

312 Gravity, Planetary Orbits, and the Hydrogen Atom

P11.33 (a) The major axis of the orbit is $2a = 50.5 \text{ AU}$ so $a = 25.25 \text{ AU}$

Further, in Figure 11.5, $a + c = 50 \text{ AU}$ so $c = 24.75 \text{ AU}$

Then
$$e = \frac{c}{a} = \frac{24.75}{25.25} = \boxed{0.980}$$

(b) In $T^2 = K_s a^3$ for objects in solar orbit, the Earth gives us

$$(1 \text{ yr})^2 = K_s (1 \text{ AU})^3 \quad K_s = \frac{(1 \text{ yr})^2}{(1 \text{ AU})^3}$$

Then
$$T^2 = \frac{(1 \text{ yr})^2}{(1 \text{ AU})^3} (25.25 \text{ AU})^3 \quad T = \boxed{127 \text{ yr}}$$

(c)
$$U = -\frac{GMm}{r} = -\frac{(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(1.991 \times 10^{30} \text{ kg})(1.2 \times 10^{10} \text{ kg})}{50(1.496 \times 10^{11} \text{ m})} = \boxed{-2.13 \times 10^{17} \text{ J}}$$

Section 11.5 Atomic Spectra and the Bohr Theory of Hydrogen

P11.34 (a) The energy of the photon is found as
$$E = E_i - E_f = \frac{-13.606 \text{ eV}}{n_i^2} - \frac{(-13.606 \text{ eV})}{n_f^2}$$

$$E = 13.606 \text{ eV} \left(\frac{1}{n_f^2} - \frac{1}{n_i^2} \right)$$

Thus, for $n = 3$ to $n = 2$ transition
$$E = 13.606 \text{ eV} \left(\frac{1}{4} - \frac{1}{9} \right) = \boxed{1.89 \text{ eV}}$$

(b) $E = \frac{hc}{\lambda}$ and
$$\lambda = \frac{6.626 \times 10^{-34} \text{ J} \cdot \text{s} (2.998 \times 10^8 \text{ m/s})}{1.89 \text{ eV} (1.602 \times 10^{-19} \text{ J/eV})} = \boxed{656 \text{ nm}}$$

(c) $f = \frac{c}{\lambda}$
$$f = \frac{3 \times 10^8 \text{ m/s}}{6.56 \times 10^{-7} \text{ m}} = \boxed{4.57 \times 10^{14} \text{ Hz}}$$

P11.35 (a) Lyman series $\frac{1}{\lambda} = R \left(1 - \frac{1}{n_i^2} \right)$ $n_i = 2, 3, 4, \dots$

$$\frac{1}{\lambda} = \frac{1}{94.96 \times 10^{-9}} = (1.097 \times 10^7) \left(1 - \frac{1}{n_i^2} \right) \quad \boxed{n_i = 5}$$

(b) Paschen series: $\frac{1}{\lambda} = R \left(\frac{1}{3^2} - \frac{1}{n_i^2} \right)$ $n_i = 4, 5, 6, \dots$

The shortest wavelength for this series corresponds to $n_i = \infty$ for ionization

$$\frac{1}{\lambda} = 1.097 \times 10^7 \left(\frac{1}{9} - \frac{1}{n_i^2} \right)$$

For $n_i = \infty$, this gives $\lambda = 820 \text{ nm}$

This is larger than 94.96 nm, so this wave length
cannot be associated with the Paschen series.

Balmer series: $\frac{1}{\lambda} = R \left(\frac{1}{2^2} - \frac{1}{n_i^2} \right)$ $n_i = 3, 4, 5, \dots$

$$\frac{1}{\lambda} = 1.097 \times 10^7 \left(\frac{1}{4} - \frac{1}{n_i^2} \right) \quad \text{with } n_i = \infty \text{ for ionization, } \lambda_{\min} = 365 \text{ nm}$$

Once again the shorter given wavelength cannot be associated with the Balmer series.

P11.36 (a) $v_1 = \sqrt{\frac{k_e e^2}{m_e r_1}}$

where $r_1 = (1)^2 a_0 = 0.00529 \text{ nm} = 5.29 \times 10^{-11} \text{ m}$

$$v_1 = \sqrt{\frac{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(1.60 \times 10^{-19} \text{ C})^2}{(9.11 \times 10^{-31} \text{ kg})(5.29 \times 10^{-11} \text{ m})}} = \boxed{2.19 \times 10^6 \text{ m/s}}$$

(b) $K_1 = \frac{1}{2} m_e v_1^2 = \frac{1}{2} (9.11 \times 10^{-31} \text{ kg})(2.19 \times 10^6 \text{ m/s})^2 = 2.18 \times 10^{-18} \text{ J} = \boxed{13.6 \text{ eV}}$

(c) $U_1 = -\frac{k_e e^2}{r_1} = -\frac{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(1.60 \times 10^{-19} \text{ C})^2}{5.29 \times 10^{-11} \text{ m}} = -4.35 \times 10^{-18} \text{ J} = \boxed{-27.2 \text{ eV}}$

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P11.37 $\Delta E = (13.6 \text{ eV}) \left(\frac{1}{n_i^2} - \frac{1}{n_f^2} \right)$

Where for $\Delta E > 0$ we have absorption and for $\Delta E < 0$ we have emission.

- (i) for $n_i = 2$ and $n_f = 5$, $\Delta E = 2.86 \text{ eV}$ (absorption)
- (ii) for $n_i = 5$ and $n_f = 3$, $\Delta E = -0.967 \text{ eV}$ (emission)
- (iii) for $n_i = 7$ and $n_f = 4$, $\Delta E = -0.572 \text{ eV}$ (emission)
- (iv) for $n_i = 4$ and $n_f = 7$, $\Delta E = 0.572 \text{ eV}$ (absorption)

(a) $E = \frac{hc}{\lambda}$ so the shortest wavelength is emitted in transition **ii**.

(b) The atom gains most energy in transition **i**.

(c) The atom loses energy in transitions **ii and iii**.

P11.38 We use

$$E_n = \frac{-13.6 \text{ eV}}{n^2}$$

To ionize the atom when the electron is in the n^{th} level,

it is necessary to add an amount of energy given by $E = -E_n = \frac{13.6 \text{ eV}}{n^2}$.

(a) Thus, in the ground state where $n = 1$, we have **$E = 13.6 \text{ eV}$** .

(b) In the $n = 3$ level,

$$E = \frac{13.6 \text{ eV}}{9} = \mathbf{1.51 \text{ eV}}$$

P11.39 (a) $r_2^2 = (0.0529 \text{ nm})(2)^2 = \mathbf{0.212 \text{ nm}}$

(b) $m_e v_2 = \sqrt{\frac{m_e k_e e^2}{r_2}} = \sqrt{\frac{(9.11 \times 10^{-31} \text{ kg})(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(1.60 \times 10^{-19} \text{ C})^2}{0.212 \times 10^{-9} \text{ m}}}$
 $m_e v_2 = \mathbf{9.95 \times 10^{-25} \text{ kg} \cdot \text{m/s}}$

(c) $L_2 = m_e v_2 r_2 = (9.95 \times 10^{-25} \text{ kg} \cdot \text{m/s})(0.212 \times 10^{-9} \text{ m}) = \mathbf{2.11 \times 10^{-34} \text{ kg} \cdot \text{m}^2/\text{s}}$

(d) $K_2 = \frac{1}{2} m_e v_2^2 = \frac{(m_e v_2)^2}{2m_e} = \frac{(9.95 \times 10^{-25} \text{ kg} \cdot \text{m/s})^2}{2(9.11 \times 10^{-31} \text{ kg})} = 5.43 \times 10^{-19} \text{ J} = \mathbf{3.40 \text{ eV}}$

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$$(e) \quad U_2 = -\frac{k_e e^2}{r_2} = -\frac{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(1.60 \times 10^{-19} \text{ C})^2}{0.212 \times 10^{-9} \text{ m}} = -1.09 \times 10^{-18} \text{ J} = \boxed{-6.80 \text{ eV}}$$

$$(f) \quad E_2 = K_2 + U_2 = 3.40 \text{ eV} - 6.80 \text{ eV} = \boxed{-3.40 \text{ eV}}$$

P11.40 Starting with $\frac{1}{2} m_e v^2 = \frac{k_e e^2}{2r}$

we have $v^2 = \frac{k_e e^2}{m_e r}$

and using $r_n = \frac{n^2 \hbar^2}{m_e k_e e^2}$

gives $v_n^2 = \frac{k_e e^2}{m_e (n^2 \hbar^2 / m_e k_e e^2)}$

or $v_n = \frac{k_e e^2}{n \hbar}$

P11.41 Each atom gives up its kinetic energy in emitting a photon,

$$\text{so } \frac{1}{2} m v^2 = \frac{h c}{\lambda} = \frac{(6.626 \times 10^{-34} \text{ J} \cdot \text{s})(3.00 \times 10^8 \text{ m/s})}{(1.216 \times 10^{-7} \text{ m})} = 1.63 \times 10^{-18} \text{ J}$$

$$v = \boxed{4.42 \times 10^4 \text{ m/s}}$$

Section 11.6 Context Connection—Changing from a Circular to an Elliptical Orbit

P11.42 The original orbit radius is $r = a = 6.37 \times 10^6 \text{ m} + 500 \times 10^3 \text{ m} = 6.87 \times 10^6 \text{ m}$. The original energy is

$$E_i = -\frac{GMm}{2a} = -\frac{(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(5.98 \times 10^{24} \text{ kg})(10^4 \text{ kg})}{2(6.87 \times 10^6 \text{ m})} = -2.90 \times 10^{11} \text{ J}.$$

We assume that the perigee distance in the new orbit is $6.87 \times 10^6 \text{ m}$. Then the major axis is $2a = 6.87 \times 10^6 \text{ m} + 2.00 \times 10^7 \text{ m} = 2.69 \times 10^7 \text{ m}$ and the final energy is

$$E_f = -\frac{GMm}{2a} = -\frac{(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(5.98 \times 10^{24} \text{ kg})(10^4 \text{ kg})}{2.69 \times 10^7 \text{ m}} = -1.48 \times 10^{11} \text{ J}.$$

The energy input required from the engine is $E_f - E_i = -1.48 \times 10^{11} \text{ J} - (-2.90 \times 10^{11} \text{ J}) = \boxed{1.42 \times 10^{11} \text{ J}}$.