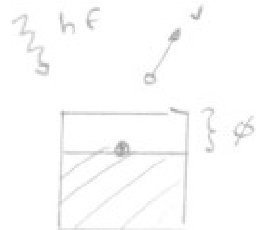
28.10

10. Electrons are ejected from a metallic surface with speeds ranging up to 4.60×10^5 m/s when light with a wavelength of 625 nm is used. (a) What is the work function of the surface? (b) What is the cutoff frequency for this surface?

Lösning:



a) $hf - \phi = \frac{1}{2} m v_{max}^2$

$\Rightarrow \phi = hf - \frac{1}{2} m v_{max}^2$

I eV: $\phi = \frac{hc}{e\lambda} - \frac{1}{2e} m v_{max}^2 = \frac{1}{1{,}6 \cdot 10^{-19}} \left(\frac{6{,}64 \cdot 10^{-34} \cdot 3 \cdot 10^8}{625 \cdot 10^{-9}} - \frac{9.11 \cdot 10^{-31} (4.6 \cdot 10^5)^2}{2} \right)$

$= 1.38$ eV. (Lågt!)

$\frac{hf_c}{e} = 1{,}38 \Rightarrow f_c = \frac{1.6 \cdot 10^{-19} \cdot 1{,}38}{6{,}64 \cdot 10^{-34}} = 3{,}34 \cdot 10^{14}$ Hz

28.38

38. An electron that has an energy of approximately 6 eV moves between rigid walls 1.00 nm apart. Find (a) the quantum number n for the energy state that the electron occupies and (b) the precise energy of the electron.

Lösning:

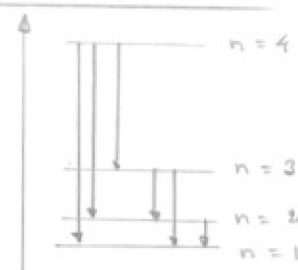
$E_n = \frac{n^2 h^2}{8m \cdot a^2} \Rightarrow n^2 = \frac{E_n \cdot 8 \cdot m \cdot a^2}{h^2} =$

$= [\text{allt i SI-enheter}] = \frac{6 \cdot 1{,}6 \cdot 10^{-19} \cdot 8 \cdot 9.11 \cdot 10^{-31} \cdot (1{,}0 \cdot 10^{-9})^2}{(6{,}64 \cdot 10^{-34})^2}$

$\Rightarrow n = 3{,}98 = \underline{\underline{4.}}$

28.39

39. Physics Now™ An electron is contained in a one-dimensional box of length 0.100 nm. (a) Draw an energy level diagram for the electron for levels up to $n = 4$. (b) Find the wavelengths of all photons that can be emitted by the electron in making downward transitions that could eventually carry it from the $n = 4$ state to the $n = 1$ state.



Lösning:

$E_n = n^2 \frac{h^2}{8ma^2}$ $\quad \Delta E = (n_i^2 - n_f^2) \frac{h^2}{8ma^2}$

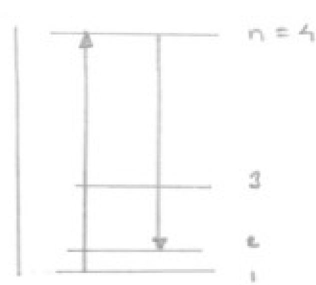
$\Delta E = hf = \frac{hc}{\lambda} \Rightarrow \lambda = \frac{hc}{\Delta E} = \frac{hc \cdot 8ma^2}{h^2} \frac{1}{n_i^2 - n_f^2} = \frac{8ma^2 c}{h} \frac{1}{n_i^2 - n_f^2} =$

$= \frac{8 \cdot 9.11 \cdot 10^{-31} \cdot (0{,}100 \cdot 10^{-9})^2 \cdot 3 \cdot 10^8}{6{,}64 \cdot 10^{-34}} \frac{1}{n_i^2 - n_f^2} = 32{,}9 \cdot 10^{-9} \frac{1}{n_i^2 - n_f^2}$

	$1/n_i^2 - n_f^2$		
$n_i = 4$			
$n_f = 3$	$1/(16-9)$	$\Rightarrow$	$\lambda = 4{,}70$ nm
$n_f = 2$	$1/(16-4)$	$\Rightarrow$	$2{,}75$
$n_f = 1$	$1/(16-1)$	$\Rightarrow$	$2{,}20$
$n_i = 3$			
$n_f = 2$	$1/(9-4)$	$\Rightarrow$	$6{,}60$
$n_f = 1$	$1/(9-1)$	$\Rightarrow$	$4{,}12$
$n_i = 2$ $n_f = 1$	$1/(4-1)$	$\Rightarrow$	11.0

28.41

41. A photon with wavelength λ is absorbed by an electron confined to a box. As a result, the electron moves from state $n = 1$ to $n = 4$. (a) Find the length of the box. (b) What is the wavelength of the photon emitted in the transition of that electron from the state $n = 4$ to the state $n = 2$?



Lösning:

$n = 1 \rightarrow n = 4 \qquad E_n = n^2 \dfrac{h^2}{8ma^2}$

$$\Delta E_{1\to 4} = 4^2 \frac{h^2}{8ma^2} - 1^2 \frac{h^2}{8ma^2} = 15 \frac{h^2}{8ma^2}$$

$$\Delta E = \frac{hc}{\lambda} \Rightarrow \lambda = \frac{hc}{\Delta E} = \frac{c\, 8ma^2}{15h} \Rightarrow a = \left(\frac{15h\lambda}{8mc}\right)^{1/2}$$

$n = 4 \rightarrow n = 2 \qquad \Rightarrow \lambda' = \dfrac{8ma^2c}{12h}$

$4^2 - 2^2 = 12 \qquad \Rightarrow \lambda' = 1{,}25\,\lambda \quad \dfrac{15}{12}$

28.46

46. The wave function for a particle confined to moving in a one-dimensional box is

$$\psi(x) = A \sin\left(\frac{n\pi x}{L}\right)$$

Use the normalization condition on ψ to show that

$$A = \sqrt{\frac{2}{L}}$$

(*Suggestion:* Because the box length is L, the wave function is zero for $x < 0$ and for $x > L$, so the normalization condition, Equation 28.23, reduces to $\int_0^L |\psi|^2\, dx = 1$.)

Lösning:

$$\int_0^L |\psi(x)|^2 \cdot dx = 1 \Rightarrow \int_0^L A^2 \cdot \sin^2\left(\frac{n\pi x}{L}\right) dx = 1$$

Skriv om $\sin^2\alpha$! $\qquad \sin^2\alpha = \dfrac{1}{2} - \dfrac{\cos 2\alpha}{2}$

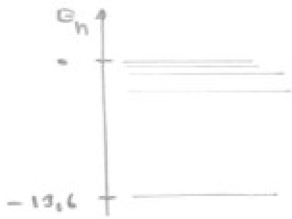
$$\Rightarrow \int_0^L A^2\left[\frac{1}{2} - \frac{\cos 2\left(\frac{n\pi x}{L}\right)}{2}\right] dx = 1 \Rightarrow A^2\left[\frac{x}{2} - \frac{\sin 2\left(\frac{n\pi x}{L}\right)}{\frac{2L}{n\pi x}}\right]_0^L = 1$$

$$\Rightarrow A^2 \cdot \frac{L}{2} = 1 \Rightarrow \underline{\underline{A = \sqrt{\frac{2}{L}}}}$$

$$\sin 2\left(\frac{n\pi L}{L}\right) = 0$$

(29.6)

6. A photon with energy 2.28 eV is barely capable of causing a photoelectric effect when it strikes a sodium plate. Suppose the photon is instead absorbed by hydrogen. Find (a) the minimum n for a hydrogen atom that can be ionized by such a photon and (b) the speed of the released electron far from the nucleus.



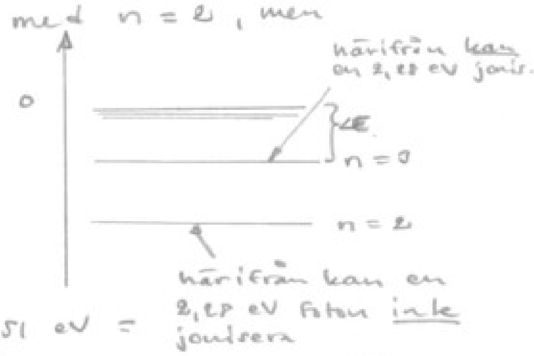
Lösning: $E_{foton} = 2,28$ eV Väte: $E_n = -13,6 \dfrac{1}{n^2}$ eV

vilket n' svarar mot $E_{n'} = -2,28$ eV $\Rightarrow n'^2 = \dfrac{13,6}{2,28} \Rightarrow n' = 2,44$

(n' ej nödvändigtvis ett heltal.)

$\therefore$ fotonen kan inte jonisera en atom med $n = 2$, men däremot en atom i $n = 3$

$E_3 = -1,51$ eV $\Rightarrow \Delta E = 1,51$ eV



Rörelseenergi: $E_k = E_{foton} - \Delta E$.

$$\Rightarrow \frac{1}{2} m v^2 = E_{foton} - \Delta E = 2,28 - 1,51 \text{ eV} = 0,769 \text{ eV} = 0,769 \cdot 1,6 \cdot 10^{-19} \text{ J}.$$

$$\Rightarrow v^2 = \frac{2 \cdot 0,769 \cdot 1,6 \cdot 10^{-19}}{9,11 \cdot 10^{-31}} (m/s)^2 \Rightarrow v = 5,20 \cdot 10^5 \text{ m/s} =$$

$$= \underline{\underline{520 \text{ km/s}}}.$$

(29.11)

11. The ground-state wave function for a hydrogen atom is

$$\psi_{1s}(r) = \frac{1}{\sqrt{\pi a_0^3}} e^{-r/a_0}$$

where r is the radial coordinate of the electron and a_0 is the Bohr radius. (a) Show that the wave function as given is normalized. (b) Find the probability of locating the electron between $r_1 = a_0/2$ and $r_2 = 3a_0/2$.

Partialintegration:

$$\int u \, dv = [uv] - \int v \cdot du$$

här: $u = r^2$

$v = -\dfrac{a_0}{2} e^{-2r/a_0}$

a) Normalisering: $\displaystyle\int_0^\infty |\psi|^2 \cdot 4\pi r^2 \cdot dr = 1$

$$\int_0^\infty r^2 \cdot e^{-2r/a_0} = \left[-\frac{a_0}{2} e^{-2r/a_0} \cdot r^2\right]_0^\infty - \int_0^\infty -\frac{a_0}{2} e^{-2r/a_0} \cdot 2r \cdot dr$$

$$\int_0^\infty a_0 e^{-2r/a_0} \cdot r \, dr = \left[-a_0\left(\frac{a_0}{2}\right) e^{-2r/a_0} \cdot r\right]_0^\infty - \int -\left(\frac{a_0^2}{2}\right) e^{-2r/a_0} dr$$

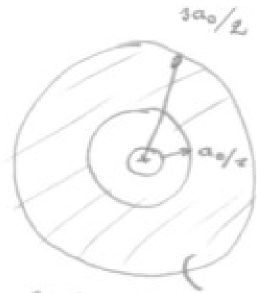
$$\int_0^\infty \frac{a_0^2}{2} e^{-2r/a_0} dr = \left[-\frac{a_0^3}{4} e^{-2r/a_0}\right]_0^\infty$$

$$\therefore \int_0^\infty |\psi|^2 4\pi r^2 \cdot dr = \frac{4\pi}{\pi a_0^3}\left[-\frac{1}{2} e^{-2r/a_0}\left(a_0 r^2 + a_0^2 r + \frac{1}{2} a_0^3\right)\right] = [\text{end. undr. gränsen} \neq 0]$$

$$= \frac{4}{a_0^3} \cdot \frac{1}{2} e^0 \left(0 + 0 + \frac{1}{2} a_0^3\right) = 1.$$

29.11 Forts.

$$P = \int_{a_0/2}^{3a_0/3} |\psi|^2 \cdot 4\pi r^2 \cdot dr$$



$$\Rightarrow P = \frac{4}{a_0{}^3}\left[-\frac{1}{2}e^{-2r/a_0}\left(a_0 r^2 + a_0^2 r + \frac{1}{2}a_0^3\right)\right]_{a_0/2}^{3a_0/2} =$$

P = sannolikhet att hitta elektronen i denna volym

$$= \frac{4}{a_0^3}\left[\frac{1}{2}\left[e^{-2\frac{a_0/2}{a_0}}\left(a_0\frac{a_0^2}{4} + a_0^2\frac{a_0}{2} + \frac{1}{2}a_0^3\right) - e^{-2\,\frac{3a_0}{2}/a_0}\left(a_0\frac{9a_0^2}{4} + a_0^2\frac{3a_0}{2} + \frac{1}{2}a_0^3\right.\right.\right.$$

$$= \frac{2}{a_0^3}\left\{e^{-1}\left[\frac{a_0^3}{4} + \frac{a_0^3}{2} + \frac{a_0^3}{2}\right] - e^{-3}\left[\frac{9a_0^3}{4} + \frac{3}{2}a_0^3 + \frac{1}{2}a_0^3\right]\right\} =$$

$$= \frac{2}{a_0^3}\left[e^{-1}\left(\frac{5}{4}a_0^3\right) - e^{-3}\left(\frac{17}{4}a_0^3\right)\right] =$$

$$= e^{-1}\frac{5}{2} - e^{-3}\frac{17}{2} = 0{,}9197 - 0{,}4232 = \underline{\underline{0{,}497}}$$