28.37

37. A free electron has a wave function

$$\psi(x) = Ae^{i(5.00\times 10^{10}x)}$$

where x is in meters. Find (a) its de Broglie wavelength, (b) its momentum, and (c) its kinetic energy in electron volts.

Lösning:

λ:

$$\psi = A \cdot e^{ikx} \qquad k = \frac{2\pi}{\lambda}$$

här $k = 5{,}00 \cdot 10^{10}$ m

$$\Rightarrow \lambda = \frac{2\pi}{k} = \frac{2\pi}{5{,}00 \cdot 10^{10}} = \underline{\underline{1{,}26 \cdot 10^{-10}\ \text{m}}}$$

---

p:

$$p = \hbar k = \frac{6{,}626 \cdot 10^{-34}}{2\pi} \cdot 5{,}00 \cdot 10^{10} = \underline{\underline{5{,}27 \cdot 10^{-24}}}\ \text{kgm/s}$$

K:

$$K = \frac{1}{2} mv^2 = \frac{1}{2} \frac{(mv)^2}{m} = \frac{1}{2} \frac{p^2}{m} =$$

$$= \frac{1}{2} \frac{(5{,}27 \cdot 10^{-24})^2}{9{,}11 \cdot 10^{-31}}\ \text{J} = 1{,}52 \cdot 10^{-17}\ \text{J} =$$

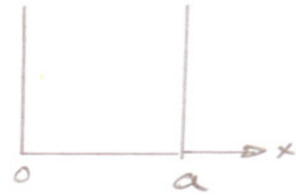
$$= \frac{1{,}52 \cdot 10^{-17}}{1{,}60 \cdot 10^{-19}}\ \text{eV} = \underline{\underline{95{,}5\ \text{eV}}}$$

Obs!

Delräkningarna
i lösn. ovan har
gjorts för att illustrera
de num. värdena.

28.41

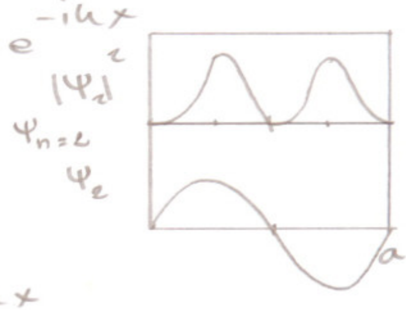
41. A photon with wavelength λ is absorbed by an electron confined to a box. As a result, the electron moves from state $n = 1$ to $n = 4$. (a) Find the length of the box. (b) What is the wavelength of the photon emitted in the transition of that electron from the state $n = 4$ to the state $n = 2$?



Lösning: Partikel i låda. $V(x) = \text{konstant}$

$$\Rightarrow \text{Schr. ekv.} \quad -\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + 0 = E\psi \quad \Rightarrow -\frac{\hbar^2}{2mE}\frac{\partial^2\psi}{\partial x^2} + \psi = 0$$

$$\Rightarrow \psi(x) = A \cdot e^{ikx} + B\, e^{-ikx}$$



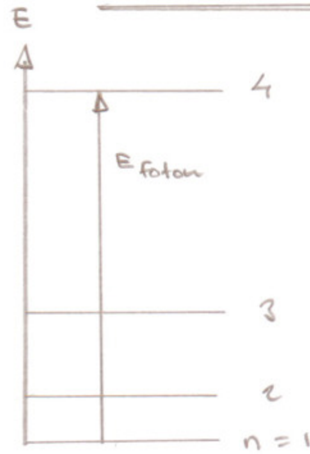
där $$k = \frac{\sqrt{2mE}}{\hbar} \quad \Rightarrow \quad E = \frac{\hbar^2}{2m}k^2$$

Randvillkor $\psi(x) = 0 \quad \Rightarrow \quad \psi(x) = C \cdot \sin kx$

$\psi(x = a) \quad \Rightarrow \quad k = n \cdot \frac{\pi}{a} \qquad n = \text{heltal}$

$$\therefore \psi(x) = C \cdot \sin\left[\left(n\frac{\pi}{a}\right) \cdot x\right]$$

$$E_n = \frac{\hbar^2}{2m}\left(n\frac{\pi}{a}\right)^2 = \frac{h^2\pi^2}{(2\pi)^2\, 2m a^2} n^2 = \underline{\underline{\frac{h^2}{8ma^2} n^2}}$$



$$E_{abs} = E_{foton} = E_4 - E_1 =$$

$$= (4^2 - 1^2)\frac{h^2}{8ma^2} = \frac{hc}{\lambda}$$

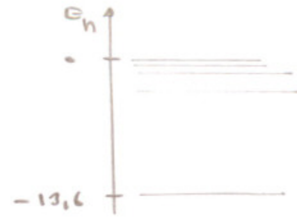
$$\Rightarrow a = \left(\frac{15\lambda}{8mc}\right)^{1/2}$$

$$E_4 - E_2 = (4^2 - 2^2)\frac{h^2}{8ma^2} = \frac{hc}{\lambda'}$$

$$\Rightarrow 12\frac{h^2}{8ma^2} = \frac{hc}{\lambda'} \qquad \therefore \frac{\lambda'}{\lambda} = \frac{15}{12} = \underline{\underline{1{,}25}}$$

29.6

6. A photon with energy 2.28 eV is barely capable of causing a photoelectric effect when it strikes a sodium plate. Suppose the photon is instead absorbed by hydrogen. Find (a) the minimum n for a hydrogen atom that can be ionized by such a photon and (b) the speed of the released electron far from the nucleus.



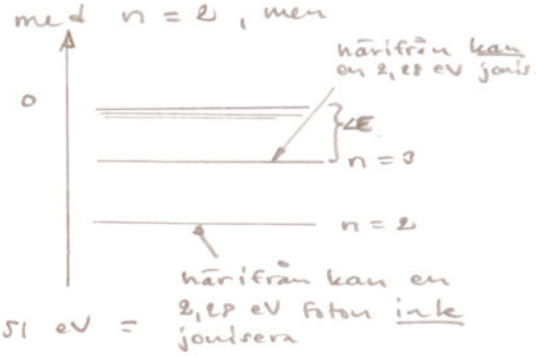
Lösning:

$E_{foton} = 2,28$ eV Väte: $E_n = -13,6 \frac{1}{n^2}$ eV

vilket n' svarar mot $E_{n'} = -2,28$ eV $\Rightarrow n'^2 = \frac{13,6}{2,28} \Rightarrow n' = 2,44$

(n' ej nödvändigtvis ett heltal.)

$\therefore$ fotonen kan inte jonisera en atom med $n = 2$, men däremot en atom i $n = 3$



$E_3 = -1,51$ eV $\Rightarrow \Delta E = 1,51$ eV

Rörelseenergi: $E_k = E_{foton} - \Delta E$.

$$\Rightarrow \frac{1}{2}mv^2 = E_{foton} - \Delta E = 2,28 - 1,51 \text{ eV} = $$
$$= 0,769 \text{ eV} = 0,769 \cdot 1,6 \cdot 10^{-19} \text{ J}.$$

$$\Rightarrow v^2 = \frac{2 \cdot 0,769 \cdot 1,6 \cdot 10^{-19}}{9,11 \cdot 10^{-31}} (m/s)^2 \Rightarrow v = 5,20 \cdot 10^5 \text{ m/s} =$$
$$\underline{\underline{= 520 \text{ km/s}}}.$$

29.11

11. The ground-state wave function for a hydrogen atom is

$$\psi_{1s}(r) = \frac{1}{\sqrt{\pi a_0^3}} e^{-r/a_0}$$

where r is the radial coordinate of the electron and a_0 is the Bohr radius. (a) Show that the wave function as given is normalized. (b) Find the probability of locating the electron between $r_1 = a_0/2$ and $r_2 = 3a_0/2$.

Partialintegration:

$$\int u\,dv = [uv] - \int v \cdot du$$

här: $u = r^2$
$v = -\frac{a_0}{2} e^{-2r/a_0}$

a) Normalisering: $\int_0^\infty |\psi^2| \cdot 4\pi r^2 \cdot dr = 1$

$$\int_0^\infty r^2 \cdot e^{-2r/a_0} = \left[-\frac{a_0}{2} e^{-2r/a_0} \cdot r^2\right]_0^\infty - \int_0^\infty -\frac{a_0}{2} e^{-2r/a_0} \cdot 2r \cdot dr$$

$$\int_0^\infty a_0 e^{-2r/a_0} \cdot r\,dr = \left[-a_0\left(\frac{a_0}{2}\right) e^{-2r/a_0} \cdot r\right]_0^\infty - \int -\left(\frac{a_0^2}{2}\right) e^{-2r/a_0} dr$$

$$\int_0^\infty \frac{a_0^2}{2} e^{-2r/a_0} dr = \left[-\frac{a_0^3}{4} e^{-2r/a_0}\right]_0^\infty$$

$$\therefore \int_0^\infty |\psi|^2 4\pi r^2 \cdot dr = \frac{4\pi}{\pi a_0^3}\left[-\frac{1}{2} e^{-2r/a_0}\left(a_0 r^2 + a_0^2 r + \frac{1}{2} a_0^3\right)\right] = [\text{end. undr. gränsen} \neq 0]$$

$$= \frac{4}{a_0^3} \frac{1}{2} e^0 \left(0 + 0 + \frac{1}{2} a_0^3\right) = 1.$$

29.11

Lösning:

a) normalisering: $\int_0^\infty |\psi|^2 4\pi r^2\,dr = 1$

Beräkna: $\int_0^\infty e^{-2r/a_0} \cdot r^2 = \overbrace{\left[r^2\left(-\frac{a_0}{2}\right)e^{-2r/a_0}\right]_0^\infty}^{A} -$

$- \int \left(-\frac{a_0}{2}\right) e^{-2r/a_0} \cdot 2r\,dr = A - \overbrace{\left[\left(\frac{a_0}{2}\right)^2 e^{-2r/a_0} \cdot 2r\right]_0^\infty}^{B} +$

$+ \int \left(\frac{a_0}{2}\right)^2 e^{-2r/a_0} \cdot 2\,dr =$

$= A - B - \left[2\left(\frac{a_0}{2}\right)^3 e^{-2r/a_0}\right]_0^\infty = 0 + 0 + \frac{a_0^3}{4}$

$\Rightarrow \frac{1}{\cancel{\pi} a_0^3} \cdot 4\cancel{\pi} \cdot \frac{a_0^3}{4} = 1$ v.s.v.

Sannolikheten att finna electronen i $\left[\frac{a_0}{2}, 3\frac{a_0}{2}\right]$:

$\int_{a_0/2}^{3a_0/2} e^{-2r/a_0} \cdot r^2 = \left[r^2\left(-\frac{a_0}{2}\right)e^{-2r/a_0}\right]_{\frac{a_0}{2}}^{\frac{3a_0}{2}} - \left[\left(\frac{a_0}{2}\right)^2 e^{-2r/a_0} \cdot 2r\right]_{\frac{a_0}{2}}^{\frac{3a_0}{2}} -$

$- \left[\frac{a_0^3}{4} e^{-2r/a_0}\right]_0^\infty =$

$\boxed{\text{Svar: } 0{,}497}$

$= \frac{9a_0^2}{4}\left(-\frac{a_0}{2}\right)e^{-3a_0/a_0} - \left(\frac{a_0^2}{4}\right)\left(-\frac{a_0}{2}\right)e^{-1} - \left(\frac{a_0}{2}\right)^2 e^{-3} \cdot 3a_0 +$

$+ \left(\frac{a_0}{2}\right)^2 e^{-1} \cdot a_0 - \frac{a_0^3}{4}e^{-3} + \frac{a_0^3}{4}e^{-1} = a_0^3 \frac{5}{8} e^{-1} - a_0^3 \frac{17}{8} e^{-3}$

$\frac{1}{\pi a_0^3} \cdot 4\pi \cdot \left(a_0^3 \frac{5}{8} e^{-1} - a_0^3 \frac{17}{8} e^{-3}\right) = \frac{20}{8} e^{-1} - \frac{68}{8} e^{-3} = 0{,}9197 - 0{,}4231 =$

29.51

Explain your reasoning.

51. Assume that three identical uncharged particles of mass m and spin $\frac{1}{2}$ are contained in a one-dimensio box of length L. What is the ground-state energy of t system?

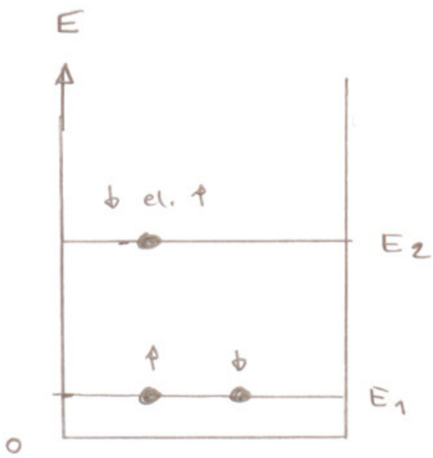
Lösning

$$\left.\begin{aligned} E &= \frac{\hbar^2}{2m} k^2 \\ k &= n\frac{\pi}{L} \end{aligned}\right\} \Rightarrow E_n = \frac{h^2}{8mL^2} n^2$$

$$E_1 = \frac{h^2}{8mL^2}$$

$$E_2 = 4E_1$$

$$E_3 = 9E_1 \quad \text{o.s.v.}$$



Pauliprincipen säger att två elektroner som befinner sig i samma system inte får ha identiska uppsättningar kvanttal $\uparrow\downarrow$

Här (1 dim. problem) är kvanttalen n och spinnet

⇒ Två elektroner kan finnas på varje energinivå. E beror här inte av spinnet

Den totala energin hos grundtillståndet (= den fördelning av el. som har lägst total energi) blir

$$2E_1 + E_2 = 2E_1 + 4E_1 = 6E_1 =$$

$$= 6\frac{h^2}{8mL^2} = \underline{\underline{\frac{3}{4}\frac{h^2}{mL^2}}}$$