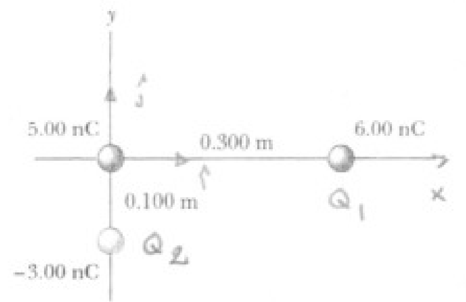
(19.13)

13. Three point charges are arranged as shown in Figure P19.13. (a) Find the vector electric field that the 6.00-nC and −3.00-nC charges together create at the origin. (b) Find the vector force on the 5.00-nC charge.



Lösning: Elektrische fältet från en punktladdning Q

$$\bar{E}(\bar{r}) = k_e \frac{Q}{r^2}\hat{r}$$

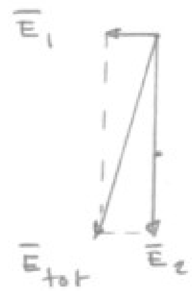
Fältet i origo:

Q_1: $$\bar{E}_1 = k_e \frac{Q_1}{r_1^2}(-\hat{\imath}) = 9\cdot 10^9 \frac{6{,}00\cdot 10^{-9}}{0{,}300^2}(-\hat{\imath})\,V/m = $$

$$= 600(-\hat{\imath})\ V/m$$

Q_2: $$\bar{E}_2 = k_e \frac{Q_2}{r_2^2}(-\hat{\jmath}) = 9\cdot 10^9 \frac{3{,}00\cdot 10^{-9}}{0{,}100^2}(-\hat{\jmath}) = $$

$$= 2700(-\hat{\jmath})\ V/m$$



$$|\bar{E}_{tot}| = \sqrt{|\bar{E}_1|^2 + |\bar{E}_2|^2} =$$

$$= \sqrt{600^2 + 2700^2} = 2766\ V/m$$

Kraft på Q_3 i origo:

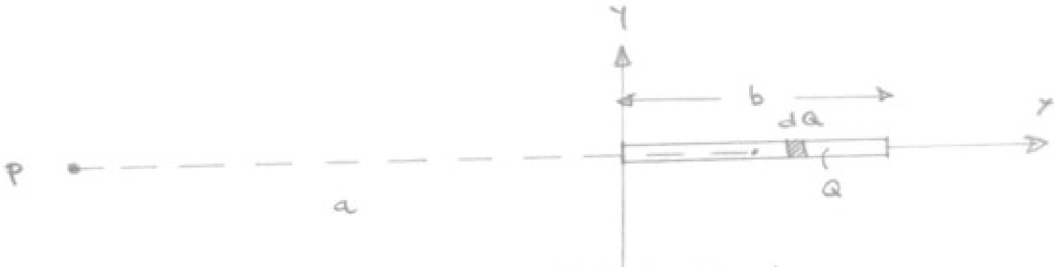
$$\bar{F}_3 = Q_3(\bar{E}_1 + \bar{E}_2) = 5{,}00\cdot 10^{-9}\left[600(-\hat{\imath}) + 2700(-\hat{\jmath})\right]$$

$$= 3{,}00\cdot 10^{-6}(-\hat{\imath}) + 13{,}5\cdot 10^{-6}(-\hat{\jmath})\quad N$$

$$|\bar{F}_3| = \sqrt{3{,}00^2 + 13{,}5^2}\cdot 10^{-6}\,N = 13{,}8\cdot 10^{-6}\,N$$

19. 17. A rod 14.0 cm long is uniformly charged and has a total charge of $-22.0\ \mu C$. Determine the magnitude and direction of the electric field along the axis of the rod at a point 36.0 cm from its center.

Lösning:



El. fältet i P : $\bar{E}_p \parallel \hat{i}$ staven är negativt laddad.

$$|\bar{E}_p| = \int_0^b k_e \frac{dQ}{(x+a)^2} = k_e \frac{Q}{b} \int_0^b \frac{dx}{(x+a)^2}$$

$$dQ = \frac{Q}{b} \cdot dx$$

$$\therefore |\bar{E}_p| = k_e \frac{Q}{b} \left[-\frac{1}{x+a} \right]_0^b =$$

$$= k_e \frac{Q}{b} \left[\frac{1}{a} - \frac{1}{a+b} \right] = k_e \frac{Q \cancel{b}}{\cancel{b}\, a(a+b)} =$$

$$= k_e Q \frac{1}{a(a+b)}$$

$b = 0{,}14$ m $\qquad a = 0{,}36 - 0{,}07 = 0{,}29$ m

$$\Rightarrow |\bar{E}_p| = 9 \cdot 10^9 \cdot 22 \cdot 10^{-6} \frac{1}{0{,}29 \cdot 0{,}43} \text{ V/m} =$$

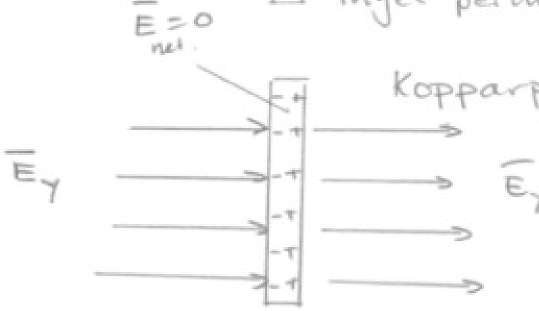
$$= \underline{\underline{1{,}59 \cdot 10^6 \text{ V/m}}}$$ riktat mot staven $(\because \parallel \hat{i})$

(19.44)

44. A square plate of copper with 50.0-cm sides has no net charge and is placed in a region of uniform electric field of 80.0 kN/C directed perpendicularly to the plate. Find (a) the charge density of each face of the plate and (b) the total charge on each face.

Lösning:

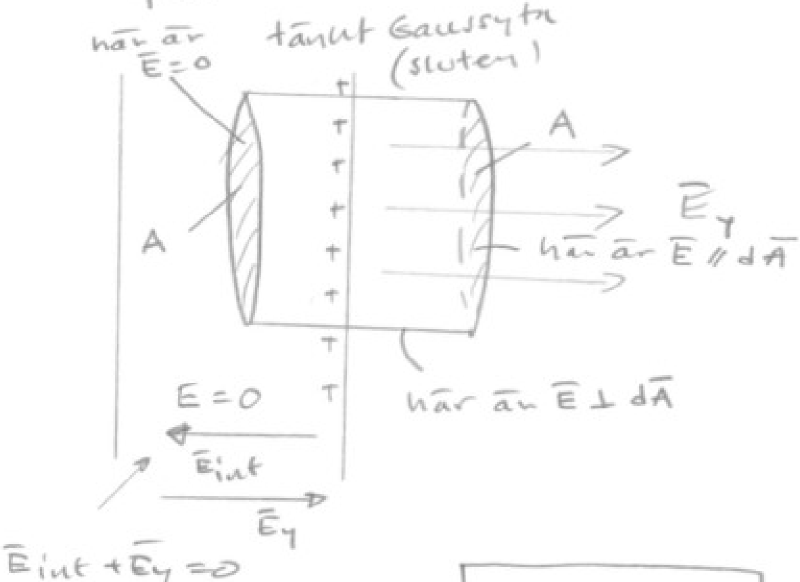
utan yttre elektriskt fält rör sig ledningselektronerna slumpmässigt
lika stor elektrontäthet överallt.
Nettoladdningen är noll!
Inget permanent elektriskt fält någonstans.



Kopparplattan i yttre el. fält.

Fortfarande inget (netto) fält inne i plattan
Omfördelning av elektronerna så att $E_{netto} = 0$

Bestäm utladdningstätheterna på kopparplattan ytor



Gauss sats.

$$\phi = \oint \bar{E} \cdot d\bar{A} = \frac{q_{in}}{\epsilon_0}$$

$$q_{in} = \sigma \cdot A$$

$$\Rightarrow E_Y \cdot A = \frac{\sigma \cdot A}{\epsilon_0}$$

$$\Rightarrow \boxed{\sigma = E\epsilon_0} = 80{,}0 \cdot 10^3 \cdot 8{,}85 \cdot 10^{-12} =$$

$$= 7{,}08 \cdot 10^{-7}\ C/m^2$$

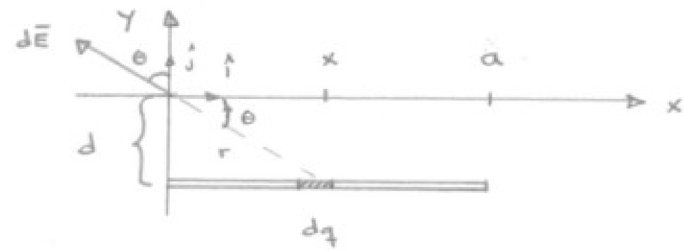
b) total laddning

$$Q = A \cdot \sigma = 0{,}500^2 \cdot 7{,}08 \cdot 10^{-7}\ C = 1{,}77 \cdot 10^{-7}\ C$$

19.64

64. A line of charge with uniform density 35.0 nC/m lies along the line $y = -15.0$ cm, between the points with coordinates $x = 0$ and $x = 40.0$ cm. Find the electric field it creates at the origin.

Lösning:



De olika delarna av staven ger dĒ:n som dels har olika belopp och dels olika riktningar.
Vår uppgift är att addera dessa dĒ:n dvs. integrera över stavens längd.

dĒ från ett dq på staven: $d\bar{E} = \frac{k_e\, dq}{r^2}\left(-\cos\theta\, \hat{i} + \sin\theta\, \hat{j}\right)$

men $dq = \frac{Q}{a} \cdot dx = \lambda \cdot dx$

$\cos\theta = \frac{x}{r}$, $\sin\theta = \frac{d}{r}$ $\qquad r^2 = d^2 + x^2$

$$\Rightarrow \quad d\bar{E} = \frac{k_e \lambda \cdot dx}{d^2 + x^2}\left[\frac{-x}{(d^2+x^2)^{1/2}}\hat{i} + \frac{d}{(d^2+x^2)^{1/2}}\hat{j}\right]$$

Totalt elektriskt fält: $\bar{E} = \int_0^a d\bar{E}$

Primitiv funktion till $\frac{1}{(d^2+x^2)^{3/2}}$ är $\frac{x}{d^2(d^2+x^2)^{1/2}}$

$\frac{-x}{(d^2+x^2)^{3/2}}$ är $\frac{1}{(d^2+x^2)^{1/2}}$

$$\therefore \quad \bar{E} = k_e \lambda \left\{\left[\frac{\hat{i}}{(x^2+d^2)^{1/2}}\right]_0^a + \left[\frac{x\,\hat{j}}{d(x^2+d^2)^{1/2}}\right]_0^a\right\} =$$

$d = 0{,}15$ m $\qquad a = 0{,}40$ m

$$= 9 \cdot 10^9 \cdot 35{,}0 \cdot 10^{-9}\left[(2{,}34 - 6{,}67)\,\hat{i} + 6{,}24\,\hat{j}\right]$$

$$\Rightarrow \quad \bar{E} = \underline{\underline{\left(-1{,}36\,\hat{i} + 1{,}96\,\hat{j}\right)\ \text{kN/C}}}$$

19.65

65. **Physics Now™** A solid, insulating sphere of radius a has a uniform charge density ρ and a total charge Q. Concentric with this sphere is an uncharged, conducting hollow sphere whose inner and outer radii are b and c as shown in Figure P19.65. (a) Find the magnitude of the electric field in the regions $r < a$, $a < r < b$, $b < r < c$, and $r > c$. (b) Determine the induced charge per unit area on the inner and outer surfaces of the hollow sphere.

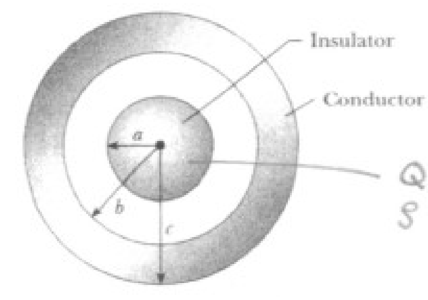


FIGURE **P19.65**

Lösning:

$$\Phi = \oint \bar{E} \cdot d\bar{A} = \frac{q_{in}}{\epsilon_0}$$

Isolerande sfären i mitten : $\rho = \dfrac{Q}{\frac{4}{3}\pi a^3}$

Gauss sats tillämplig pga hög symmetri.

a) E :

$r < a$ $\qquad E \cdot 4\pi r^2 = \dfrac{\frac{4}{3}\pi r^3 \frac{Q}{\frac{4}{3}\pi a^3}}{\epsilon_0} \Rightarrow E = \dfrac{Q}{4\pi\epsilon_0 a^3} r$

$a < r < b$ $\qquad E \cdot 4\pi r^2 = \dfrac{Q}{\epsilon_0} \Rightarrow E = \dfrac{Q}{4\pi\epsilon_0}\dfrac{1}{r^2}$

$b < r < c$ (ledare) $\qquad E = 0$

$r > c$ $\qquad E \cdot 4\pi r^2 = \dfrac{Q}{\epsilon_0} \Rightarrow E = \dfrac{Q}{4\pi\epsilon_0}\dfrac{1}{r^2}$

---

b) Inducerad laddningstäthet σ på den ledande ihåliga sfären :

Ytterradien $\qquad E \cdot 4\pi c^2 = \dfrac{\sigma \cdot 4\pi c^2}{\epsilon_0}$

men $E = \dfrac{Q}{4\pi\epsilon_0 c^2}$ $\qquad \Rightarrow$

$$\Rightarrow \frac{Q}{4\pi\cancel{\epsilon_0} c^2} \cdot \cancel{4\pi c^2} = \frac{\sigma \cdot \cancel{4\pi c^2}}{\cancel{\epsilon_0}} \Rightarrow \sigma_c = \frac{Q}{4\pi c^2}$$

Innerradien (negativ laddning)

p.s.s. $\quad \sigma_b = -\dfrac{Q}{4\pi b^2}$

fältet går in i Gaussytan : −

20.25

25. A rod of length L (Fig. P20.25) lies along the x axis with its left end at the origin. It has a nonuniform charge density $\lambda = \alpha x$, where α is a positive constant. (a) What are the units of α? (b) Calculate the electric potential at A.

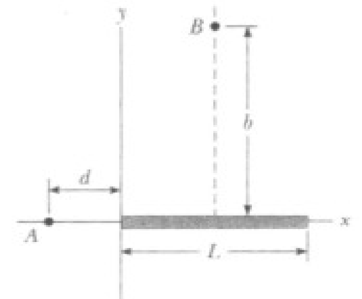


FIGURE **P20.25** Problems 20.25 and 20.26.

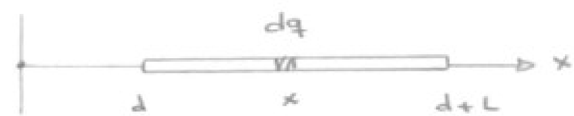
26. For the arrangement described in Problem 20.25, calculate the electric potential at point B that lies on the perpendicular bisector of the rod a distance b above the x axis.

Lösning:

Allmänt: potentialen på avståndet r från en punktladdning q

$$V(r) = k_e \frac{q}{r}$$

i) Potentialen i A om λ = konstant

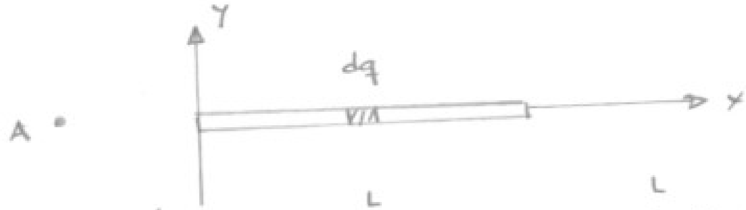


$$V_A = k_e \int_d^{d+L} \frac{dq}{x} =$$

$$dq = \lambda \cdot dx$$

$$= k_e \lambda \int_d^{d+L} \frac{dx}{r} = k_e \lambda \left[\ln(d+L) - \ln d \right]$$

ii) Potentialen i A med $\lambda = \alpha x$ (origo i stavens vä ände)



$$V_A = k_e \int_0^L \frac{dq}{x+d} = k_e \int_0^L \frac{\lambda \cdot dx}{x+d} = \alpha k_e \int_0^L \frac{x \cdot dx}{x+d}$$

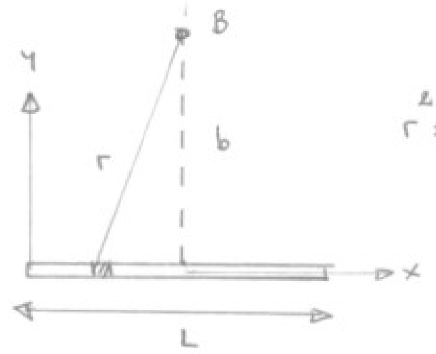
$z = x + d \Rightarrow x = z - d$ och $dx = dz$

$$\Rightarrow V_A = k_e \cdot \alpha \int_d^{L+d} \frac{(z-d)\,dz}{z} = k_e \alpha \int_d^{L+d} \left(1 - \frac{d}{z}\right) dz =$$

$$= k_e \alpha \left\{ \left[(L+d) - d\right] - d\left[\ln(L+d) - \ln d\right] \right\} =$$

$$= k_e \alpha \left[L - d \ln\left(\frac{L}{d} + 1\right) \right]$$

20.26



$$r^2 = b^2 + \left(\frac{L}{2} - x\right)^2$$

B :

$$V = k_e \int_0^L \frac{\alpha x \, dx}{\left[b^2 + \left(\frac{L}{2} - x\right)^2\right]^{1/2}}$$

Inför $z = \frac{L}{2} - x \Rightarrow dx = -dz$ och $x = \frac{L}{2} - z$

$$\therefore \quad V = k_e \alpha \int_{L/2}^{-L/2} \frac{\left(\frac{L}{2} - z\right)(-dz)}{(b^2 + z^2)^{1/2}} =$$

$$= -\frac{k_e \alpha L}{2} \int_{L/2}^{-L/2} \frac{dz}{(b^2 + z^2)^{1/2}} + k_e \alpha \int_{L/2}^{-L/2} \frac{z \cdot dz}{(b^2 + z^2)^{1/2}}$$

$\ln\left[z + \sqrt{z^2 + b^2}\right]$ är primitiv fkn till $\frac{1}{(b^2 + z^2)^{1/2}}$

$(b^2 + z^2)^{1/2}$ —"— $\frac{z}{(b^2 + z^2)^{1/2}}$

$$\therefore \quad V = -\frac{k_e \alpha L}{2} \left[\ln\left(z + \sqrt{z^2 + b^2}\right)\right]_{L/2}^{-L/2} +$$

$$+ k_e \alpha \left[\sqrt{b^2 + z^2}\right]_{L/2}^{-L/2} =$$

$$= -\frac{k_e \alpha L}{2} \ln \frac{-\frac{L}{2} + \sqrt{\left(\frac{L}{2}\right)^2 + b^2}}{\frac{L}{2} + \sqrt{\left(\frac{L}{2}\right)^2 + b^2}} + k_e \alpha \left[\sqrt{b^2 + \left(\frac{L}{2}\right)^2} - \sqrt{b^2 + \frac{L}{2}}\right] =$$

$$= -\frac{k_e \alpha L}{2} \ln \frac{\sqrt{b^2 + \frac{L^2}{4}} - \frac{L}{2}}{\sqrt{b^2 + \frac{L^2}{4}} + \frac{L}{2}} > 0$$