13.16

16. **Review problem.** A light string with a mass per unit l
of 8.00 g/m has its ends tied to two walls separated by
tance equal to three-fourths the length of the s
(Fig. P13.16). An object of mass *m* is suspended
the center of the string, putting a tension in the st
(a) Find an expression for the transverse wave speed i
string as a function of the mass of the hanging ol
(b) What should be the mass of the object suspe
from the string so as to produce a wave speed of 60.0 m/s

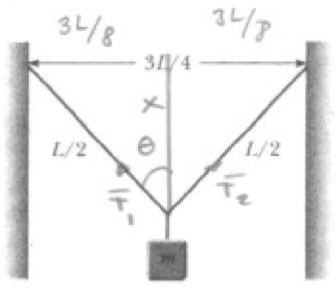


FIGURE P13.16

Lösning:

$$2T \cdot \cos\theta = mg$$

$$\cos\theta = \frac{x}{L/2}$$

$$x^2 + \left(\frac{3L}{8}\right)^2 = \left(\frac{L}{2}\right)^2 \Rightarrow x^2 + \frac{9}{64}L^2 = \frac{16}{64}L^2 \Rightarrow x = \frac{\sqrt{7}}{8}L$$

$$\Rightarrow \cos\theta = \frac{\sqrt{7}}{4}.$$

$$\Rightarrow T = \frac{mg}{2 \cdot \cos\theta} = \frac{mg \cdot 4}{2 \cdot \sqrt{7}}$$

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{2 \cdot g}{\sqrt{7} \cdot 8 \cdot 10^{-3}}}\sqrt{m} = 30{,}4\sqrt{m} \quad m/s$$

$v = 60{,}0$ m/s :

$$m = \left(\frac{v}{30{,}4}\right)^2 = \left(\frac{60{,}0}{30{,}4}\right)^2 = \underline{3{,}89 \text{ kg}}$$

13.48

13.47

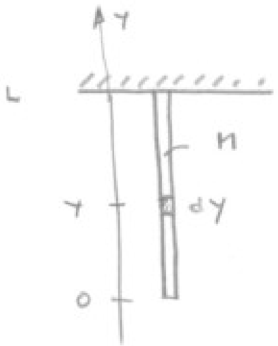
48. Assume that an object of mass M is suspended from the bottom of the rope in Problem 13.47. (a) Show that the time interval for a transverse pulse to travel the length of the rope is

$$\Delta t = 2\sqrt{\frac{L}{mg}}\left(\sqrt{M+m}-\sqrt{M}\right)$$

(b) Show that this expression reduces to the result of Problem 13.47 when $M = 0$. (c) Show that for $m \ll M$, the expression in part (a) reduces to

$$\Delta t = \sqrt{\frac{mL}{Mg}}$$

Lösning.



$$v(y) = \sqrt{\frac{T(y)}{\mu}} = \sqrt{\frac{T(y)}{m/L}}$$

spännkraften varierar

$$T(y) = \frac{y}{L} Mg$$

Tid för passage av dy på höjden y

$$dt = \frac{dy}{v(y)} = \frac{dy}{\sqrt{\frac{T(y)}{\mu}}} = \left(\frac{L\mu}{mg}\right)^{1/2} y^{-1/2}\, dy$$

Tid för passage av hela repet

$$t = \int_0^t dt = \left(\frac{\cancel{L}\,\cancel{m}}{\cancel{m}\,\cancel{L}\,g}\right)^{1/2} \int_0^L y^{-1/2}\, dy = g^{-1/2}\left[2y^{1/2}\right]_0^L =$$

$$= \underline{\underline{2\sqrt{\frac{L}{g}}}}$$

Med massan M hängande i änden:

$$T(y) = \left(\frac{y}{L} m + M\right) g$$

$$v(y) = \left(\frac{y/L \cdot m + M}{m/L}\, g\right)^{1/2} = \left[\left(y + \frac{LM}{m}\right) g\right]^{1/2}$$

$$dt = \frac{dy}{v(y)} = \left[\left(y + \frac{LM}{m}\right) g\right]^{-1/2} dy$$

$$\Rightarrow t = 2\frac{1}{\sqrt{g}}\left[\sqrt{L + \frac{LM}{m}} - \sqrt{\frac{LM}{m}}\right] =$$

$$= 2\frac{\sqrt{L}}{\sqrt{mg}}\left[\sqrt{m+M} - \sqrt{M}\right]$$

$$(m+M)^{1/2} = \sqrt{M}\left(1+\frac{m}{M}\right)^{1/2} \approx \sqrt{M}\left(1 + \frac{1}{2}\frac{m}{M} + \dots\right) \Rightarrow t = 2\frac{L}{\sqrt{mg}}\,\frac{1}{2}\,\frac{m}{M} =$$

$$= \sqrt{\frac{mL}{Mg}}$$

(14.10)

10. Two speakers are driven by the same oscillator of frequency f. They are located a distance d from each other on a vertical pole. A man walks straight toward the lower speaker in a direction perpendicular to the pole as shown in Figure P14.10. (a) How many times will he hear a minimum in sound intensity? (b) How far is he from the pole at these moments? Let v represent the speed of sound and assume that the ground does not reflect sound.

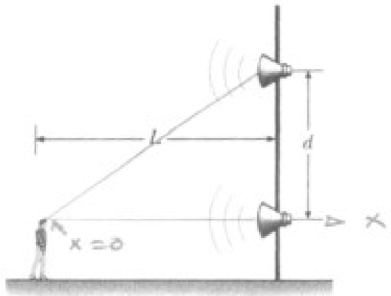


FIGURE **P14.10**

Lösning: minimum när $\Delta r = \sqrt{x^2 + d^2} - x = (2n-1)\frac{\lambda}{2}$

$$\Rightarrow \sqrt{x^2+d^2} - x = \left(n - \frac{1}{2}\right)\frac{v}{f} \qquad n = 1, 2, 3, \ldots$$

$$\Rightarrow d^2 - \left(n-\frac{1}{2}\right)^2\left(\frac{v}{f}\right) = 2\left(n-\frac{1}{2}\right)\left(\frac{v}{f}\right)\cdot x$$

sätt in $n = 1, 2, 3, \ldots$ och bestäm x.

Om $x >> d$ är $\Delta r \approx 0$

antalet minima bestäms då av den maximala vägskillnaden $= d$ som ni får då $x = 0$

$$\Rightarrow d = \left(n^* - \frac{1}{2}\right)\frac{v}{f} \quad \Rightarrow \quad n^* = d\,\frac{f}{v} + \frac{1}{2}$$

varför n^* och inte n?

Jo det är ju inte säkert att $d = (2\cdot\text{heltal} - 1)\frac{\lambda}{2}$

dvs att d är lika med ett udda antal halva λ.

Om $n = 20{,}3$ noteras 20 minima mellan $x = \infty$ och $x = 0$

(14.16)

16. Two waves simultaneously present in a long string are given by the wave functions

$$y_1 = A\sin(kx - \omega t + \phi) \quad \text{and} \quad y_2 = A\sin(kx + \omega t)$$

In the case $\phi = 0$, the chapter text shows that they add to a standing wave. Demonstrate (a) that the addition of the arbitrary phase constant ϕ changes only the position of the nodes and, in particular, (b) that the distance between nodes is still one half the wavelength.

Lösning:

om vi överlagrar $Y_1 = A\sin(kx - \omega t)$ och

$$Y_2 = A\sin(kx + \omega t)$$

får vi $Y_1 + Y_2 = 2A\sin kx \cos\omega t$

denna stående våg har en nod i $x = 0$.

Vad händer när vi överlagrar

$$Y_1 = A\cdot\sin(kx - \omega t + \phi) \quad \text{och} \quad Y_2 = A\sin(kx + \omega t)$$

Skriv om detta som

$$Y_1 = A\sin\left[\left(kx + \frac{\phi}{2}\right) - \left(\omega t + \frac{\phi}{2}\right)\right]$$

och

$$Y_2 = A\sin\left[\left(kx + \frac{\phi}{2}\right) + \left(\omega t - \frac{\phi}{2}\right)\right]$$

$$\Rightarrow Y_1 + Y_2 = 2A\sin\left(kx + \frac{\phi}{2}\right)\cos\left(\omega t - \frac{\phi}{2}\right)$$

noder: $kx + \frac{\phi}{2} = n\pi \Rightarrow x = \frac{1}{k}\left(n\pi - \frac{\phi}{2}\right) =$

$$= \frac{\lambda}{2\pi}\left(n\pi - \frac{\phi}{2}\right) =$$

$$= n\frac{\lambda}{2} + \frac{\lambda\cdot\phi}{4\pi}$$

om vi vill ha en nod i $x' = 0$ flyttar vi nollan

$$kx + \frac{\phi}{2} = kx' \Rightarrow x' = x + \frac{\phi}{2k} = x + \frac{\phi\cdot\lambda}{2\cdot 2\pi}$$

$$\phi = -\pi \Rightarrow x' = x - \frac{\pi\cdot\lambda}{4\pi} = \frac{1}{4}\lambda$$

(14.40)

40. While attempting to tune the note C at 523 Hz, a piano tuner hears 2.00 beats/s between a reference oscillator and the string. (a) What are the possible frequencies of the string? (b) When she tightens the string slightly, she hears 3.00 beats/s. What is the frequency of the string now? (c) By what percentage should the piano tuner now change the tension in the string to bring it into tune?

Lösning:

a) $f = f_c \pm 2{,}00$ → 525 Hz, 521 Hz

b) $v = f \cdot \lambda$

svävningsfrekvensen ökar när T ökar

v ökar när T ökar $\quad v = \sqrt{\dfrac{T}{\mu}}$

λ = konst.

$\therefore$ f ökar när T ökar

$\therefore$ f = 526 Hz d.v.s 523 + 3 = 526 Hz.

c) $f \sim \sqrt{T}$

$$\frac{f_2}{f_1} = \frac{523}{526} = \sqrt{\frac{T_2}{T_1}} \Rightarrow \frac{T_2}{T_1} = 0{,}9886$$

Procentuell ändring

$$\frac{T_1 - T_2}{T_1} = 1 - 0{,}9886 = 0{,}0114$$

dvs. 1,14% lägre T

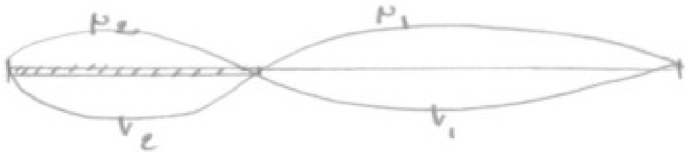
14.53

53. Two wires are welded together end to end. The wires are made of the same material, but the diameter of one is twice that of the other. They are subjected to a tension of 4.60 N. The thin wire has a length of 40.0 cm and a linear mass density of 2.00 g/m. The combination is fixed at both ends and vibrated in such a way that two antinodes are present, with the node between them being right at the weld. (a) What is the frequency of vibration? (b) How long is the thick wire?

Lösning:

$d_2 = 2d_1$

$\Rightarrow \mu_2 = 4 \cdot \mu_1$



$T = 4{,}60$ N

$l_1 = 40{,}0$ cm

$\mu_1 = 2{,}00$ g/m

f:

Två bukar och en nod i skarven.

$l_1 = \frac{\lambda}{2}$ $\qquad v_1 = \sqrt{\frac{T}{\mu_1}}$

$$f = \frac{v}{\lambda} = \frac{\sqrt{T/\mu_1}}{2l_1} = \frac{\sqrt{4{,}60/2{,}00 \cdot 10^{-3}}}{2 \cdot 0{,}40} \text{ Hz} = \underline{\underline{59{,}9 \text{ Hz.}}}$$

l_2:

$l_2 = \frac{\lambda}{2}$ $\qquad v_2 = \sqrt{\frac{T}{4\mu_1}}$ $\qquad f = \frac{\sqrt{T/\mu_1}}{2l_1}$

$$\lambda = 2l_2 = \frac{\sqrt{T/4\mu_1}}{\sqrt{T/\mu_1}} 2l_1 \Rightarrow l_2 = \frac{l_1}{2} \quad \text{d.v.s.} \quad \underline{\underline{20{,}0 \text{ cm}}}$$