WAVELET FRACTAL

1) Fractal dimension.

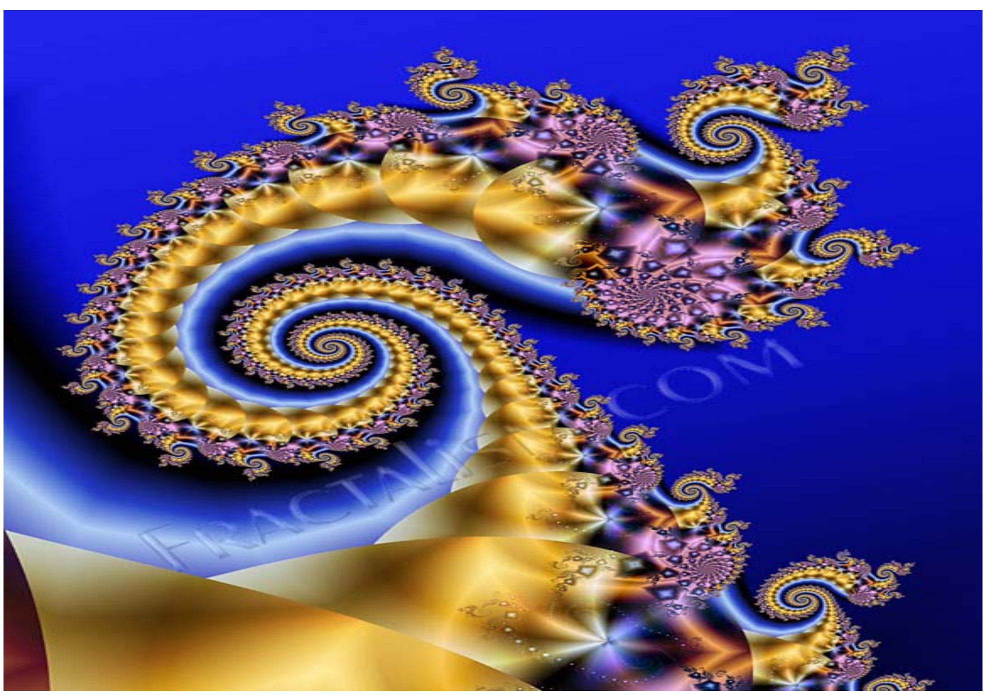
→ $2\pi r^2$ assume r=$\lim_{\infty} 1/e$

$N(e) = K(1/e)^{-2}$ ------→ k = *2* π for circle*

$$\ln N(e) = ln(k) - 2 * \ln(e)$$

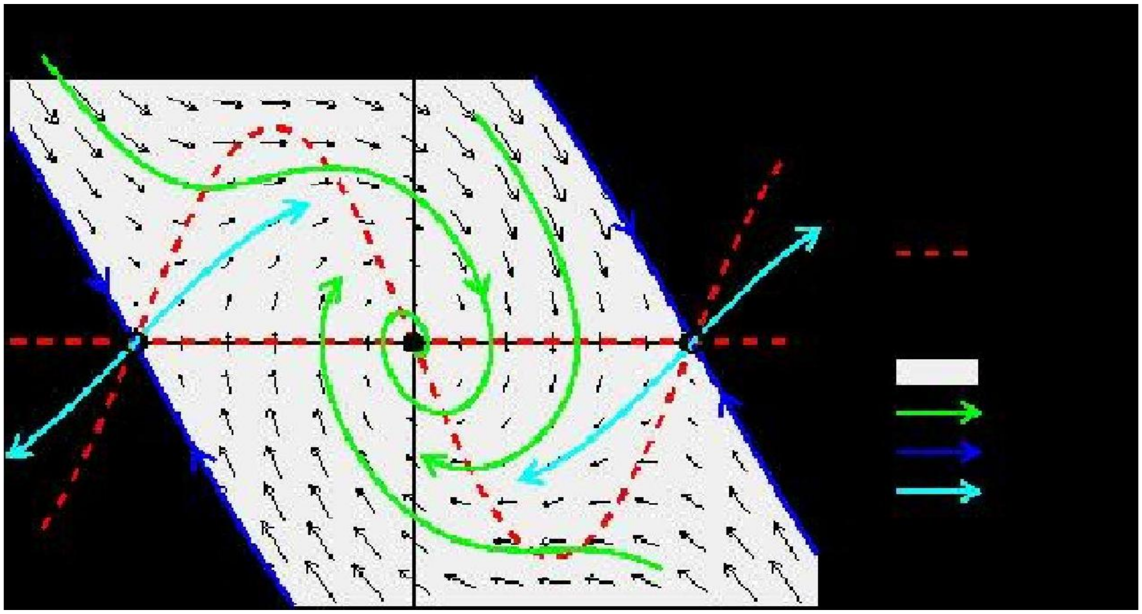
Dimension → $2 = \frac{\ln N(e)}{\ln(1/e)}$

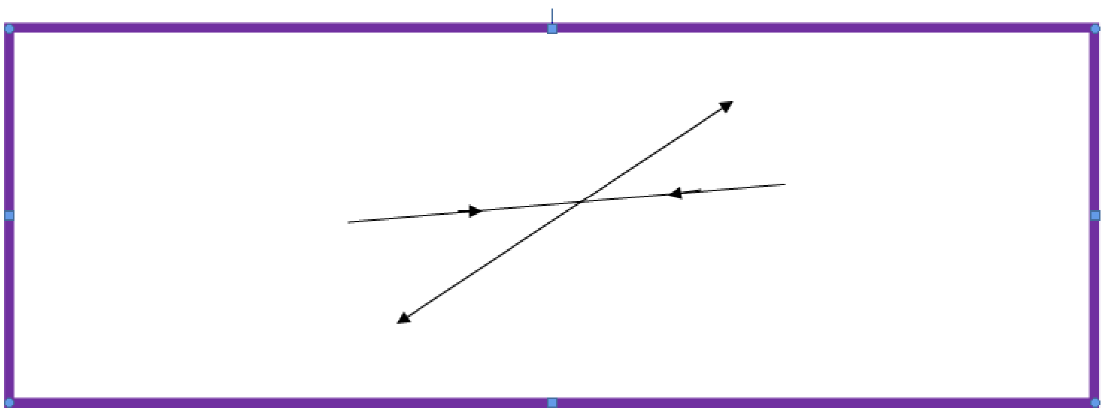
$$(R^n \quad \rightarrow \quad R^{n-1})$$

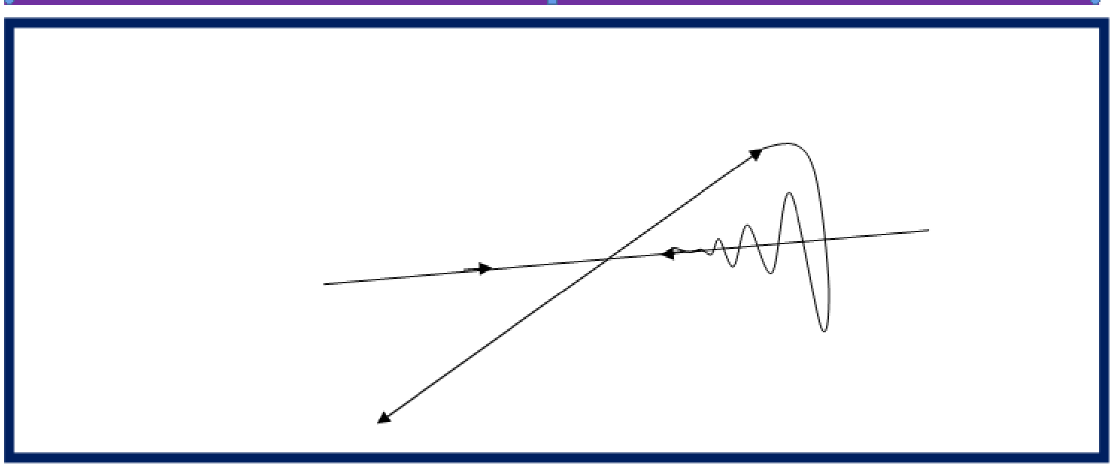


2) Why turbulence created and goes as fractal, (to small scales)?

x= c* $\dot{x}$ same direction







3) To describe the turbulence first begins with flow motion:

$$\partial u/_{\partial t} + (u.\nabla)u + \frac{1}{p\nabla P} = v\Delta u + F$$

$$\nabla . u = 0$$

(1) Advection $(u.\nabla)u$
(2) dissipation $v\Delta u$
(3) u velocity field
(4) density
(5) v kinematic viscosity
(6) F external force

Turbulence=the divide 1 by 2 (Re) which higher the ratio signifying larger turbulence.
>10^3 result UL/V, L is the characteristic of dimension →then in astrophysics this feature is important
On the other hand, importance of scales
Kolmogorov -> energy distributed by power law: E (k) =$ke^{2/3}k^{-5/3}$

K=kolmogorov *constant, e=range of energy transfer, second k is wave number 2pi/lambda*
Not valid generally: large scales injected to small ones as dissipation to heat occur in small scales (assume nature of fractal).

Problems with this assumption.
coherent structure ,large Re by using E(K) resulted to long time which is not right due to turbulence statistics also Valid in inertial range of intermediate k .
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Wavelet

Wavelet is : an oscillation which is localized in time with zero average .

Why wavelet:
(1) it is finite signal -> cosine functions goes to infinity which is not possible to implement by computers -> window effect is inevitable ,in the other the power of wavelet is analysis of no prosodic signals in comparison with ft.
(2) Ft loose the temporal behavior of signal as phase information is impossible to interpret

Individually, but by wavelet techniques it is possible to locate the frequency characteristics of signal in time .other words how function behave in time.
Figure page 10

$$Wf(a,b) = \int \varphi(t) f(t) dt$$

Mexican hat= $\frac{1}{a} e^{(-(\frac{t-b}{a})^2)}$

(3) wavelet kernel can be almost any function ,but zero in average also so many features make it more powerful to analysis ,orthogonal to polynomials and,,,,

Mexican hat good at finding maximum and minimum in the signal and not the scale or frequencies.

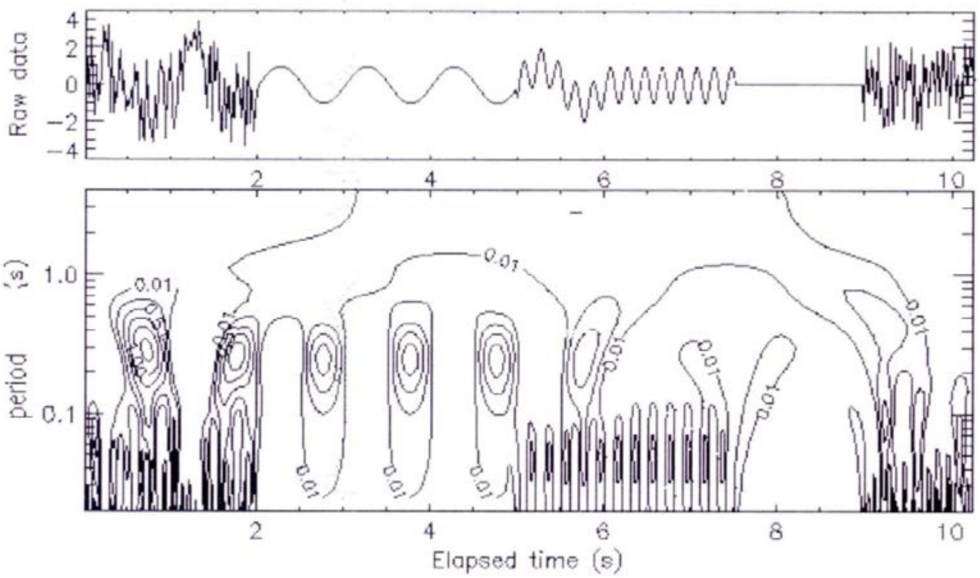
Morlet$\varphi = \exp(iwt).\exp(-0.5.t^2)$ figure 6,

Correlation:$Cor(\tau) = \int f(t)g(t+\tau)dt$ finding how much [f] behavior is similar to [g]

Auto-cor= $R(\tau) = \int f(t)f(t+\tau)dt$ finding how much f behaviour similar to itself.
Remembering turbulence that has large scales that energy goes to small scales and the fractal nature of turbulence, thus Auto correlation resulted to describing the turbulence behavior.
f→Wf(t).
Definition: WC(a, τ) = $\int Wf(a,\tau)Wf(a,b+\tau)dt$

Steps to interpretation: normalizing with C(0) and or WC(a,0) ,finding the phase by $\tan^{-1}(im(Wc)/real(Wc))$

Briefing : We try to find the self similarity by wavelet kernel feature behavior ,on the other hand self similarity means periodicity(not just usual periodicity also up to period two signal) in signal -> by wavelet the scales of periodicities are under control by (a) , the location of the (b) and the feature manner by wavelet kernel .



Raw data
4
2
0
−2
−4
2
4
6
8
10
period (s)
1.0
0.1
0.01
Elapsed time (s)

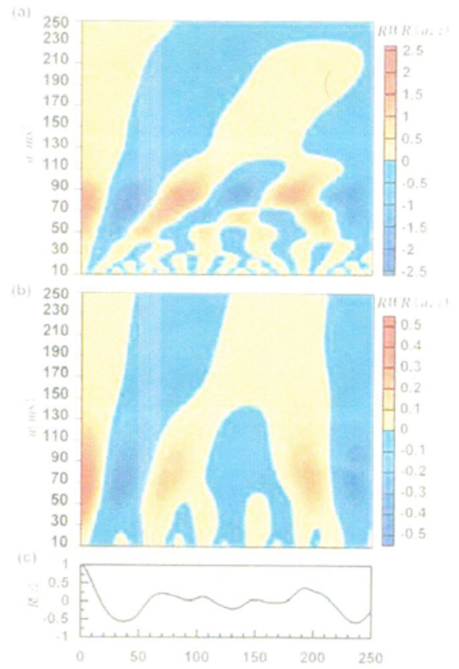


Figure 10: Measurement on centerline at $x = 4d = 100$ mm. a) Wavelet auto-correlation coefficients $RWR(a, \tau)$ using the Morlet wavelet. b) Ditto but using the Mexican hat wavelet. c) Traditional normalized auto-correlation $R(\tau)$. From Li (1998).

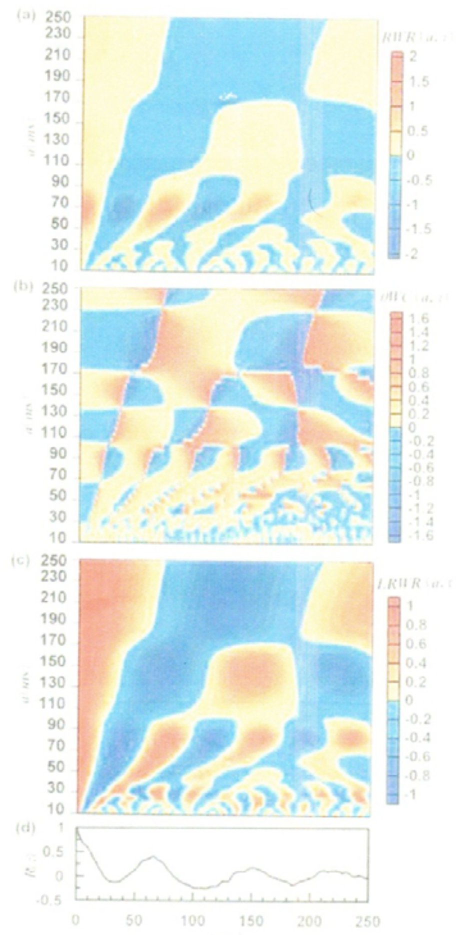


Figure 11: Measurement off the centerline at $y = d = 25$ mm, $x = 5d = 125$ mm. a) Wavelet auto-correlation $RWR(a,\tau)$. b) Phase of wavelet auto-correlation, $\theta RWR(a,\tau)$. c) Local wavelet auto-correlation, $LRWR(a,\tau)$. d) Traditional normalized auto-correlation $R(\tau)$. From Li (1998).

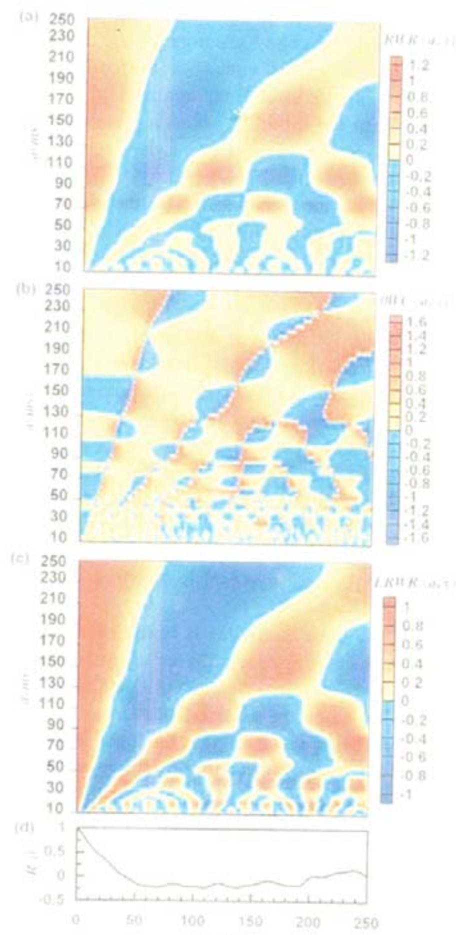


Figure 12: Measurement off the centerline at $y = 2.5d = 62.5$ mm, $x = 10d = 250$ mm. a) Wavelet auto-correlation $RWR(a, \tau)$. b) Phase of wavelet auto-correlation, $\theta RWR(a, \tau)$. c) Local wavelet auto-correlation, $LRWR(a, \tau)$. d) Traditional normalized auto-correlation $R(\tau)$. From Li (1998).

Singularity spectrum.

$$\|f(x) - Pn(x - x0)\| < C|x - x0|^h$$
$$h\ddot{o}lder\ dimentoins$$

Wf(a,x0) = $1/a\int C(x - x0)^{h(x)}\varphi\left(\frac{x-x0}{a}\right)dx = C|a|\,\hat{}\,h(x0)\int x\,\hat{}\,h(x0)\varphi(x)dx$

Therefore Wf(a,x0)~$a^{h(x0)}$

[h] which coming from Hölder equation resulted to Hausdorrf dimension .

Problem, experimental data do not contain information at infinitely small scales a. then the wavelet transform will not only involve the point x0 for which we want the singular behavior of nearby points will mix with the point x0.

Instead of observing the dependence of the wavelet transform on the scale a for a local point x we make a global approach where we make use of a partition function.

Z(q,a)=$\int(Wf)^q$dx problem negative q may cause of divergence for small Wf then if focus on maxium to describe the behavior ,integral can be change by $\sum(Wf)^q$
Result:

If (a) goes to zero the Z(q,a) goes to $a^{\tau(q)}$

By Legendre transform and put the constrains as left equation is minimum regarding then -> D(h) =q*h-τ(q) then by derivation :h=$\partial\frac{\tau}{q}$ which is holder exponent ,which lead to fractal dimension ,notice that $\tau(q)$ is linear regarding q

➔ There is one single holder exponent .-> characterize the turbulence