}{4}-\frac{(-13.6\text{ eV})}{1}=10.2\text{ eV}=\boxed{1.63\times10^{-18}\text{ J}}.$

(b) $E=\frac{3}{2}k_BT$ or $\qquad T=\frac{2E}{3k_B}=\frac{2(1.63\times10^{-18}\text{ J})}{3(1.38\times10^{-23}\text{ J/K})}=\boxed{7.88\times10^{4}\text{ K}}.$

P29.45 $r_{av}=\int_0^\infty rP(r)\,dr=\int_0^\infty\left(\frac{4r^3}{a_0^{\ 3}}\right)\left(e^{-2r/a_0}\right)dr$

Make a change of variables with $\frac{2r}{a_0}=x$ and $dr=\frac{a_0}{2}dx$.

Then $r_{av}=\frac{a_0}{4}\int_0^\infty x^3e^{-x}dx=\frac{a_0}{4}\left[-x^3e^{-x}+3\left(-x^2e^{-x}+2e^{-x}(-x-1)\right)\right]_0^\infty=\boxed{\frac{3}{2}a_0}.$

***P29.46** The average squared separation distance is

$$\langle r^2\rangle=\int_{\text{all space}}\psi_{1s}^{*}r^2\psi_{1s}dV=\int_{r=0}^{\infty}\frac{1}{\sqrt{\pi a_0^3}}e^{-r/a_0}r^2\frac{1}{\sqrt{\pi a_0^3}}e^{-r/a_0}4\pi r^2dr=\frac{4\pi}{\pi a_0^3}\int_0^\infty r^4e^{-2r/a_0}dr.$$

We use $\int_0^\infty x^ne^{-ax}dx=\frac{n!}{a^{n+1}}$ from Table B.6.

$$\langle r^2\rangle=\frac{4}{a_0^3}\frac{4!}{\left(\frac{2}{a_0}\right)^5}=\frac{a_0^2 96}{32}=3a_0^2$$

$$\Delta r=\sqrt{\langle r^2\rangle-\langle r\rangle^2}=\left(3a_0^2-\left(\frac{3a_0}{2}\right)^2\right)^{1/2}=\left(3a_0^2-\frac{9a_0^2}{4}\right)^{1/2}=\boxed{\left(\frac{3}{4}\right)^{1/2}a_0}$$

P29.47 $hf=\Delta E=\frac{4\pi^2mk_e^2e^4}{2h^2}\left(\frac{1}{(n-1)^2}-\frac{1}{n^2}\right)$

$$f=\frac{2\pi^2mk_e^2e^4}{h^3}\left(\frac{2n-1}{(n-1)^2n^2}\right)$$

As n approaches infinity, we have f approaching $\qquad \frac{2\pi^2mk_e^2e^4}{h^3}\frac{2}{n^3}$

continued on next page

The classical frequency is $f=\dfrac{v}{2\pi r}=\dfrac{1}{2\pi}\sqrt{\dfrac{k_e e^2}{m}}\dfrac{1}{r^{3/2}}$

where $r=\dfrac{n^2\hbar^2}{4\pi m k_e e^2}$

Using this equation to eliminate r from the expression for f, $f=\dfrac{2\pi^2 m k_e^2 e^4}{h^3}\dfrac{2}{n^3}$

*P29.48 (a) Probability $=\int_r^\infty P_{1s}(r')dr'=\dfrac{4}{a_0^3}\int_r^\infty r'^2 e^{-2r'/a_0}dr'=\left[-\left(\dfrac{2r'^2}{a_0^2}+\dfrac{2r'}{a_0}+1\right)e^{-2r'/a_0}\right]_r^\infty$,

using integration by parts,

$=\boxed{\left(\dfrac{2r^2}{a_0^2}+\dfrac{2r}{a_0}+1\right)e^{-2r/a_0}}$

(b)

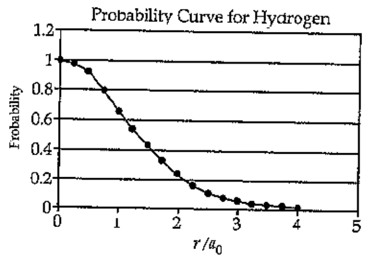

FIG. P29.48(b)

(c) The probability of finding the electron inside or outside the sphere of radius r is $\dfrac{1}{2}$

$\therefore\left(\dfrac{2r^2}{a_0^2}+\dfrac{2r}{a_0}+1\right)e^{-2r/a_0}=\dfrac{1}{2}$ or $z^2+2z+2=e^z$ where $z=\dfrac{2r}{a_0}$

One can home in on a solution to this transcendental equation for r on a calculator, the result being $r=\boxed{1.34a_0}$ to three digits.

P29.49 The wave function for the 2s state is given by Eq. 29.8: $\psi_{2s}(r)=\dfrac{1}{4\sqrt{2\pi}}\left(\dfrac{1}{a_0}\right)^{3/2}\left[2-\dfrac{r}{a_0}\right]e^{-r/2a_0}$

(a) Taking $r=a_0=0.529\times10^{-10}$ m

we find $\psi_{2s}(a_0)=\dfrac{1}{4\sqrt{2\pi}}\left(\dfrac{1}{0.529\times10^{-10}\text{ m}}\right)^{3/2}|2-1|e^{-1/2}=\boxed{1.57\times10^{14}\text{ m}^{-3/2}}$.

continued on next page

(b) $|\psi_{2s}(a_0)|^2=(1.57\times10^{14}\text{ m}^{-3/2})^2=\boxed{2.47\times10^{28}\text{ m}^{-3}}$

(c) Using Equation 29.6 and the results to (b) gives $P_{2s}(a_0)=4\pi a_0^2|\psi_{2s}(a_0)|^2=\boxed{8.69\times10^8\text{ m}^{-1}}$

P29.50 From Figure 29.12, a typical ionization energy is 8 eV. For internal energy to ionize most of the atoms we require

$\dfrac{3}{2}k_BT=8$ eV: $T=\dfrac{2\times8(1.60\times10^{-19}\text{ J})}{3(1.38\times10^{-23}\text{ J/K})}\boxed{\sim\text{ between }10^4\text{ K and }10^5\text{ K}}$.

P29.51 The fermions are described by the exclusion principle. Two of them, one spin-up and one spin-down, will be in the ground energy level, with

$d_{NN}=L=\dfrac{1}{2}\lambda$, $\lambda=2L=\dfrac{h}{p}$, and $p=\dfrac{h}{2L}$ $\qquad K=\dfrac{1}{2}mv^2=\dfrac{p^2}{2m}=\dfrac{h^2}{8mL^2}$.

The third must be in the next higher level, with

$d_{NN}=\dfrac{L}{2}=\dfrac{\lambda}{2}$, $\lambda=L$, and $p=\dfrac{h}{L}$ $\qquad K=\dfrac{p^2}{2m}=\dfrac{h^2}{2mL^2}$.

The total energy is then $\dfrac{h^2}{8mL^2}+\dfrac{h^2}{8mL^2}+\dfrac{h^2}{2mL^2}=\boxed{\dfrac{3h^2}{4mL^2}}$.

P29.52 $\Delta z=\dfrac{at^2}{2}=\dfrac{1}{2}\left(\dfrac{F_z}{m_0}\right)t^2=\dfrac{\mu_z(dB_z/dz)}{2m_0}\left(\dfrac{\Delta x}{v}\right)^2$ and $\mu_z=\dfrac{e\hbar}{2m_e}$

$\dfrac{dB_z}{dz}=\dfrac{2m_0(\Delta z)v^2(2m_e)}{\Delta x^2 e\hbar}=\dfrac{2(108)(1.66\times10^{-27}\text{ kg})(10^{-3}\text{ m})(10^4\text{ m}^2/\text{s}^2)(2\times9.11\times10^{-31}\text{ kg})}{(1.00\text{ m}^2)(1.60\times10^{-19}\text{ C})(1.05\times10^{-34}\text{ J}\cdot\text{s})}$

$\dfrac{dB_z}{dz}=\boxed{0.389\text{ T/m}}$

P29.53 We use $\psi_{2s}(r)=\dfrac{1}{4}(2\pi a_0^3)^{-1/2}\left(2-\dfrac{r}{a_0}\right)e^{-r/2a_0}$

By Equation 29.6. $P(r)=4\pi r^2\psi^2=\dfrac{1}{8}\left(\dfrac{r^2}{a_0^3}\right)\left(2-\dfrac{r}{a_0}\right)^2e^{-r/a_0}$

(a) $\dfrac{dP(r)}{dr}=\dfrac{1}{8}\left[\dfrac{2r}{a_0^3}\left(2-\dfrac{r}{a_0}\right)^2-\dfrac{2r^2}{a_0^3}\left(\dfrac{1}{a_0}\right)\left(2-\dfrac{r}{a_0}\right)-\dfrac{r^2}{a_0^3}\left(2-\dfrac{r}{a_0}\right)^2\left(\dfrac{1}{a_0}\right)\right]e^{-r/a_0}=0$

or $\dfrac{1}{8}\left(\dfrac{r}{a_0^3}\right)\left(2-\dfrac{r}{a_0}\right)\left[2\left(2-\dfrac{r}{a_0}\right)-\dfrac{2r}{a_0}-\dfrac{r}{a_0}\left(2-\dfrac{r}{a_0}\right)\right]e^{-r/a_0}=0$.

The roots of $\dfrac{dP}{dr}=0$ at $r=0$, $r=2a_0$ and $r=\infty$ are minima with $P(r)=0$.

continued on next page

Therefore we require $[\ldots\ldots] = 4 - \left(\frac{6r}{a_0}\right) + \left(\frac{r}{a_0}\right)^2 = 0$

with solutions $r = (3 \pm \sqrt{5})a_0$.

We substitute the last two roots into $P(r)$ to determine the most probable value:

When $r = (3 - \sqrt{5})a_0 = 0.763\,9a_0$, $P(r) = \frac{0.051\,9}{a_0}$

When $r = (3 + \sqrt{5})a_0 = 5.236a_0$, $P(r) = \frac{0.191}{a_0}$

Therefore, the most probable value of r is $(3 + \sqrt{5})a_0 = \boxed{5.236a_0}$.

(b) $\int_0^\infty P(r)dr = \int_0^\infty \frac{1}{8}\left(\frac{r^2}{a_0^3}\right)\left(2 - \frac{r}{a_0}\right)^2 e^{-r/a_0}dr$

Let $u = \frac{r}{a_0}$, $dr = a_0 du$,

$$\int_0^\infty P(r)dr = \int_0^\infty \frac{1}{8}u^2(4 - 4u + u^2)e^{-u}dr = \int_0^\infty \frac{1}{8}(u^4 - 4u^3 + 4u^2)e^{-u}du = -\frac{1}{8}(u^4 + 4u^2 + 8u + 8)e^{-u}\Big|_0^\infty = 1$$

$\boxed{\text{This is as required for normalization.}}$

*P29.54 (a) The system of a separated proton and electron puts out energy 13.606 eV to become a hydrogen atom in its ground state. This decrease in its rest energy appears also as a decrease in mass: the mass is $\boxed{\text{smaller}}$.

(b) $|\Delta m| = \frac{|E|}{c^2} = \frac{13.6 \text{ eV}}{(3 \times 10^8 \text{ m/s})^2}\left(\frac{1.6 \times 10^{-19} \text{ J}}{1 \text{ eV}}\right) = 2.42 \times 10^{-35} \text{ kg}$

$= 2.42 \times 10^{-35} \text{ kg}\left(\frac{1 \text{ u}}{1.66 \times 10^{-27} \text{ kg}}\right) = \boxed{1.46 \times 10^{-8} \text{ u}}$

(c) $\frac{1.46 \times 10^{-8} \text{ u}}{1.007\,825 \text{ u}} = 1.45 \times 10^{-8} = \boxed{1.45 \times 10^{-6}\%}$

(d) Table A.3 lists 1.007 825 u as the atomic mass of hydrogen. This correction of 0.000 000 01 u is on the order of 100 times $\boxed{\text{too small}}$ to affect the values listed.

P29.55 With one vacancy in the K shell, excess energy

$$\Delta E \approx -(Z-1)^2(13.6 \text{ eV})\left(\frac{1}{2^2} - \frac{1}{1^2}\right) = 5.40 \text{ keV}.$$

We suppose the outermost 4s electron is shielded by 22 electrons inside its orbit:

$$E_{\text{ionization}} \approx \frac{2^2(13.6 \text{ eV})}{4^2} = 3.40 \text{ eV}.$$

Note the experimental ionization energy is 6.76 eV.

$$K = \Delta E - E_{\text{ionization}} \approx \boxed{5.39 \text{ keV}}.$$

P29.56 $E = \frac{hc}{\lambda} = \frac{1\,240 \text{ eV}\cdot\text{nm}}{\lambda} = \Delta E$

$\lambda_1 = 310$ nm, so $\Delta E_1 = 4.00$ eV

$\lambda_2 = 400$ nm, $\Delta E_2 = 3.10$ eV

$\lambda_3 = 1\,378$ nm, $\Delta E_3 = 0.900$ eV

and the ionization energy = 4.10 eV.

The energy level diagram having the fewest levels and consistent with these energies is shown at the right.

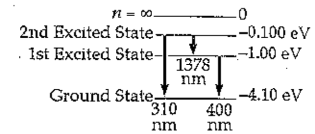

FIG. P29.56

P29.57 $P = \int_{2.50a_0}^\infty \frac{4r^2}{a_0^3}e^{-2r/a_0}dr = \frac{1}{2}\int_{5.00}^\infty z^2e^{-z}dz$ where $z \equiv \frac{2r}{a_0}$

$P = -\frac{1}{2}(z^2 + 2z + 2)e^{-z}\Big|_{5.00}^\infty = -\frac{1}{2}[0] + \frac{1}{2}(25.0 + 10.0 + 2.00)e^{-5} = \left(\frac{37}{2}\right)(0.006\,74) = \boxed{0.125}$

P29.58 (a) One molecule's share of volume

Al: $V = \frac{\text{mass per molecule}}{\text{density}} = \left(\frac{27.0 \text{ g/mol}}{6.02 \times 10^{23} \text{ molecules/mol}}\right)\left(\frac{1.00 \times 10^{-6} \text{ m}^3}{2.70 \text{ g}}\right) = 1.66 \times 10^{-29} \text{ m}^3$

$\sqrt[3]{V} = \boxed{2.55 \times 10^{-10} \text{ m} \sim 10^{-1} \text{ nm}}$.

U: $V = \left(\frac{238 \text{ g}}{6.02 \times 10^{23} \text{ molecules}}\right)\left(\frac{1.00 \times 10^{-6} \text{ m}^3}{18.9 \text{ g}}\right) = 2.09 \times 10^{-29} \text{ m}^3$

$\sqrt[3]{V} = \boxed{2.76 \times 10^{-10} \text{ m} \sim 10^{-1} \text{ nm}}$.

(b) The outermost electron in any atom sees the nuclear charge screened by all the electrons below it. If we can visualize a single outermost electron, it moves in the electric field of net charge, $+Ze - (Z-1)e = +e$, the charge of a single proton, as felt by the electron in hydrogen. So the Bohr radius sets the scale for the outside diameter of every atom. An innermost electron, on the other hand, sees the nuclear charge unscreened, and the scale size of its (K-shell) orbit is $\frac{a_0}{Z}$.

P29.59 $\Delta E = 2\mu_B B = hf$

so $2(9.27\times10^{-24}\ \mathrm{J/T})(0.350\ \mathrm{T}) = (6.626\times10^{-34}\ \mathrm{J\cdot s})f$

and $f = \boxed{9.79\times10^{9}\ \mathrm{Hz}}$

P29.60 $\psi_{2s} = \frac{1}{4}(2\pi)^{-1/2}\left(\frac{1}{a_0}\right)^{3/2}\left(2-\frac{r}{a_0}\right)e^{-r/2a_0} = A\left(2-\frac{r}{a_0}\right)e^{-r/2a_0}$ $\qquad \frac{d\psi}{dr} = Ae^{-r/2a_0}\left(-\frac{2}{a_0}+\frac{r}{2a_0^2}\right)$

$$\frac{d^2\psi}{dr^2} = \left(\frac{Ae^{-r/2a_0}}{a_0^2}\right)\left(\frac{3}{2}-\frac{r}{4a_0}\right)$$

Substituting into Schrödinger's equation and dividing by $Ae^{-r/2a_0}$, we will have a solution if

$$-\frac{5}{4}\frac{\hbar^2}{m_e a_0^2}+\frac{k_e e^2}{a_0}+\frac{\hbar^2 r}{8m_e a_0^3}+\frac{2\hbar^2}{m_e a_0 r}-\frac{2k_e e^2}{r} = 2E-\frac{Er}{a_0}$$

Now with $a_0 = \frac{\hbar^2}{m_e e^2 k_e}$, this reduces to

$$-\frac{m_e e^4 k_e^2}{8\hbar^2}\left(2-\frac{r}{a_0}\right) = E\left(2-\frac{r}{a_0}\right).$$

This is true, so ψ_{2s} is a solution to the Schrödinger equation, provided $E = \frac{1}{4}E_1 = -3.40\ \mathrm{eV}$.

P29.61 (a) Suppose the atoms move in the $+x$ direction. The absorption of a photon by an atom is a completely inelastic collision, described by

$$mv_i\hat{\mathbf{i}}+\frac{h}{\lambda}(-\hat{\mathbf{i}}) = mv_f\hat{\mathbf{i}} \qquad \text{so} \qquad v_f - v_i = -\frac{h}{m\lambda}$$

This happens promptly every time an atom has fallen back into the ground state, so it happens every $10^{-8}\ \mathrm{s} = \Delta t$. Then,

$$a = \frac{v_f - v_i}{\Delta t} = -\frac{h}{m\lambda\Delta t} \sim -\frac{6.626\times10^{-34}\ \mathrm{J\cdot s}}{(10^{-25}\ \mathrm{kg})(500\times10^{-9}\ \mathrm{m})(10^{-8}\ \mathrm{s})} \sim \boxed{-10^{6}\ \mathrm{m/s^2}}.$$

(b) With constant average acceleration,

$$v_f^2 = v_i^2 + 2a\Delta x \qquad 0 \sim (10^3\ \mathrm{m/s})^2 + 2(-10^6\ \mathrm{m/s^2})\Delta x$$

so $$\Delta x \sim \frac{(10^3\ \mathrm{m/s})^2}{10^6\ \mathrm{m/s^2}} \boxed{\sim 1\ \mathrm{m}}.$$

***P29.62** $$\left\langle\frac{1}{r}\right\rangle = \int_0^\infty \frac{4r^2}{a_0^3}e^{-2r/a_0}\frac{1}{r}dr = \frac{4}{a_0^3}\int_0^\infty re^{-(2/a_0)r}dr = \frac{4}{a_0^3}\frac{1}{\left(\frac{2}{a_0}\right)^2} = \boxed{\frac{1}{a_0}}$$

We compare this to $\frac{1}{\langle r\rangle} = \frac{1}{\frac{3a_0}{2}} = \frac{2}{3a_0}$, and find that the average reciprocal value is $\boxed{\text{NOT}}$ the reciprocal of the average value.

ANSWERS TO EVEN PROBLEMS

P29.2 (a) 56.8 fm; (b) 11.3 N

P29.4 (a) 1.94 μm; (b) $\lambda_{CB} = \frac{1}{\frac{1}{\lambda_{CA}}-\frac{1}{\lambda_{BA}}}$

P29.6 (a) 3; (b) 520 km/s

P29.8 (a) see the solution; (b) 0.179 nm

P29.10 see the solution

P29.12 $4a_0$

P29.14 797 times

P29.16 (a) $\sqrt{6}\,\hbar$; (b) $\sqrt{12}\,\hbar$

P29.18 $\sqrt{6}\,\hbar$

P29.20 $L = \sqrt{6}\,\hbar$, $L_z = -2\hbar, -\hbar, 0, \hbar, 2\hbar$, $\theta = 145°$, $114°$, $90.0°$, $65.9°$, and $35.3°$

P29.22 $3\,\hbar$

P29.24 (a) $1s^2 2s^2 p^4$;

(b)

n	ℓ	m_ℓ	m_s
1	0	0	1/2
1	0	0	–1/2
2	0	0	1/2
2	0	0	–1/2
2	1	1	1/2
2	1	0	1/2
2	1	–1	1/2
2	1	–1	–1/2

P29.26 see the solution

P29.28 (a) see the solution; (b) 36

P29.30 $1s^2 2s^2 2p^6 3s^2 3p^6 3d^{10} 4s^2 4p^6 4d^{10} 4f^{14} 5s^2 5p^6 5d^{10} 5f^{14} 6s^2 6p^6 6d^8 7s^2$

P29.32 (a) $\ell = 0$ and $m_\ell = 0$, or $\ell = 1$ and $m_\ell = -1$, 0, or 1, or $\ell = 2$ and $m_\ell = -2, -1$, 0, 1, or 2; (b) –6.05 eV

P29.34 see the solution

P29.36 L shell = 11.8 keV, M shell = 10.2 keV, N shell = 2.47 keV, see the solution

P29.38 (a) parallel spins, with antiparallel magnetic moments; (b) 5.89 μeV; (c) 1.04×10^{-30} eV

P29.40 (a) –8.16 eV, –2.04 eV, –0.902 eV, –0.508 eV, –0.325 eV; (b) 1090 nm, 811 nm, 724 nm, 609 nm; (c) see the solution; (d) The spectrum could be that of hydrogen, Doppler shifted by motion away from us at speed 0.471c.

P29.42 (a) 487 nm; (b) 0.814 m/s

P29.44 (a) 1.63×10^{-18} J; (b) 7.88×10^{4} K

P29.46 $\sqrt{\frac{3}{4}}a_0 = 0.866a_0$

P29.48 (a) Probability $= \left(\frac{2r^2}{a_0^2} + \frac{2r}{a_0} + 1\right)e^{-2r/a_0}$; (b) see the solution; (c) $1.34a_0$

P29.50 between 10^4 K and 10^5 K

P29.52 0.389 T/m

P29.54 (a) smaller; (b) 1.46×10^{-8} u; (c) 1.45×10^{-6}%; (d) no

P29.56 see the solution

P29.58 (a) For Al, about 0.255 nm ~ 0.1 nm. For U, about 0.276 nm ~ 0.1 nm. For an outer electron, the nuclear charge is screened by the interior electrons.
(b) For an inner-shell electron, the nuclear charge is unscreened. The distance scale of the wave function, representing the orbital size, is proportional to $\frac{a_0}{Z}$.

P29.60 see the solution

P29.62 $\frac{1}{a_0}$, no

Chapter 30
Nuclear Physics

CHAPTER OUTLINE

ANSWERS TO QUESTIONS

Q30.1 Because of electrostatic repulsion between the positively-charged nucleus and the $+2e$ alpha particle. To drive the α-particle into the nucleus would require extremely high kinetic energy.

Q30.2 Nuclei with more nucleons than bismuth-209 are unstable because the electrical repulsion forces among all of the protons is stronger than the nuclear attractive force between nucleons.

Q30.3 Extra neutrons are required to overcome the increasing electrostatic repulsion of the protons. The neutrons participate in the net attractive effect of the nuclear force, but feel no Coulomb repulsion.

Q30.4 The nuclear force favors the formation of neutron-proton pairs, so a stable nucleus cannot be too far away from having equal numbers of protons and neutrons. This effect sets the upper boundary of the zone of stability on the neutron-proton diagram. All of the protons repel one another electrically, so a stable nucleus cannot have too many protons. This effect sets the lower boundary of the zone of stability.

Q30.5 ^{4}He, ^{16}O, ^{40}Ca, and ^{208}Pb.

Q30.6 If one half the number of radioactive nuclei decay in one year, then one half the remaining number will decay in the second year. Three quarters of the original nuclei will be gone, and one quarter will remain.

Q30.7 Since the samples are of the same radioactive isotope, their half-lives are the same. When prepared, sample A has twice the activity (number of radioactive decays per second) of sample B. After 5 half-lives, the activity of sample A is decreased by a factor of 2^5, and after 5 half-lives the activity of sample B is decreased by a factor of 2^5. So after 5 half-lives, the ratio of activities is still 2:1.

Q30.8 The statement is false. Both patterns show monotonic decrease over time, but with very different shapes. For radioactive decay, maximum activity occurs at time zero. Cohorts of people now living will be dying most rapidly perhaps forty years from now. Everyone now living will be dead within less than two centuries, while the mathematical model of radioactive decay tails off exponentially forever. A radioactive nucleus never gets old. It has constant probability of decay however long it has existed.