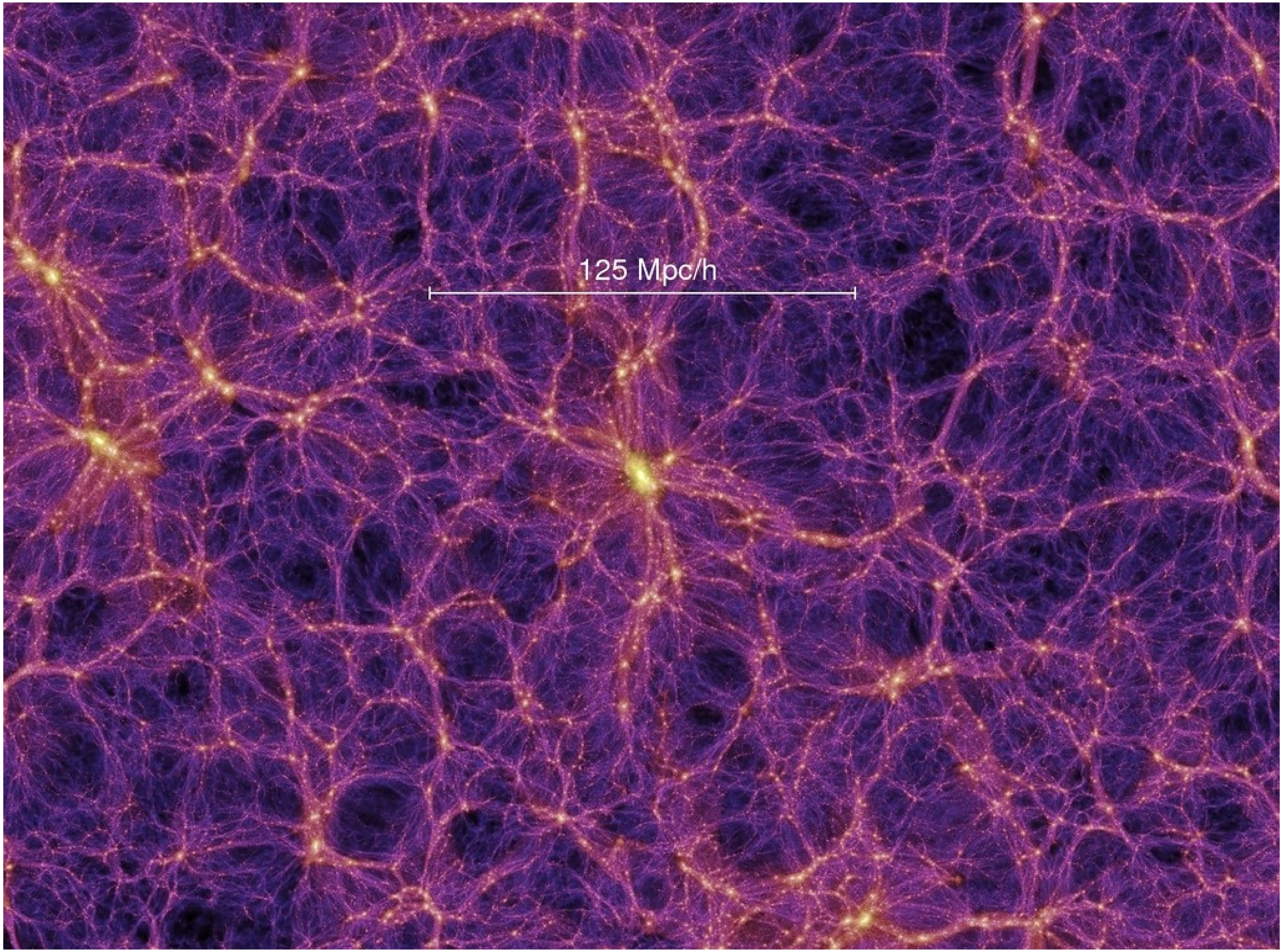
Simulation Methods for Fluids



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Overview

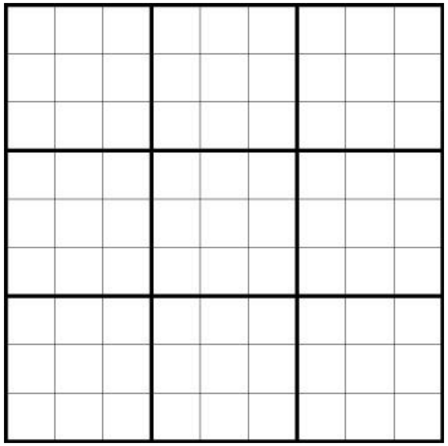
- Introduction
- Eulerian versus Laplacian approaches
- Smothered Particle Hydrodynamics
- Future developments
- Conclusions

Introduction

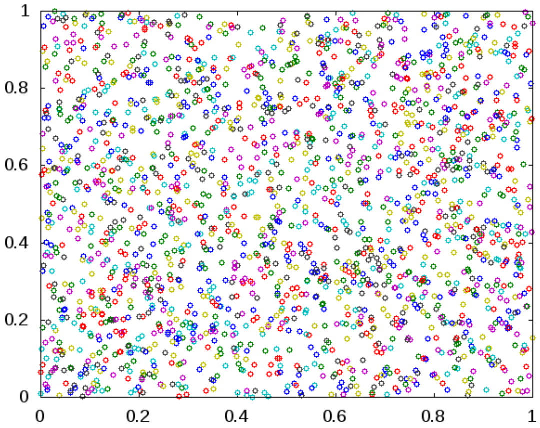
- Limitations of theoretical models in non linear regimes
- Follow a systems evolution
- Computational power of current computers very limited
    - Must reduce the physical systems to less information

Eulerian versus Laplacian

- Eulerian methods
  - Divides space in static volume elements
  - More researched
  - Waste resolution in regions where nothing happen



- Laplacian methods
  - Divides matter into particles with fixed mass that flow freely
  - Resolution follows matter



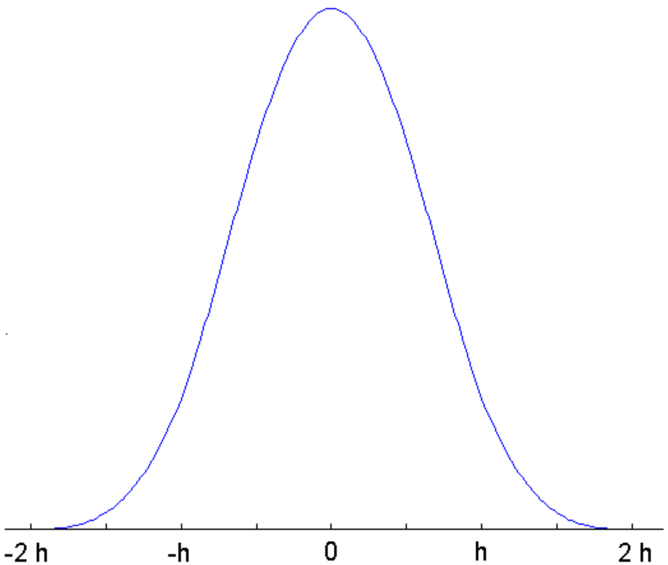
Smoothed Particle Hydrodynamics

- Divides the matter into Particle with equal mass
- Smooths the particles to achieve fluid like behavior
- Density in a point will be deciding by

$$\rho(\boldsymbol{r}) = \sum m_i W\left(\left|\boldsymbol{r} - \boldsymbol{r_i}\right|, h_i\right)$$

- Likewise another property A will be decided by

$$A(\boldsymbol{r}) = \sum A_i \frac{m_i}{\rho(\boldsymbol{r_i})} W\left(\left|\boldsymbol{r} - \boldsymbol{r_i}\right|, h_i\right)$$



How do we choose h?

- Large h → Very slow computations in high density regions
- Smaller h → Bad behavior in low density regions since particles there “see” to few other particles
- Solution:

  Vary h according to density: $h \propto \left(\frac{m}{\rho} \right)^{1/3}$

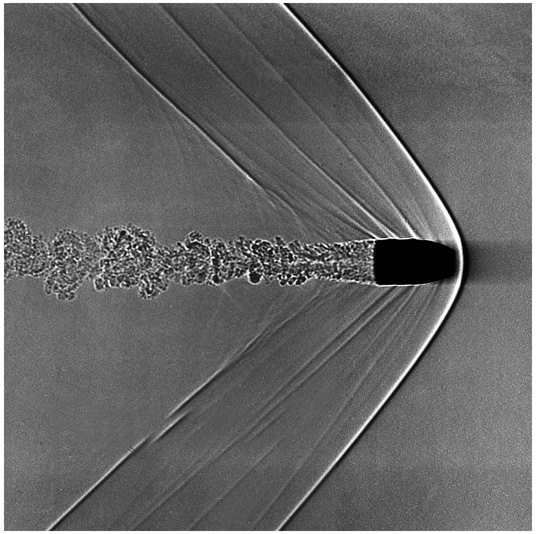
- Number of particles covered in a volume h^3 varies little in absence of discontinuities

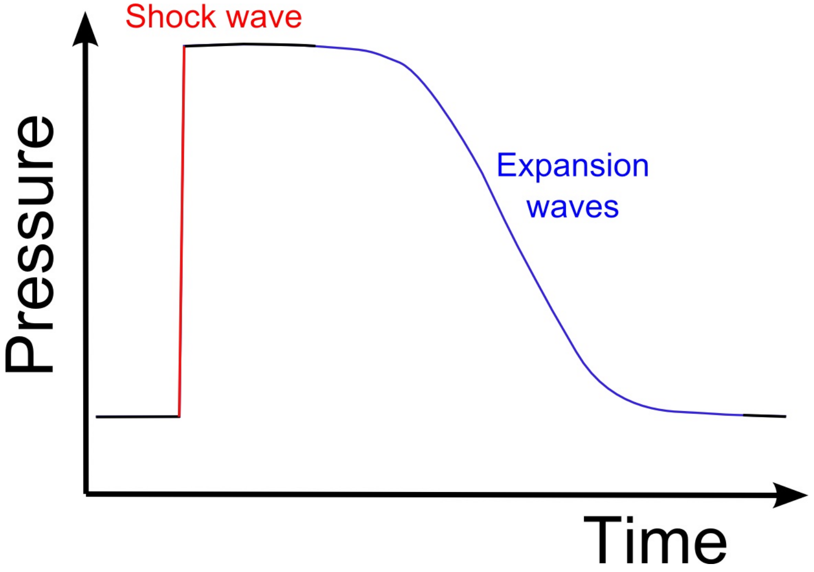
Evolving the system

- Particles evolved separately
  - Position
  - Velocity
  - Energy
- Equations of motion derived from a discretized version of the Lagrangian changing the integral over space to a sum over all particles
- Equations derived using Euler's equations

Shocks in SPH

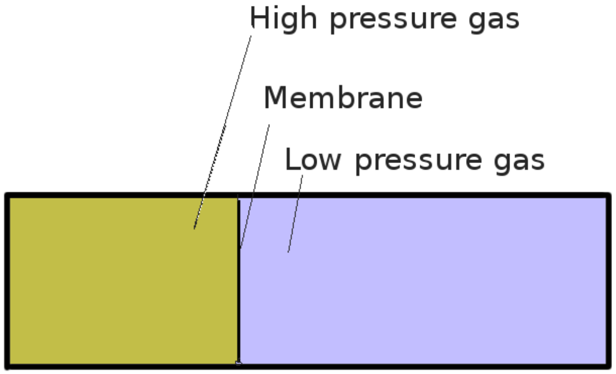
- No viscosity!
- Shocks will not propagate and evolve properly
- Add an artificial viscosity term in the momentum equation



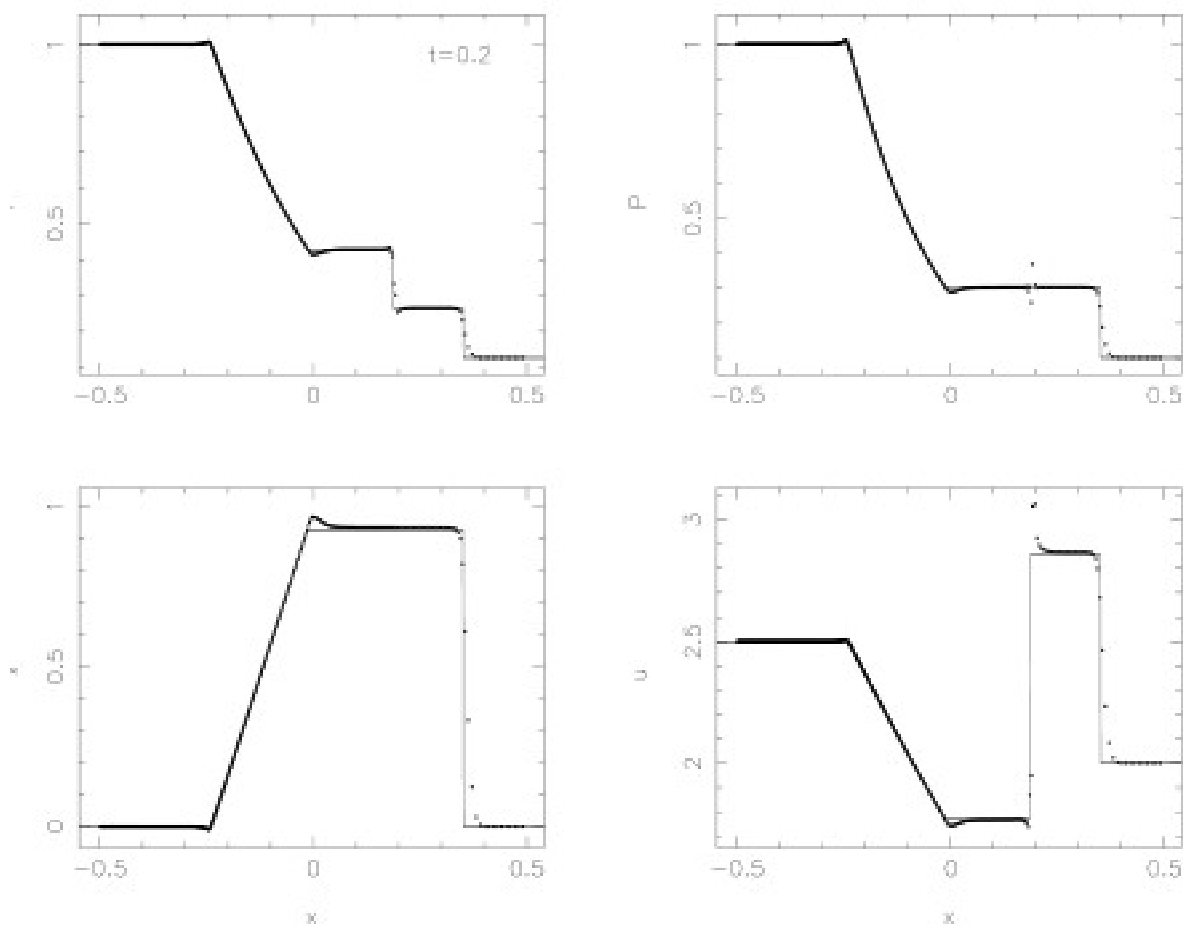


Testing shocks in the SPH

- Recreating the shock tube experiment
- Standard test for shocks in fluid simulations
- Problem was thoroughly studied by Gary Sod



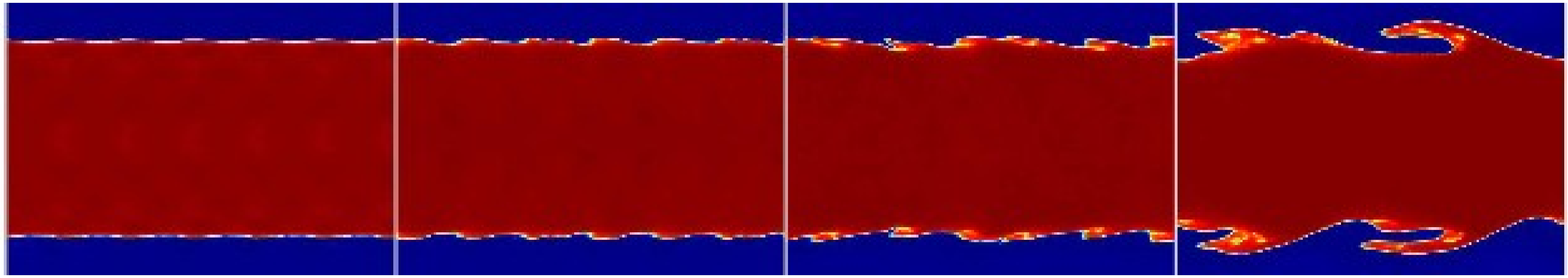
Results for SPH with AV



Simulation of shock tube done by Daniel Price

More on discontinuities in SPH

- Important physics for astrophysical dynamics in these regions
  - Kelvin-Helmholtz instabilities
  - Rayleigh-Taylor instabilities
- SPH doesn't treat these instabilities well



Simulation of Kelvin-Helmholtz instability done by Daniel Price

Root of the problem

- Studying the simulations dispersions relation will reveal several new instabilities
- Local Mixing Instability
  - Particles with different density will not mix properly
- Clumping Instability and Banding Instability
  - Fluid behavior becomes more particle like in some regions
  - Due to the particle representation of a fluid

Solution to the problem

- Local Mixing Instability
  - Smooth out entropy over discontinuities by adding heat conduction
- Clumping Instability and Banding Instability
  - Modifying the kernel function

Future developments

- The particle interactions in SPH determined by the kernel functions behavior
- Particles with other schemes for working out particle-particle interactions
- Tessellation techniques

Thank you!

Any questions?