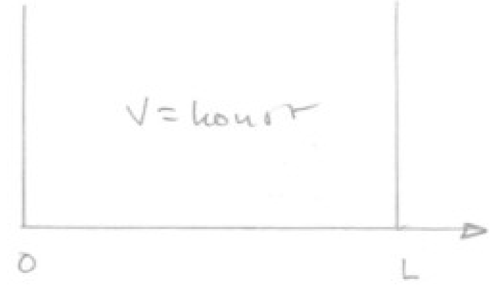
Partikel i låda.



$$\Psi = C \cdot \sin kx$$

$$k = n\frac{\pi}{L} \qquad \sin kL = 0$$

$$E = \frac{\hbar^2}{2m}k^2 = \frac{h^2}{8mL^2} \qquad \Psi = C \cdot \sin\left(n\frac{\pi}{L}x\right)$$

Övning:

1) Bestäm C! normering: $\int_0^L |\Psi|^2 \cdot dx = 1$

$$\Rightarrow \int_0^L C^2 \sin^2 kx \cdot dx = 1 \qquad \boxed{\sin^2\alpha = \frac{1}{2} - \frac{1}{2}\cos 2\alpha}$$

$$\Rightarrow \int_0^L C^2\left(\frac{1}{2} - \frac{1}{2}\cos 2kx\right)dx = 1$$

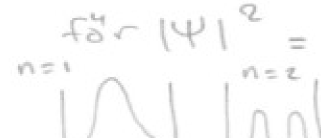
$$\Rightarrow C^2\left[\frac{x}{2} - \frac{1}{2}\frac{1}{2k}\sin 2kx\right]_0^L =$$

$$= C^2\left[\frac{L}{2} - \frac{1}{4k}\underbrace{\sin 2kL}_{=0}\right] = C^2\frac{L}{2} = 1$$

$$\Rightarrow C = \sqrt{\frac{2}{L}} \quad \text{oberoende av } k.$$

$$\Rightarrow \begin{cases} \Psi_1 = \sqrt{\frac{2}{L}}\sin\left(\frac{\pi}{L}x\right) \\ \Psi_2 = \sqrt{\frac{2}{L}}\sin\left(2\frac{\pi}{L}x\right) \\ \Psi_3 = \sqrt{\frac{2}{L}}\sin\left(3\frac{\pi}{L}x\right) \\ \text{osv.} \end{cases}$$

Antalet max för $|\Psi|^2 = n$

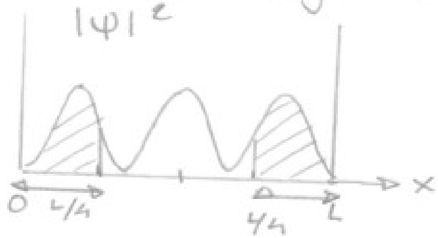


Ytorna under kurvorna = 1

n = 3 se nästa exempel

2) Bestäm sannolikheten att finna elektronen inom avståndet $L/4$ från någon av väggarna. om $n = 3$.

$$\psi = \sqrt{\frac{2}{L}} \sin\left(3\frac{\pi}{L}x\right)$$



Den efterfrågade sannolikheten är den streckade ytan i förhållande till den totala ytan under de tre puckarna.
Grafiskt ser vi att sannolikheten ligger i intervallet $[0{,}5 \,,\, 1]$

$$P = 2\int_0^{L/4} |\psi|^2 dx = 2\sqrt{\frac{2}{L}}^2 \int_0^{L/4} \sin^2\left(3\frac{\pi}{L}x\right) dx =$$

$$= \frac{4}{L}\int_0^{L/4} \left[\frac{1}{2} - \frac{1}{2}\cos\left(2\,\frac{3\pi}{L}x\right)\right] dx =$$

$$= \frac{4}{L}\left[\frac{x}{2} - \frac{1}{2\left(2\,\frac{3\pi}{L}\right)}\sin\left(2\,\frac{3\pi}{L}x\right)\right]_0^{L/4} =$$

$$= \frac{4}{L}\left[\frac{L/4}{2} - \frac{1}{12\pi/L}\sin 2\,\frac{3\pi}{L}\frac{L}{4}\right] =$$

$$= \frac{4}{L}\left[\frac{L}{8} - \frac{L}{12\pi}\underbrace{\sin\frac{6\pi}{4}}_{= \sin 3\frac{\pi}{2} = -1}\right] = \frac{1}{2} + \frac{1}{3\pi} =$$

$$= 0{,}5 + 0{,}106 = \underline{\underline{0{,}606}}$$