Cosmological fractals

Cosmological Principle

- Milne → *whole world-picture as seen by one observer is similar to the world-picture seen by any other observer*

- Isotropy + 'Copernican principle' → Homogeneity *** (assumes smooth density)

Cosmological Principle

De Vaucouleurs →

- 30's: galaxies but for a few clusters
- 50's: cluster centers
- Now: superclusters – last scale of clustering

Cosmological Principle

- $\log N(m) = 0.2\mathcal{D}m + k \quad \rightarrow$ '0.6m–law'

- Local isotropy:
  - Cosmic microwave background radiation
  - Radio sources and X–ray emmiters

Fractal structure

- ISM
  - Recursive Jean's instability and fragmentation
- Galaxies
  - Multifractality; transition to homogeneity
- Self-gravity
  - Gravity creates fractal order from chaos

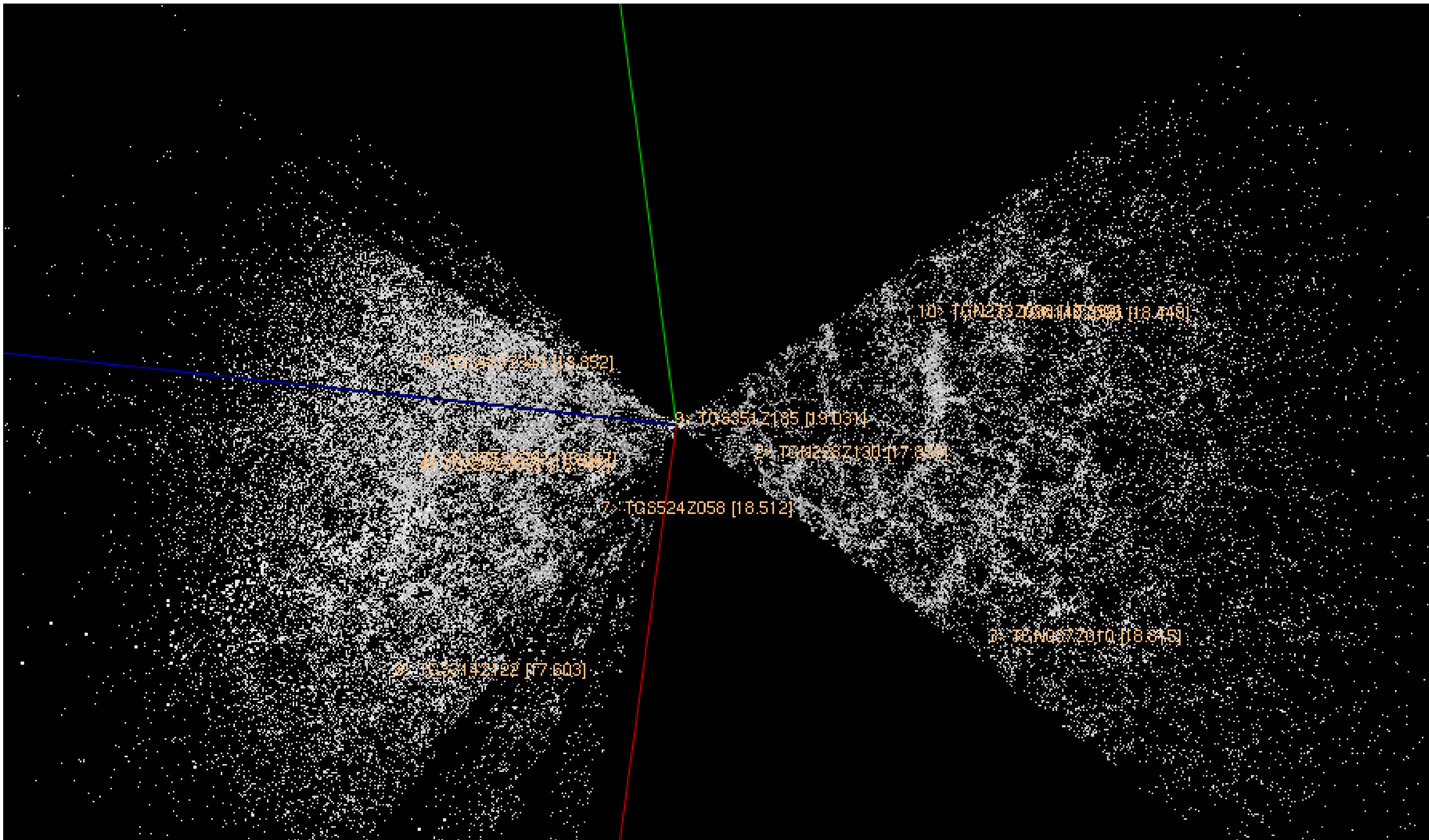
Fractal Structure

Number density of galaxies in clusters:

- Carpenter → “Nonuniform but nonrandom”
  - $n \propto N/R^3 \propto R^{-1.5}$
- Vaucouleurs: $n \propto R^{-1.7}$
  - All observers will find that <n> decreases
    - Distinguishes fractality from clumpiness
- Fractality carries uniformity

3D space maps

2dF Galaxy redshift survey



Correlation length

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$$\xi(r) = \frac{\langle n(r_i)\cdot n(r_i + r)\rangle}{\langle n\rangle^2}$$

- Crossover between the scale of strong clustering and an essentially smooth distribution
  - $\mathcal{D} \approx 1.2$ and $R_{max} < 10$ Mpc

Conditional density

- Pietronero: log C(ε) vs log ε *correlation dimension
  - $\mathcal{D} \approx 2$ and R_{max} > 100 Mpc
  - $n = N/V \propto R^{\mathcal{D}-3}$
  - $M(R) \propto R^{\mathcal{D}}$
  - $\mathcal{D}=2$: critical fractal dimension – eq between large-scale and short-scale fluctuations

10's vs. 100's MPc

10's

Lahav

- *No dynamical theory which could have produced strongly fluctuating fractal structures on large scales from the very smooth initial state.*
- *Isotropic sky distribution of X-ray sources and background radiation*

100's

Labini

- *Non-uniform fractals may look rather smooth in the sky even for hundreds of Mpc (Lacunarity)*
- *Ratio of dark matter*

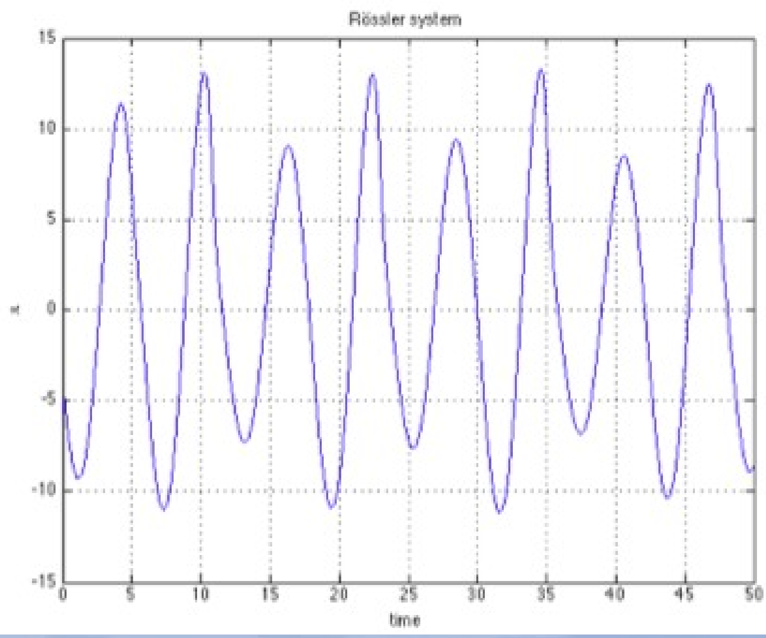
So, when does uniformity begin?

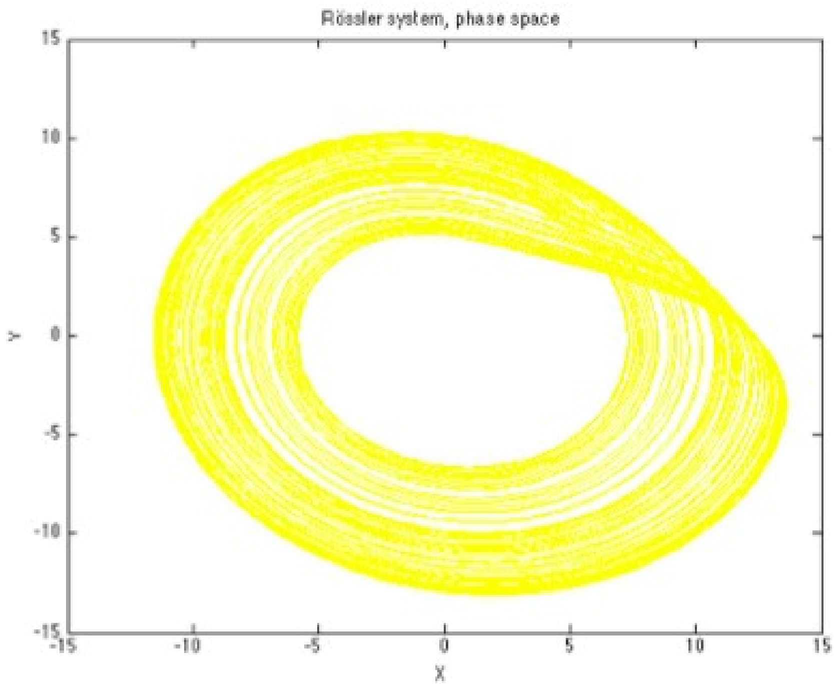
- At the crossover of galaxy fractals to uniform density; or maybe much sooner
- When fractal density falls below the density of one of the uniform components (physical vacuum?)

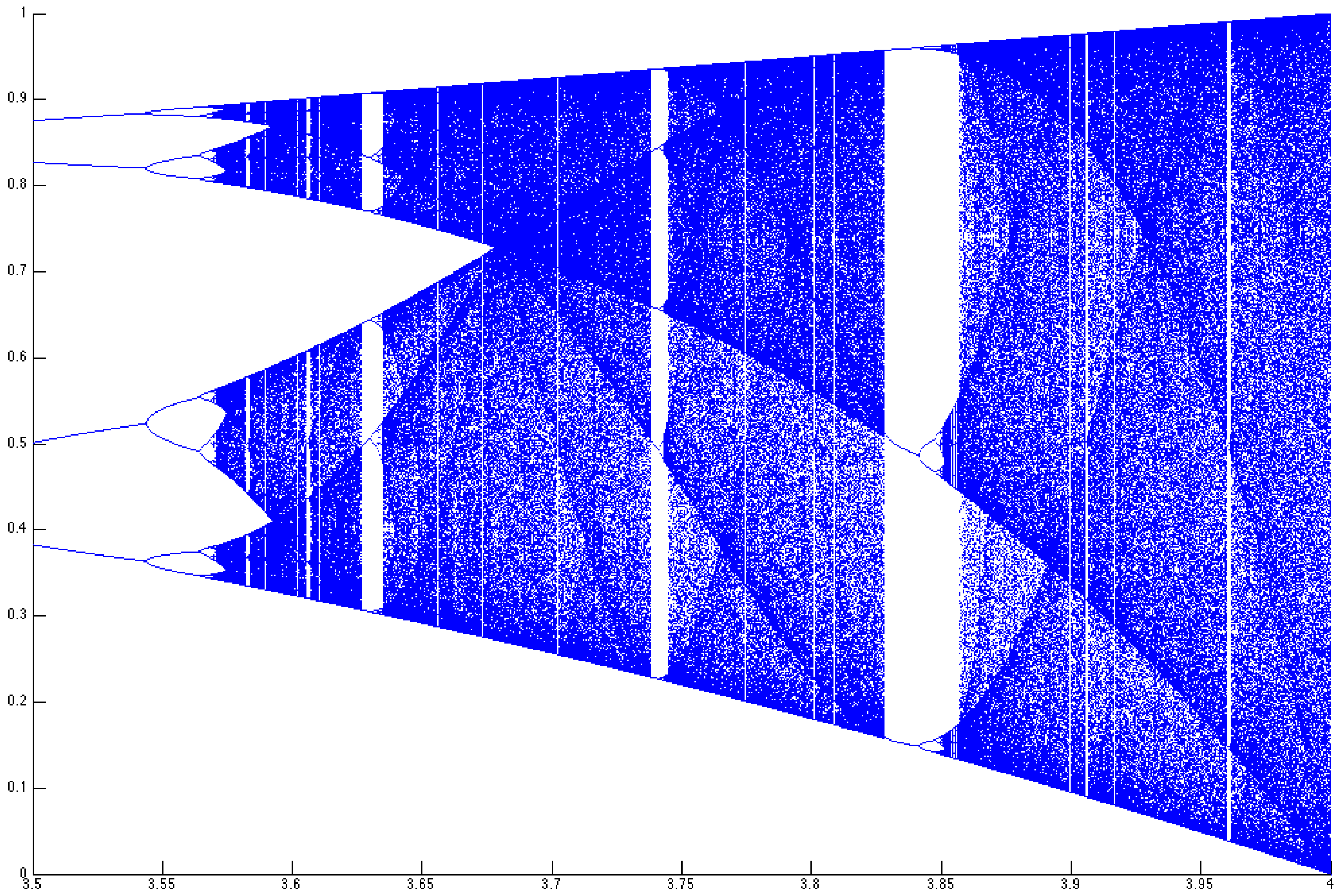
Universality

- Logistic map: $x_{n+1} = r \cdot x_n(1-x_n)$
- Sine map: $x_{n+1} = r \cdot \sin(\pi x_n)$
- Rössler system (Lorentz map):

$$\dot{x} = -y - z$$
$$\dot{y} = x + 0.15y$$
$$\dot{z} = 0.4 + z(x - 8.5)$$







1
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0.7
0.6
0.5
0.4
0.3
0.2
0.1
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3.6
3.65
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3.75
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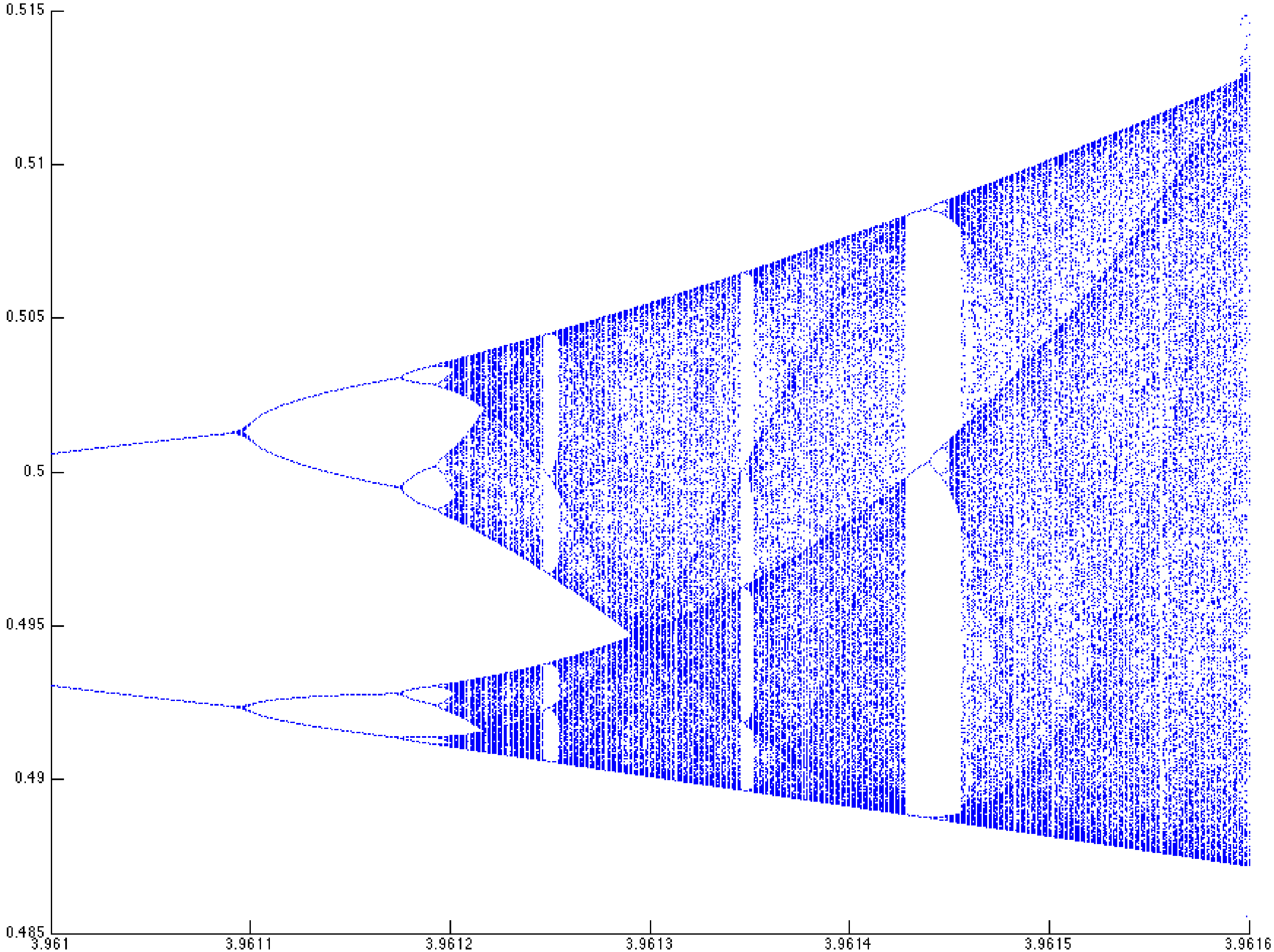
3.85
3.9
3.95
4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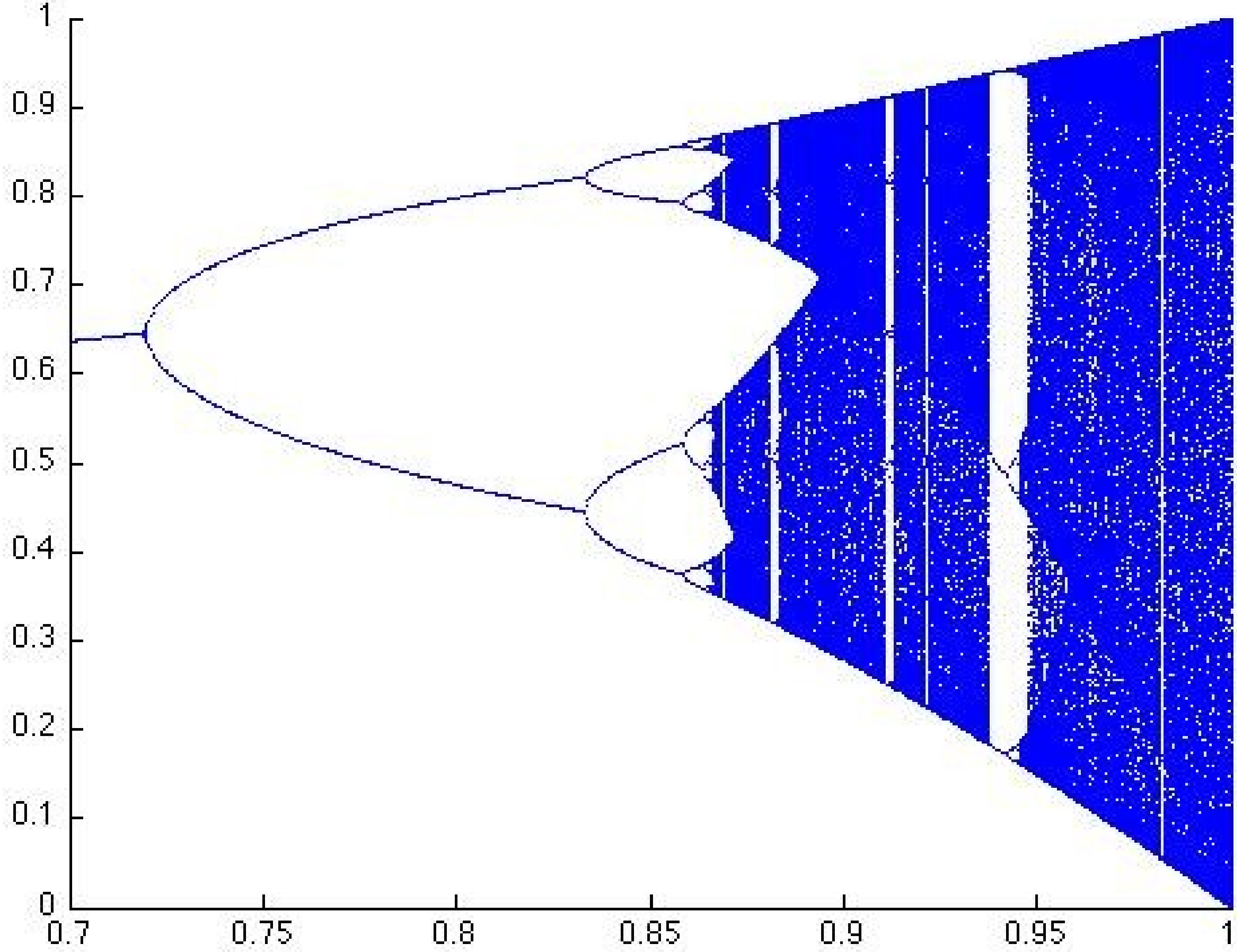

$R_n/R_{n+1} = 4.669$ Libchaber experiment

Universality

- Second order phase-transitions

  $C \alpha | T - Tc |^{-a}$

  - Critical point
  - Critical exponent
  - Order parameter
- Universality classes
  - Dimensionality d of space and D of the order parameter

Universality

Statistical self-gravity

- Order parameter: Correlation length
- D=3, D=1

- Ising model d=3

Questions?