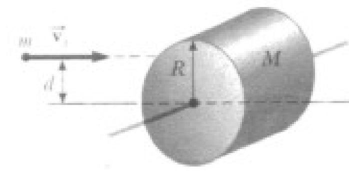
72. A wad of sticky clay with mass m and velocity $\vec{\mathbf{v}}_i$ is fired at a solid cylinder of mass M and radius R (Fig. P10.72). The cylinder is initially at rest and is mounted on a fixed horizontal axle that runs through its center of mass. The line of motion of the projectile is perpendicular to the axle and at a distance $d < R$ from the center. (a) Find the angular speed of the system just after the clay strikes and sticks to the surface of the cylinder. (b) Is mechanical energy of the clay–cylinder system conserved in this process? Explain your answer.



a) ω_f :

Låt lerklump och cylinder bilda ett slutet system.
Inga externa krafter som ger vrid. moment
m.a.p. axeln O.

$$\bar{L} = \bar{r} \times (m\bar{v})$$

$$\Rightarrow \bar{L}_i = \bar{L}_f$$

$$\bar{L}_i = d\,\hat{j} \times (m v_i\,\hat{i}) = d m v_i (\underbrace{\hat{j} \times \hat{i}}_{-\hat{z}}) = -d m v_i\,\hat{k}$$

$$\bar{L}_f = \bar{\omega}_f \cdot I = \bar{\omega}_f \left(\frac{1}{2}MR^2 + mR^2\right)$$

$$\bar{\omega}_f = \omega_f \cdot \hat{z}$$

kom ihåg: $\omega_f > 0$ innebär rot. moturs
$\omega_f < 0$ — " — medurs.

$$\bar{L}_i = \bar{L}_f \Rightarrow -d m v_i\,\hat{k} = \omega_f \left(\frac{1}{2}MR^2 + mR^2\right)\hat{k}$$

$$\Rightarrow \omega_f = \frac{-d m v_i}{\left(\frac{1}{2}MR^2 + mR^2\right)} < 0 \Rightarrow$$ medurs

---

träff på undersidan:

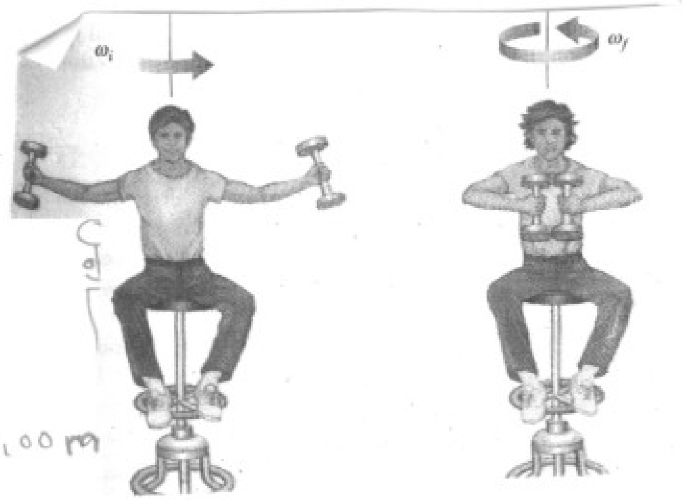
$$\Rightarrow L_i = (-d\hat{j}) \times m v\,\hat{i}$$
$$= +d m v\,\hat{k} \ \ldots$$

$$\ldots \Rightarrow \omega_f = \frac{+d m v_i}{\frac{1}{2}MR^2 + mR^2} > 0 \Rightarrow$$ moturs.

10.48

11.30

A student sits on a freely rotating stool holding two weights, each of mass 3.00 kg (Figure P11.30). When his arms are extended horizontally, the weights are 1.00 m from the axis of rotation and he rotates with an angular speed of 0.750 rad/s. The moment of inertia of the student plus stool is 3.00 kg · m² and is assumed to be con-



Givet: $m = 3{,}00$ kg $\quad r_1 = 1{,}00$ m

$\omega_i = 0{,}750$ rad/s

$I_0 = 3{,}00$ kgm² (student + stol)

$r_2 = 0{,}300$ m

Sökt: ω_f

Inga yttre vridande moment på systemet

$$\Rightarrow L_i = L_f$$

$$L_i = I_0 \omega_i + 2 m r_1^2 \cdot \omega_i$$

$$L_f = I_0 \omega_f + 2 m r_2^2 \cdot \omega_f$$

$$\therefore \quad \omega_f = \frac{I_0 + 2mr_1^2}{I_0 + 2mr_2^2}\,\omega_i = \frac{3{,}00 + 2 \cdot 3{,}00 \cdot 1{,}00^2}{3{,}00 + 2 \cdot 3{,}00 \cdot 0{,}300^2}\,0{,}75 =$$

$$= \underline{1{,}91 \text{ rad/s}}$$

Rot. energier:

$$K_i = \frac{1}{2}\left(I_0 + 2mr_1^2\right) \cdot \omega_i^2 = 2{,}53 \text{ J}$$

$$K_f = \frac{1}{2}\left(I_0 + 2mr_2^2\right) \cdot \omega_f^2 = 6{,}44 \text{ J}$$

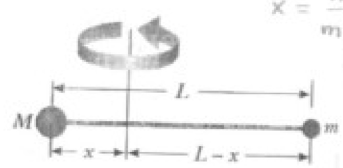
$\therefore$ $6{,}44 - 2{,}53 = 3{,}91$ J uträttas av muskelarbetet

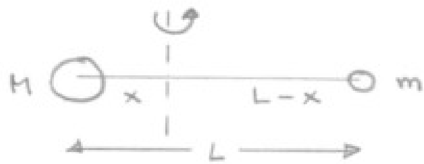
(10, 22)

22. Two balls with masses M and m are connected by a rigid rod of length L and negligible mass as in Figure P10.22. For an axis perpendicular to the rod, show that the system has the minimum moment of inertia when the axis passes

$x = \frac{m}{m+M} L$

finns inte
i Principles...





Visa att $I_{min} = \mu L^2$ där $\mu = \frac{mM}{m+M}$

$$I = Mx^2 + m(L-x)^2 =$$

$$= Mx^2 + mL^2 - 2mLx + mx^2 \quad \Rightarrow \quad \frac{dI}{dx} = 2Mx - 2mL + 2mx$$

$$\frac{dI}{dx} = 0 \quad \text{om} \quad \boxed{x = \frac{m}{M+m} L}$$

$$\Rightarrow I_{min} = M\left(\frac{m}{M+m}\right)^2 L^2 + m\left(L - \frac{m}{M+m}L\right)^2 =$$

$$= M\left(\frac{m}{M+m}\right)^2 + m\left(\frac{M+m-m}{M+m}\right)^2 L^2 =$$

$$= \left[\frac{M^2 m}{(M+m)^2} + \frac{m^2 M}{(M+m)^2}\right] L^2 =$$

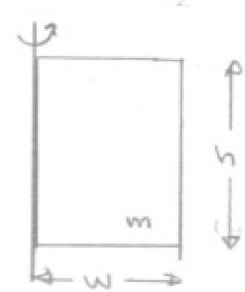
$$= \frac{Mm(M+m)}{(M+m)^2} L^2 = \underline{\underline{\frac{Mm}{M+m} L^2}} \quad \text{V.S.V.}$$

(10, 25)

A uniform thin solid door has height 2.20 m, width 0.870 m, and mass 23.0 kg. Find its moment of inertia for rotation on its hinges. Is any piece of data unnecessary?

$$I = \frac{1}{3} m w^2 = \frac{1}{3}\, 23{,}0 \cdot 0{,}87^2 =$$

$$\underline{\underline{= 5{,}80 \text{ kgm}^2}}$$

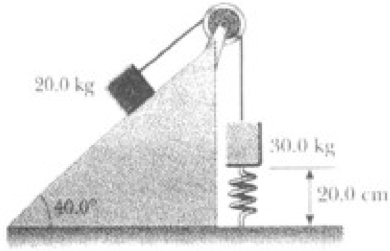


höjden 'onödig'

7 59

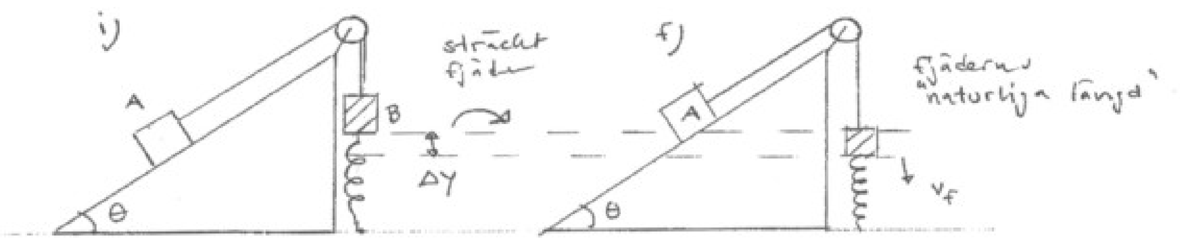
8.59

A 20.0-kg block is connected to a 30.0-kg block by a string that passes over a light frictionless pulley. The 30.0-kg block is connected to a spring that has negligible mass and a force constant of 250 N/m, as shown in Figure P8.59. The spring is unstretched when the system is as shown in the figure, and the incline is frictionless. The 20.0-kg block is pulled 20.0 cm down the incline (so that the 30.0-kg block is 40.0 cm above the floor) and released from rest. Find the speed of each block when the 30.0-kg block is 20.0 cm above the floor (that is, when the spring is unstretched).



Lösning:

friktionsfritt



Givet: $\theta = 40°$ $m_A = 20{,}0$ kg $m_B = 30{,}0$ kg $k = 250$ N/m
$\Delta y = 0{,}20$ m $v_i = 0$ f: fjädern relaxerad.

Sökt: v_f

Den mekaniska energin bevaras eftersom de krafter som är relevanta är konservativa.

$$\Rightarrow \Delta E_{mek} = E_{mek,f} - E_{mek,i} \quad \text{dvs "(efter) − (före)"} = 0$$

$$\Delta E_{mek} = \Delta E_{mek,A} + \Delta E_{mek,B} + \Delta E_{mek,fjäder} = 0$$

$$\Delta E_{mek,A} = \frac{1}{2} m_A v_f^2 + m_A g \Delta y \cdot \sin\theta$$

$$\Delta E_{mek,B} = \frac{1}{2} m_B v_f^2 - m_B g \Delta y - 0$$

$$\Delta E_{mek,fj} = 0 - \frac{1}{2} k (\Delta y^2)$$

$$\Rightarrow \frac{1}{2} m_A v_f^2 + m_A g \Delta y \sin\theta + \frac{1}{2} m_B v_f^2 - m_B g \Delta y - \frac{1}{2} k (\Delta y)^2$$

$$\Rightarrow v_f = \sqrt{\frac{2 g \Delta y \left(\frac{m_B}{m_A} - \sin\theta\right) + \frac{k}{m_A} (\Delta y)^2}{1 + \frac{m_B}{m_A}}} =$$

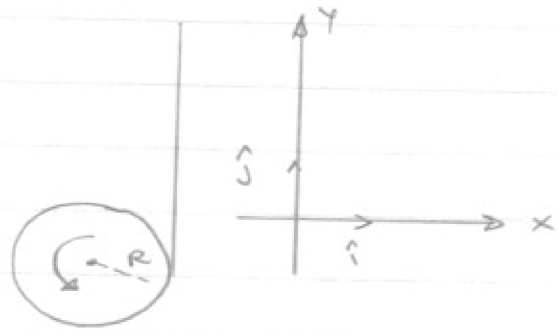
$$= \sqrt{\frac{2 \cdot 9{,}81 \cdot 0{,}20 \left(\frac{30}{20} - \sin 40\right) + \frac{250}{20} (0{,}20)^2}{1 + \frac{30}{20}}} = \underline{\underline{1{,}24 \text{ m/s}}}$$

8

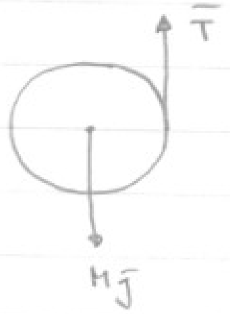
En enkel jo-jo består av ett snöre som är lindat runt en solid cylinder med massa M och radie R. Om man håller änden av snöret stilla och släpper cylindern så kommer den att sänkas under rotation.
Bestäm accelerationen hos cylinderns tyndpunkt och spännkraften i snöret. Cylinderns tröghetsmoment är 1/2 MR². (4 p)

1) $\sum_i \bar{F}_i = M\bar{a}_{cm}$

2) $\sum_i \bar{\tau}_i = I\bar{\alpha}$



1)

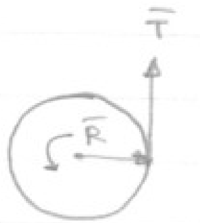


$M\bar{g} + \bar{T} = M\bar{a}_{cm}$

$Mg(-\hat{j}) + T(\hat{j}) = M a_{cm}\hat{j}$

$\therefore \boxed{T - Mg = M a_{cm}}$ (1)

2)



$\sum_i \bar{\tau}_i = \bar{R}\times\bar{T} = (R\hat{i})\times T(\hat{j}) =$

$= RT\hat{k} = I\bar{\alpha} = I\alpha\hat{k}$

T och R positiva $\bar{\tau} \parallel \hat{k}$ rot. moturs.

$\boxed{RT = I\alpha}$ (2)

samband mellan a_{cm} och α ; moturs rot. $\Rightarrow a_{cm} < 0$
trissan sänks nedåt.

$\therefore a_{cm} = -\alpha R$ el. $\boxed{\alpha = -\frac{a_{cm}}{R}}$ (3)

$(-R)\cdot(1) \Rightarrow MgR - RT = -MRa_{cm}$

(2) och (3) $\rightarrow RT = -\frac{1}{2}MR^2\frac{a_{cm}}{R}$

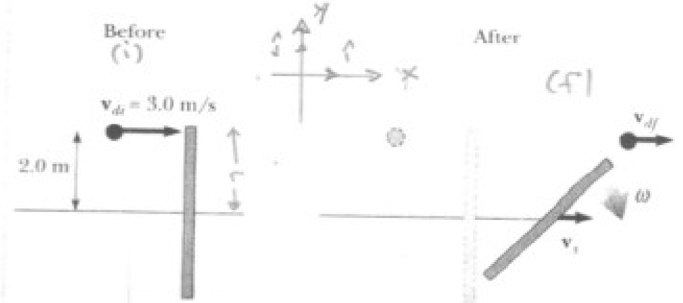
$\Rightarrow g = -\frac{3}{2}a_{cm}$

$\Rightarrow \underline{a_{cm} = -\frac{2}{3}g}$

(1): $T = Mg + Ma_{cm} = Mg + M\left(-\frac{2}{3}g\right) = \frac{1}{3}Mg$

Example 11.10 Disk and Stick

A 2.0-kg disk traveling at 3.0 m/s strikes a 1.0-kg stick of length 4.0 m that is lying flat on nearly frictionless ice, as shown in Figure 11.13. Assume that the collision is elastic and that the disk does not deviate from its original line of motion. Find the translational speed of the disk, the translational speed of the stick, and the angular speed of the stick after the collision. The moment of inertia of the stick about its center of mass is 1.33 kg · m².



a) Elastisk stöt : <u>sökt</u> v_{df} och v_s

<u>givet</u> : $m_d = 2{,}0$ kg $r = 2{,}0$ m

$v_{di} = 3{,}0$ m/s $I_{CM} = 1{,}33$ kgm²

$m_s = 1{,}0$ kg.

<u>Lösning:</u> konserverade storheter : K, P & L

P : Inga externa krafter verkar, $\Rightarrow P_i = P_f$

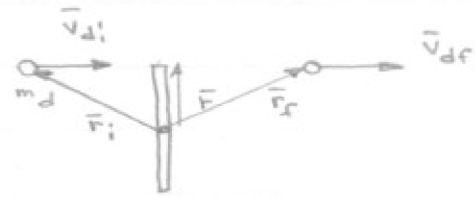
$P_i = m_d v_{di} + 0$ $P_f = m_d v_{df} + m_s v_s$

allt // $\hat{i}$

$$\Rightarrow \boxed{v_s = \frac{m_d v_{di} - m_d v_{df}}{m_s}} \quad (1)$$

v_{df} okänt.

L : Inga externa vridande moment verkar på systemet (skiva + pinne) (d + s)



$$\bar{L}_i = \bar{r}_i \times m_d \bar{v}_{di} + 0 =$$

$$= r\hat{j} \times m_d v_{di}\hat{i} = r m_d v_{di}(-\hat{k})$$

$$\bar{L}_f = \bar{L}_{fd} + \bar{L}_{fs} = \bar{r}_f \times m_d \bar{v}_{df} + I_{CM}\bar{\omega} =$$

$$= \bar{r} \times m_d \bar{v}_{df} + I\bar{\omega} = r\hat{j} \times m_d v_{df}\hat{i} + I\bar{\omega} =$$

$$= r m_d v_{df}(-\hat{k}) + I\omega\hat{k}$$

Här hade vi kunnat skriva $\bar{L}_{fs} = I\omega(-\hat{k})$ ②
eftersom vi vet att pinnen kommer att
rotera medurs.

$$\bar{L}_{fs} = I\omega\,\hat{k} \Rightarrow \omega < 0$$
$$\bar{L}_{fs} = I\omega(-\hat{k}) \Rightarrow \omega > 0$$

$$\bar{L}_f = \bar{L}_{fd} + \bar{L}_{fs} \Rightarrow$$

$$\boxed{-r m_d v_{di} = -r m_d v_{df} + I\omega} \quad (2)$$

K: $K_i = K_f$ eftersom stöten är elastisk

$$\Rightarrow \boxed{\frac{1}{2} m_d v_{di}^2 = \frac{1}{2} m_d v_{df}^2 + \frac{1}{2} m_s v_s^2 + \frac{1}{2} I\omega^2} \quad (3)$$

3 obekanta och 3 ekvationer ∴ lösbart

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(2) ger $v_{df} = \frac{I\omega}{r m_d} + v_{di} \quad (2')$

Beräkning se appendix ger $\omega = -2$ rad/s

$$\Rightarrow v_{df} = -\frac{1{,}33 \cdot 2}{2 \cdot 2} + 3{,}0 = \underline{\underline{2{,}33 \text{ m/s}}}$$

samt

$$v_s = \frac{2 \cdot 3 - 2 \cdot 2{,}33}{1{,}0} \text{ m/s} = \underline{\underline{1{,}34 \text{ m/s.}}}$$

Utregning av ω: ③

$$\frac{1}{2} m_d v_{di}^2 = \frac{1}{2} m_d \left(\frac{I\omega}{r m_d} + v_{di} \right)^2 + \frac{1}{2} m_s \left(\frac{I\omega}{m_s r} \right)^2 + \frac{1}{2} I\omega^2$$

$$\Rightarrow m_d v_{di}^2 = m_d \left(\frac{I^2\omega^2}{r^2 m_d^2} + v_{di}^2 + 2 \frac{I\omega \cdot v_{di}}{r \cdot m_d} \right) +$$

$$+ m_s \frac{I^2\omega^2}{m_s^2 r^2} + I\omega^2$$

$$\Rightarrow 0 = \frac{I^2\omega^2}{m_d r^2} + 2 \frac{I\omega v_{di}}{r} + \frac{I^2\omega^2}{m_s r^2} + I\omega^2$$

$$\Rightarrow 0 = \frac{I\omega}{m_d r^2} + 2 \frac{v_{di}}{r} + \frac{I\omega}{m_s r^2} + \omega$$

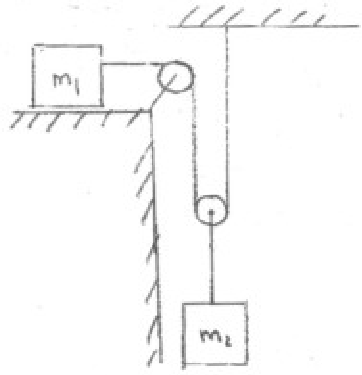
$$\Rightarrow \omega \left(\frac{I}{m_d r^2} + \frac{I}{m_s r^2} + 1 \right) = -\frac{2 v_{di}}{r}$$

$$\Rightarrow \omega = \frac{-2 v_{di}}{r \left(\frac{I}{m_d r^2} + \frac{I}{m_s r^2} + 1 \right)} =$$

$$= \frac{-2 \cdot 3}{2 \left(\frac{1{,}33}{2 \cdot 4} + \frac{1{,}33}{1 \cdot 4} + 1 \right)} =$$

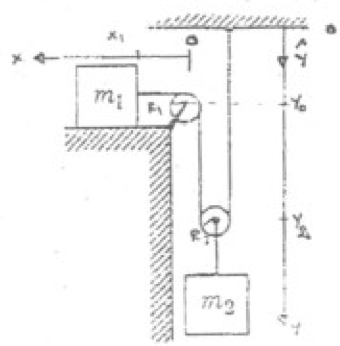
$$= \frac{-3}{1{,}33 \left(\frac{1}{8} + \frac{1}{4} \right) + 1} = -2 \text{ rad/s}$$

5 Bestäm accelerationen (uttryck gärna i procent av g och ta dig en ordentlig funderare på om svaret är rimligt) hos de båda kropparna i figuren nedan om $m_1 = 4{,}0$ kg och $m_2 = 6{,}0$ kg. Trissan är såväl friktionsfri som masslös och snöret är otänjbart och masslöst. Underlaget som m_1 glider på är friktionsfritt.



Lösning:

Relation mellan accelerationen hos m_1 och m_2:



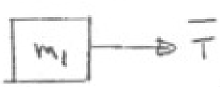
snörets längd = l

$$l = x_1 + \frac{1}{2}\pi R_1 + (y_2 - y_0) + y_2$$

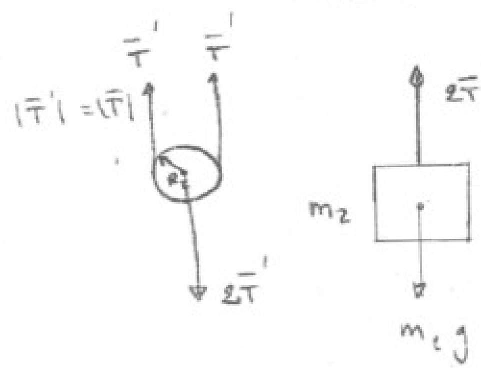
$$\Rightarrow 0 = \ddot{x}_1 + 2\ddot{y}_2 \quad \Rightarrow \quad \ddot{x}_1 = -2\ddot{y}_2$$

blocket m_2 har samma acceleration som trissan med radien R_2

Friläggning:



$$\left.\begin{array}{l} \bar{T} = -T\,\hat{x} \\ \ddot{\bar{x}}_1 = \ddot{x}_1\,\hat{x} \end{array}\right\} \Rightarrow -T = m_1\ddot{x}_1 = -2m_1\ddot{y}_2 \quad (1)$$



y-axeln pekar neråt: $2\bar{T}' = 2T'(-\hat{y})$, $m\bar{g} = mg\,\hat{y}$

$$m_2 g - 2T = m_2\ddot{y}_2 \quad (2)$$

men ekv. (1) ger $-2T = 2\cdot(-2m_1\ddot{y}_2)$

insättning i (2): $m_2 g - 4m_1\ddot{y}_2 = m_2\ddot{y}_2$

$$\Rightarrow \ddot{y}_2 = \frac{m_2}{m_2 + 4m_1}\,g = \frac{6{,}0}{6{,}0 + 4\cdot 4{,}0} =$$

$$= \underline{2{,}7 \text{ m/s}^2}$$

$$\Rightarrow \ddot{x}_1 = \underline{-5{,}4 \text{ m/s}^2}$$

$$|T| = |m_1\ddot{x}_1| = 4\cdot 5{,}4 = \underline{21{,}6 \text{ N}}$$