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Scaling in an Agent Based Model of an artificial stock market

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Abstract. In this project, Bak's agent based model (BPS model) is reproduced and implemented in an artificial stock market to investigate whether price variation in simulation result of this Agent Based Model may have similar scaling phenomena as those in a real stock index.

The probability distribution of price variations in HuShen 300 index in 2012 is analyzed, it is found that it follows the Levy stable distribution and the return probability has power law dependence on time interval with -0.75. Using scaling variable in probability distribution, those distribution curves collapse on the same one.

Different kinds of updating rules are defined in the agent based model, we discover that the agent based model with agents having imitation and positive feedback has the similar scaling behavior as real stock index.

1. Introduction

The recent financial crises raise doubts about standard economic models which tend to use Gaussian distribution to represent the price variation in financial markets [1, 2]. Those theories work, but only in long time intervals. High frequency operation in the stock market is becoming more and more popular among traders, which means that the distribution in short time intervals is important. Actually the price variations in the real financial markets are similar to Levy stable distribution not Gaussian distribution, a conclusion is brought up by Mandelbrot when he studied cotton market [3].

Agent based model is a powerful mathematical tool for stochastic process analysis in complex system, being able to show dynamic properties with interacting particles. For social systems, agent based model may take into account many details in the simulation process to find social actions emerging by defining the interacting or updating rules for individual agents. Researchers use agent based model to approach non-equilibrium systems and to explain the non-Gaussian distribution and scaling behavior in stock market [4, 5, 6, 7, 8, 9].

In this paper, we will analyze the probability distribution of price variations in the HuShen 300 Index in 2012 over different time scales, then build agent-based models with agents defined by different interacting and updating rules. Scaling in simulation results of the interacting agents in an artificial stock market will be compared to it in real stock markets. Then it will explain somehow what kinds of behaviors will lead to non-Guassian distribution of the price variation in stock market.

2. Analysis of real stock index

The analysis of stock markets by physical methods has been practiced for a long time [3, 10, 11, 13]. In this study, we take HuShen 300 index in 2012 as analysis objects. We will study the probability distribution of price variations in different time intervals.

2.1. Price variation of stock index

For a time series $S(t)$ of stock index prices, the variation $V_{\Delta t}(t)$ over a time scale Δt is defined as the forward change in the natural logarithm of $S(t)$ [11, 12].

$$V_{\Delta t}(t) = \ln\left(\frac{S(t+\Delta t)}{S(t)}\right) = \ln\left(1 + \frac{S(t+\Delta t)-S(t)}{S(t)}\right) \quad (1)$$

From (1), if the time interval is small, the variation will be quite small, then

$$V_{\Delta t}(t) = \ln\left(\frac{S(t+\Delta t)}{S(t)}\right) = \ln\left(1 + \frac{S(t+\Delta t)-S(t)}{S(t)}\right) \approx \frac{S(t+\Delta t)-S(t)}{S(t)} \quad (2)$$

where (2) is the return function for economic theory.

In Figure 1, 1 minute data of the HuShen 300 index in 2012 is illustrated, and we use the return of stock price as variation. The price variation of index is shown in Fig.2. Some economic theories try to claim that price variations of stock market should follow the process of random walk, which is correct only in long time intervals. Many stock crises are happening these days, which gives doubt about those theories, since in Gaussian distribution extreme conditions (large variation) should not happen so frequently. As shown in Figure 3, this Gaussian noise plot which has same mean and variation with 1 minute data does not have that many extreme events as they are in historical 1 minute data.

2.2. Scaling in probability distribution of price variation

In 1966, Mandelbrot used Levy stable distribution to represent the price variation of the cotton market [3], and also Mantegna and Stanley tell us that stable non-Gaussian

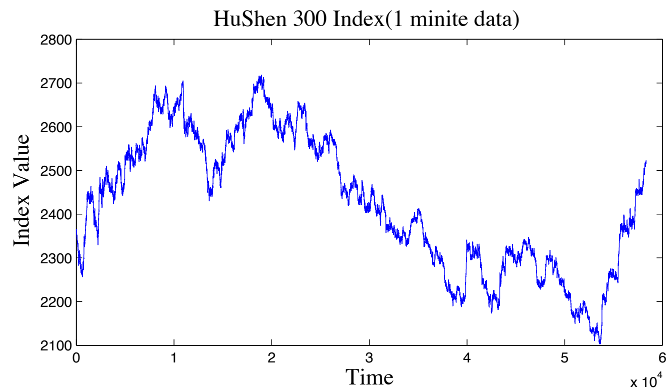


Figure 1. 1 minute data of HuShen 300 index 2012(Data is from BiaoPuYongHua Data Center)

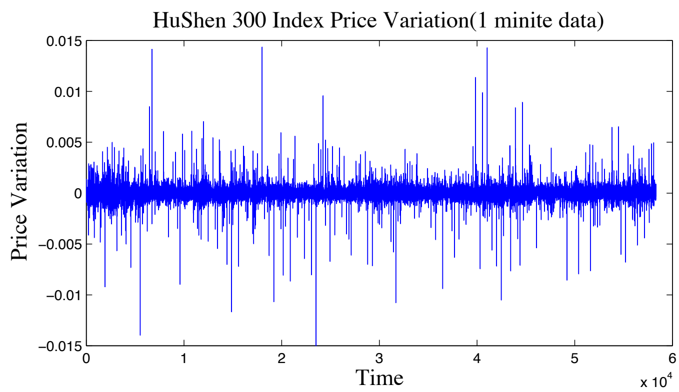


Figure 2. Price variation of 1 minute data of HuShen 300 index 2012

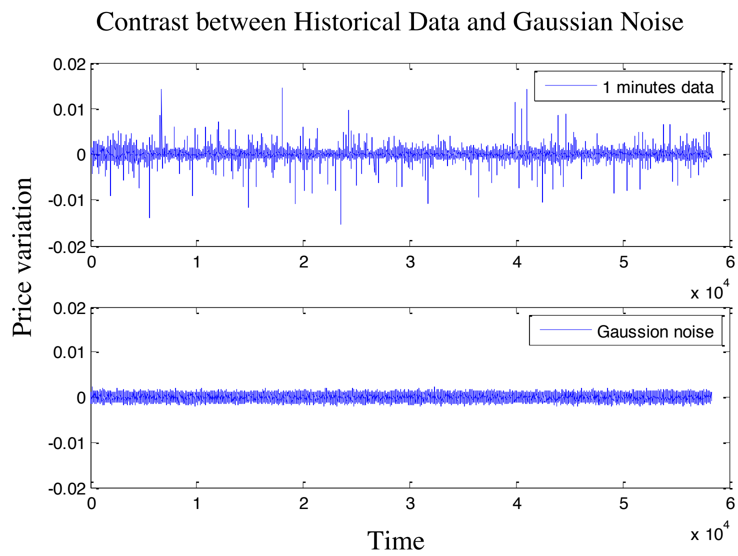


Figure 3. Contrast between variation of Gaussian Noise with 1 min historical data

distributions are suitable for financial analysis because they still obey limit theorems [12]. Also Levy stable distribution can better explain that fat-tail and leptokurtic phenomena in stock market. Figure 4 shows the probability distribution curves of price variation in different time intervals (Δt varies from 1 minute to 10 minute).

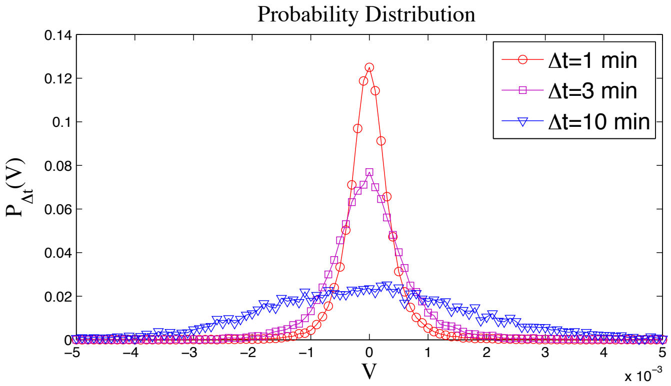


Figure 4. Probability distribution of price variation in different time intervals

Here we use the approach as Mantegna and Stanley did in [10], the return probability, which is $P_{\Delta t}(0)$ as a function of Δt, is studied. This is because $P_{\Delta t}(0)$ is the data least affected by noise. Plot of $P_{\Delta t}(0)$ versus Δt in log-log figure is in Figure 5. We observe that slope of the data in a large interval fits -0.75, which is not a Gaussian distribution since the slope is not 1/2 [13]. This conclusion also quite well agrees with the Levy stable distribution [14].

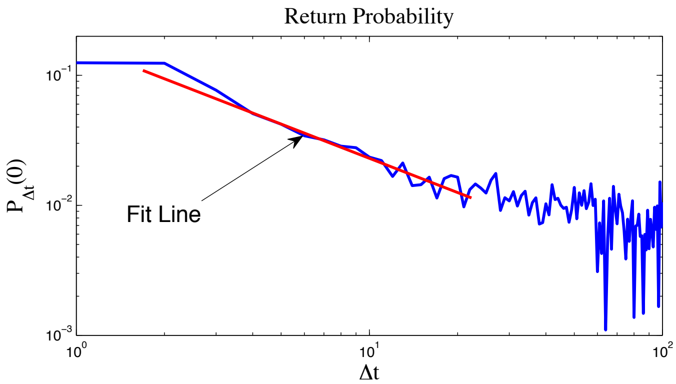


Figure 5. Return probability $P_{\Delta t}(0)$ versus Δt in log-log plot and fit slope.

In figure 5, some points do not quite fit the line. This is because our data is limited, noise can not be avoided. A Levy stable symmetrical distribution [10] is described as

$$L_\alpha (V, \Delta t) = \frac{1}{\pi} \int_0^\infty \exp\left(-\gamma \Delta t q^\alpha\right) \cos(qV) dq \qquad (3)$$

this probability of return $P_{\Delta t}(0)$ is given by

$$P_{\Delta t}(0) = L_\alpha\left(0, \Delta t\right) = \frac{1}{\pi}\int_0^\infty \exp\left(-\gamma \Delta t q^\alpha\right) dq = \frac{\Gamma(1/\alpha)}{\pi\alpha(\gamma\Delta t)^{1/\alpha}} \tag{4}$$

As shown in Figure 5, slope is -0.75, the parameter α is as follows

$$-\frac{1}{\alpha} = -0.75, \alpha \approx 1.33 \tag{5}$$

Also Levy stable symmetrical distribution rescales under transformation as following

$$V_s = \frac{V}{(\Delta t)^{1/\alpha}} \tag{6}$$

$$L_\alpha\left(V_s, 1\right) = L_\alpha\left(V, \Delta t\right)\left(\Delta t\right)^{1/\alpha} \tag{7}$$

So we know that scaling in the entire interval have the similar behavior as return probability $P_{\Delta t}(0)$. As in Figure 6, with index $\alpha = 1.33$ in (5), we may get that the probability distribution of price variation in different time intervals collapse on same curve by using the transformation equation (6,7).

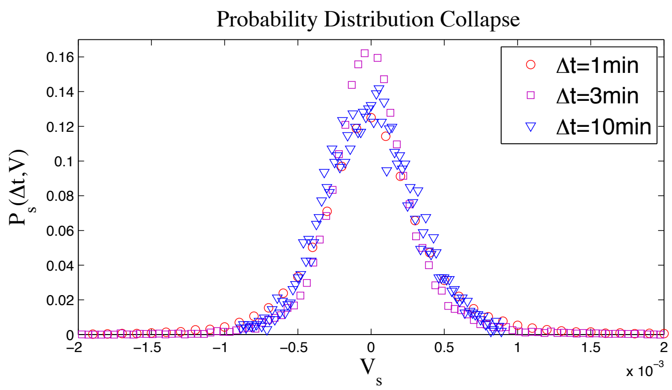


Figure 6. Probability distribution of price variation in different time intervals collapse on same curve by using scaling transformation equation.

In this section, by analyzing the real index HuShen 300 index 2012, it can be concluded that the price variation in short time intervals does not behave like random walk, it quite fits into Levy stable symmetrical distribution with scaling parameter $\alpha = 1.33$. Next I will discuss that whether agent based models with agents who have different kinds of behaviors may have the similar probability distribution as the real market.

3. Agent-Based Model in Artificial Stock Market

In 1997, an agent based model(BPS models) was introduced to simulate price variation in stock market by P. Bak, M. Paczuski, M. Shubik [4]. In the following parts, several simplified BPS models will be set up to discuss what kinds of trading behaviors give probability distributions of price variation in real stock market that are non-Gaussian distribution.

3.1. BPS model in an Artificial Stock Market

Here definitions, simplifications and restrictions of the BPS model in an artificial stock market are introduced.

(i) There are N agents and $N/2$ shares in our artificial stock market, each agent can only own one share at most. It means that in every time step, there are $N/2$ agents who own shares and are potential sellers, while the $N/2$ agents who do not own shares are potential buyers. If the agent i is a potential sellers, he will have a sell price $P_s(i)$, also if the he is a potential buyers, he will have a bid price $P_b(i)$

(ii) Initially we randomly set the buyers bid price P_b and sellers sell price P_s in a symmetric range of prices distributed uniformly. Also stock price limitation is set as: maximum price is $maxP$, and minimum price is 1.

(iii) In each step one agent is randomly chosen, it may be a potential buyer or seller. Price updating rules are as follows.

  (a) His price $P_s(i)$ or $P_b(i)$ is updated according to rules: approaching ΔP to instant stock price, which is also the price in last step $P(t-1)$, with probability $(1 + D)/2$, stepping away ΔP from instant stock price with probability $(1 - D)/2$. ΔP differs in different models. Here the drift parameter D is used to give agents a direction to update their price.

  (b) After updating his price, this agent will look into the entire market. If this agent owns a share, and observes that the price that one or more buyers who are willing to give is more than his sell price $P_b(i)$, he will sell his share to the buyer who offers the highest price $P_s(j)$. While if he wants to buy a share, and one or more buyers are willing to sell shares at less than his bid price $P_s(i)$, he buys from the seller offering the lowest price $P_b(j)$. If there is no price for which this agent wants to sell or buy, the procedure will go to the next step, and the stock price $P(t)$ stays the same as $P(t-1)$. When a transaction happens, that price is the new stock $P(t)$, which is bid or sell price of agent j.

  (c) After the transaction, seller in this transaction becomes a potential buyer, he will have a bid price P_b. And buyer becomes a potential seller, he will have a sell price P_s. New P_b and P_s are defined differently in different models.

3.2. Market only with zero-intelligence agents

In this model, at each update step the chosen agent updates his price by one unit randomly, which means $\Delta P = 1$. Once there is a transaction, new buyer and seller will randomly choose their new prices in the range as $0 < P < P(t)$ or $P(t) < P < maxP$. These ranges are defined like this because agents should not lose money at the first step after transaction.

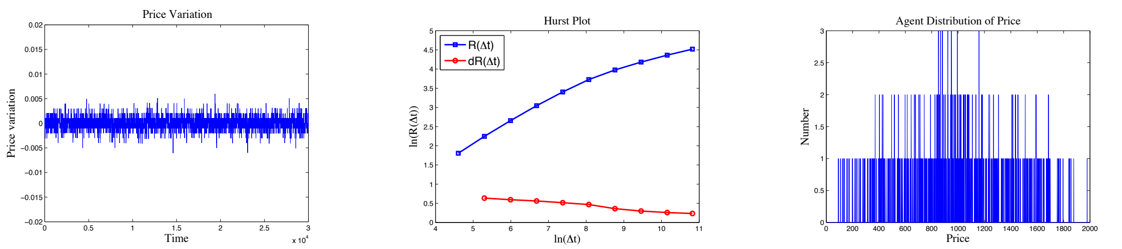
This process is according to Bak [4] similar with reaction-diffusion process in [15], in which there are two kinds of particles in a one dimensional tube. Each kind of particles will be jetted into the tube from the each end. Particles will update their position randomly. When two kinds of particle meet with each other, they will react, and then they will be replaced by new particles. Authors in [15] show that the position variation $P_{\Delta t}$ in time interval Δt will follow:

$$V_{\Delta t} \sim \Delta t^{1/4} (\ln \Delta t)^{1/2} \tag{8}$$

Since this reaction-diffusion process has similar interacting rules as our Agent Based Model, so we expect that there is some similarity between its "position" and our "price". So the price variation in our model is represent as $R(\Delta t)$, which is maximum variation in non-overlapping time intervals Δt over its time intervals Δt, averaged by the entire simulation time. And also $R(\Delta t)$ can be represented as a Hurst plot [16], whose slope of its local derivatives of its logarithmic function gives exponent H. Based on the theoretical result in (8) from similar process in [15], we expect that our price variation $R(\Delta t)$ will follow the similar equation as

$$\ln (R (\Delta t)) \sim \tfrac{1}{4} \ln(\Delta t) + \tfrac{1}{2} \ln (\ln(\Delta t)) \tag{9}$$

Simulation parameters are set as agent number $N = 500$, maximum price $maxP = 2000$ and drift parameter $D = 0.3$. Buyers' bid price P_b and sellers' sell price P_s are assigned in the distributed uniformly intervals $\frac{1}{3}maxP < P_b < \frac{1}{2}maxP$ and $\frac{1}{2}maxP < P_s < \frac{2}{3}maxP$ initially. Also in our simulation results, we use one step to represent N times agents' updating times. The reason we integrate this is because there is barely transactions happening during the whole stochastic process, so we take the average time that all agents can update once as one step of simulation results.



(a) Stock price variation

(b) Hurst plot of $R(\Delta t)$ and its local derivatives

(c) A snapshot of agent number distribution of Price

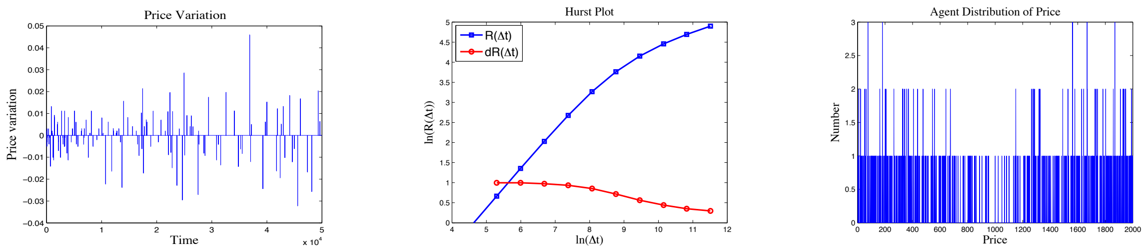
Figure 7. Simulation of zero-intelligence agents with drift parameter

As it is shown in Figure 7, in long time interval H exponent goes to $\frac{1}{4}$, which quite agrees quite well with previous analytic work.

3.3. Simulation impact of drift parameter D

We should discuss that whether drift parameter D has influence on our simulation result, so we keep the same parameter as the model in 3.2 except we set drift parameter to $D = 0$.

Based on the comparison of simulation results between models with and without drift parameter D, we can tell that in 7(a) transactions happen more frequently than in 8(a), also in 7(c) agents price are closer to each side than in 8(c). But in 8(b) and 7(b), both local derivative curves converge to $\frac{1}{4}$ in long time interval, but obviously in 7(b), it converges faster. Then we may conclude that the drift parameter D can accelerate the trading process without changing the scaling behavior. So drift parameter D will be kept in the following models.



(a) Stock price variation

(b) Hurst plot of $R(\Delta t)$ and its local derivatives

(c) A snapshot of agent number distribution of Price

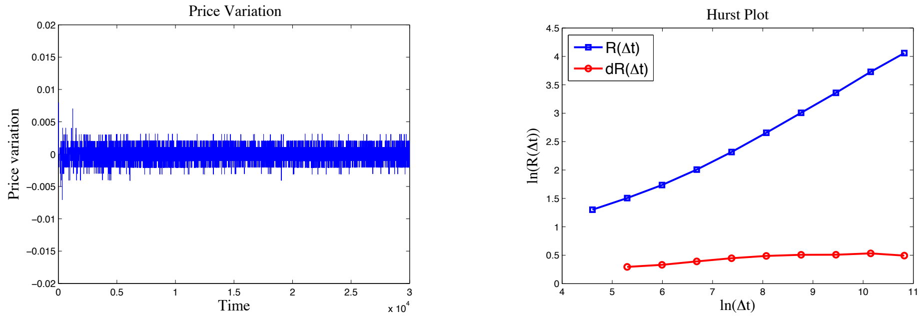
Figure 8. Simulation of zero-intelligence agents without drift parameter

As it is shown above, we show that price variations in agent based model may not necessarily follow Gaussian distribution. In the following sections, we will give the agents some behaviors according to real trading behaviors in real markets to discuss whether agent based model will reproduce similar scaling results as the analysis of real markets.

3.4. Market with imitating agents

It has been argued that in real stock market imitation trading universally exists [17], which also causes 'crowd effect'. Bak discussed whether this imitating behavior is a reason that leads to non-Gaussian phenomenon. In that model, imitating behavior is defined as: after a transaction new potential buyer will randomly copy some potential buyer's price $P_b(m)$ as his new bid price $P_b(i) = P_b(m)$. Similarly when he is a new potential seller, his new sell price is randomly copied from other seller agent n, $P_s(i) = P_s(n)$. Obviously, after a long time, agents will emerge to choose the market prices are offered by more agents. In our simulation, parameters of the model are the same with the "zero-intelligence agent with drift" model, the only difference is this model has imitating behavior.

The simulation result, shown in Figure 9, is quite interesting because exponent H will converge to 0.5 in long time interval, which means that it has random walk behavior. This fits quite well to the financial theories, that in long time interval price variation distribution is Gaussian distribution while in short time it is not.



(a) Stock price variation

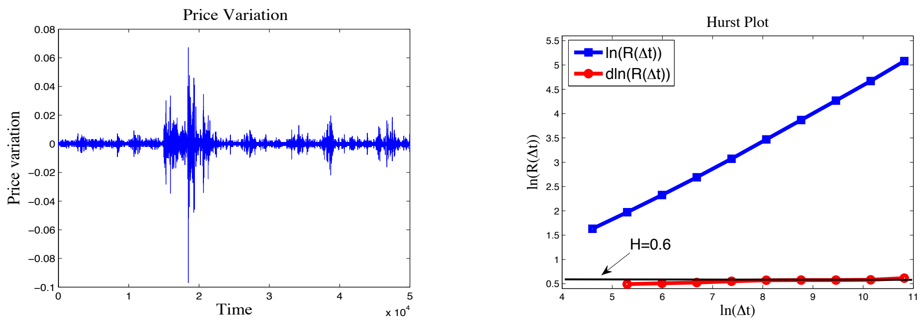
(b) Hurst plot of $R(\Delta t)$ and its local derivatives

Figure 9. Simulation of market with imitating agents

3.5. Price updating by positive feedback volatility

Time series in real stock markets has similar behavior to earthquakes and floods which has long-term positive autocorrelation, meaning that after a big variation will likely follow another big change [18]. Here positive feedback is introduced to modify the update value ΔP. In this model ΔP is equal to largest price variation in last T time steps.

Also In this part parameter T equals to 5, and other parameters of this simulation are the same as the "zero-intelligence agent with drift" model. Differences between that and the present simulation are imitating behavior and positive feedback volatility.



(a) Stock price variation

(b) Hurst plot of $R(\Delta t)$ and its local derivatives

Figure 10. Simulation of market with imitating agents updating price by volatility positive feedback, $T = 5$

Comparison between Figure 9 and Figure 10 shows that volatility positive feedback behavior can enlarge the price variation in short time intervals. This conclusion agrees with the observation of big variation in short time intervals and long-term positive autocorrelation in real stock market.

3.6. Scaling in Agent Based Model

We will study scaling behavior in the model which has imitating agents updating price with volatility positive feedback. As shown in Figure 10, the simulation result shows us that the exponent H may be represented as a constant $H = 0.6$. Since $H = 0.6$ is constant, it means that

$$\ln (R (\Delta t)) \sim H \ln(\Delta t) + C \tag{10}$$

Then we set the scaling variable as

$$z = \frac{V}{\Delta t^H} \tag{11}$$

which has the similar transformation equation as (6).

We have

$$R(t) = \int V P (V, \Delta t) dV = C \Delta t^H \tag{12}$$

If we follow (11), distribution curves can collapse, then we have

$$\begin{aligned} R(t) &= \int |V| \, P (V, \Delta t) dV / \int P (V, \Delta t) dV \\ &= \int |z| \, \Delta t^H P \left(z \Delta t^H, \Delta t \right) dz \Delta t^H / \int P \left(z \Delta t^H, \Delta t \right) dz \Delta t^H \\ &= \Delta t^H \left(\int |z| \, F (z) dz / \int F (z) dz \right) \end{aligned} \tag{13}$$

So we use similar methods to analyze the simulation results as we use in HuShen 300 index, in section 2.

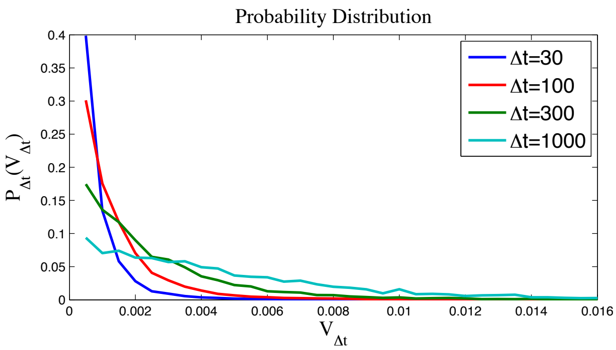


Figure 11. Probability distribution of price variation in different time intervals in agent based model with imitation behavior and positive feedback

In Figure 11, probability distribution curves of price variation in different time intervals are plotted. Since the number of times that zero occurs is quite related to the parameters in this model and also zero variation happens much more frequently than the other values, during this analyzing process, we only use the data for non-zero variation. The time intervals we choose are from 30 to 3000.

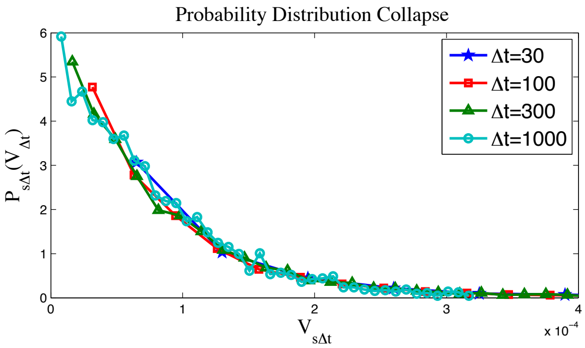


Figure 12. Probability distribution of price variation in different time intervals collapse on same curve by using scaling transformation equation in Agent Based Model

In Figure 12, we can see that after the transformation by scaling variable and normalization, distribution of price variation in different time intervals collapse on the same curve, shows that it has scaling behavior. So we can get

$$P(V, \Delta t) \sim F\left(V/\Delta t^{H}\right) \tag{14}$$

It has the similar scaling as (7). This kind of behavior was observed by Mandelbrot, who suggested that the scaling function F is a Levy stable distribution [4]. Our results quite agree with his analytical work.

4. Conclusion

It has been argued that Levy stable distribution universally exits in financial market. By analyzing HuShen 300 index, we find that the scaling behavior in this index fits the theoretical analysis of Levy stable symmetrical distribution. Implanting two kinds of behavior imitation and positive feedback to an agent based model, we also find such scaling in this model. The results may imply that imitation which causes "crowd effect" in financial market and positive feedback which gives long-term positive autocorrelation may be related to the dramatic evolution of financial markets.

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References

[1] Michele Costa, Giuseppe Cavaliere, Stefano Iezzi, The Role of the Normal Distribution in Financial Markets, *New Developments in Classification and Data Analysis, 2005, Springer US*

[2] V Akgiray, GG Booth, The Siable-Law Model of Stock Returns, *Journal of Business* & *Economic Statistics, Vol.6(1988), 51-57*

[3] B. Mandelbrot, The variation of some other speculative prices, *The Journal of Business of the University of Chicago, Vol.40(1967), 393-413*

[4] P. Bak, M. Paczuski, M. Shubik, Price variations in a stock market with many agents, *Physica A, Vol.246(1997), 430-453*

[5] T Lux, M Marchesi, Scaling and criticality in a stochastic multi-agent model of a financial market, *Nature, Vol.397(1999), 498-500*

[6] JP Da Cruz, PG Lind, Self-organized criticality in a network of economic agents with finite consumption, *Physica A, Vol.391(2012), 1445-1452*

[7] M Ebert, W Paul, Trading leads to scale-free self-organization, *Physica A, Vol.391(2012), 6033-6038*

[8] Y Li, W Zhang, Y Zhang, X Zhang, X Xiong, Calibration of the agent-based continuous double auction stock market by scaling analysis, *Information Sciences, 2012*

[9] B Tth, J Kertsz, JD Farmer, Studies of limit order book around large price changes, *The European Physical Journal B, Vol.71(2009), 499-510*

[10] RN Mantegna, HE Stanley, Scaling behavior in dynamics of an economic index, *Nature, Vol.376(1995), 46-49*

[11] P Gopikrishnan, V Plerou, LAN Amaral, M Meyer, HE Stanley, Scaling of the distribution of fluctuations of financial market indices, *Physical Review E, Vol.60(1999), 5305C5316*

[12] RN Mantegna, HE Stanley, An introduction to Econophysics: correlation and complexity in Finance, *Cambridge Press 1999*

[13] G Samorodnitsky, MS Taqqu, Stable non-Gaussian random process, *Econometric Theory, Vol.13(1997), 133-142*

[14] MF Shlesinger, GM Zaslavsky, J Klafter, Strange kinetics, *Nature,Vol.363(1993), 31-37*

[15] GT Barkema, MJ Howard, JL Cardy, Reaction-diffusion front for $A + B \to \emptyset$ in one dimension, *Physical Review E, Vol.53(1996), 2017-2020*

[16] J Feder, P Bak, Fractals, *Physics Today, Vol.42(1989), 90*

[17] M Griffiths, B Smith, D Turnbull, W White, Evidence of Imitation Trading, *SSRN 1997*

[18] M Ruth, B Hannon, Positive Feedback in the Economy, *Modeling Dynamic Economic Systems, 2012, Springer US*