Acceleration of the universe

and

dark energy

A hideously short review of general relativity

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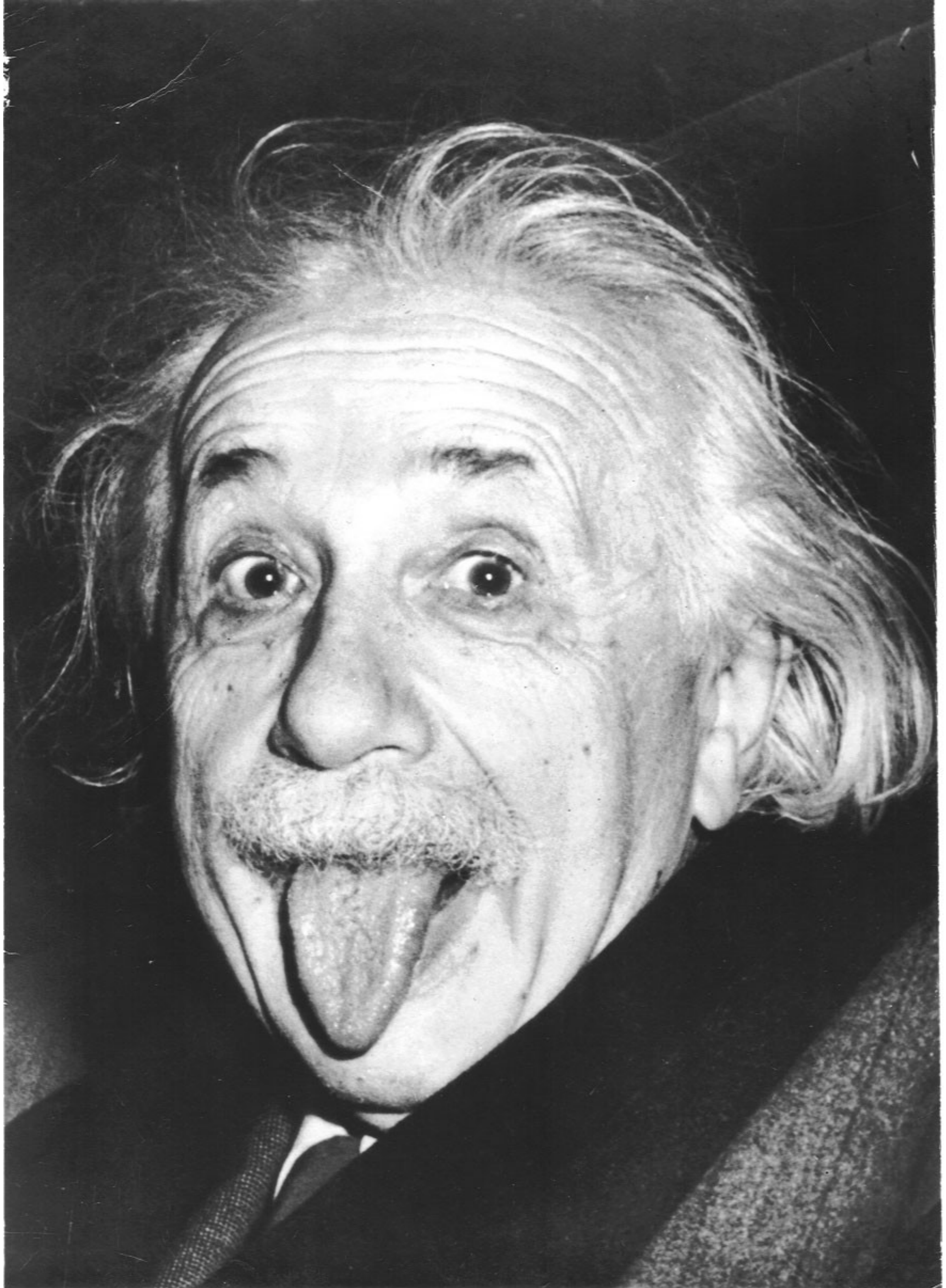

A hideously short review of general relativity

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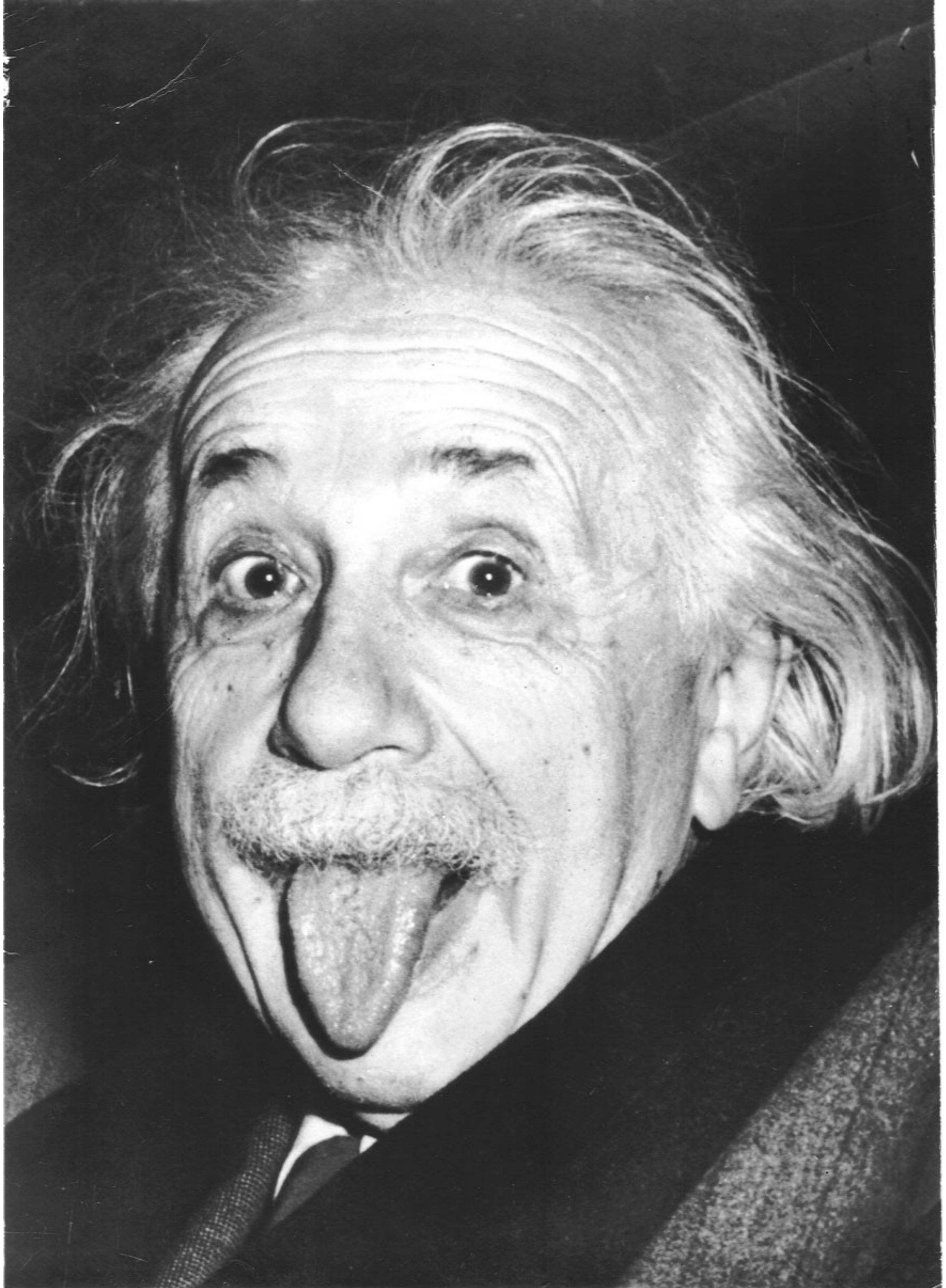
Weak equivalence principle: There is no difference between a gravitational mass and an inertial mass.

In other words – there is no way to tell the difference between a 'normal' acceleration and an acceleration due to gravity.



To describe things mathematically you work with differential geometry

The metric is an expression for distance between points

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For a plane in d = 3 it's simply the Pythagorean theorem everybody learned in kindergarten

$$ds^2 = dx^2 + dy^2 + dz^2$$

When you want to describe some interesting things (e.g a rotating, charge black hole) these can become very complicated.

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$$\frac{1}{4}\sum_{\lambda,\sigma,\rho,\eta} g^{\eta\sigma}g^{\lambda\rho}(\partial_\lambda g_{\mu\sigma}+\partial_\mu g_{\lambda\sigma}-\partial_\sigma g_{\lambda\mu})(\partial_\eta g_{\nu\rho}+\partial_\nu g_{\eta\rho}-\partial_\rho g_{\eta\nu}) \;-$$

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"Do not worry about your difficulties in Mathematics. I can assure you mine are still greater."
A. Einstein

Now lets do stuff!

Cosmology

We can actually model the universe with this! In order to do it properly we make two big but well founded assumptions: we say that it is

Isotropic

Everything looks the same in all directions

Homogeneous

Everything looks the same in any given point

If one sits down and thinks about what these facts mean when constructing a suitable metric (it should only take a couple of years) you end up with the FLRW-metric and the standard model of cosmology.

Friedmann-Lemaître-Robertson-Walker metric

$$ds^2 = -dt^2 + a(t)^2 \left(\frac{dr^2}{1-kr^2} + r^2 \, d\theta^2 + r^2 \sin(\theta) \, d\varphi^2 \right)$$

t, r, θ, φ are our coordinates

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k is a geometric parameter which can only take the values...

$$k = \begin{matrix} 1 \\ 0 \\ -1 \end{matrix}$$

1 Positive curvature (a sphere)
0 No curvature (a plane)
−1 Negative curvature (a hyperbolic surface)

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a(t) is the scale of the universe. In other words it is vital in deciding how (global) distances change over time.

So far we've avoided to mention the rhs of the Einstein equation

$$rhs = 8\pi G T_{\mu\nu}$$

T is called the energy-momentum tensor and contains the energy density and flux of momentum. To have something to work with we assume a perfect fluid with zero bulk velocity, which means

$$T_{\mu\nu} = \begin{pmatrix} \rho & 0 & 0 & 0 \\ 0 & p\, g_{22} & 0 & 0 \\ 0 & 0 & p\, g_{33} & 0 \\ 0 & 0 & 0 & p\, g_{44} \end{pmatrix}$$

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Furthermore we can define an equation of state

$$p = w\rho$$

The value of w depends on what the universe consists of. If matter is dominant (in the form of dust) then w = 0. If instead radiation is supreme then w = 1/3. Today radiation is essentially negligible in relation to dust.

Having figured out the rhs we can apply the Einstein equation to the FRLW-metric. Turning the mathematical crank leads to

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2}$$

$$\frac{\ddot{a}}{a} = \frac{-4\pi G}{3}(\rho + 3\mathrm{p})$$

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In addition there is the conservation of energy

$$D_\mu T^{\mu\nu} = 0 \qquad \Rightarrow \qquad \dot{\rho} + 3\frac{\dot{a}}{a}(\rho + p) = 0$$

This we can solve and get ρ as a function of a to be used in the eq's above

$$\rho = C\,a(t)^{-3(1+w)}$$

Giving the solution

$$a = a_0\left(\frac{t}{t_0}\right)^{\frac{2}{3(1+w)}}$$

Dark energy

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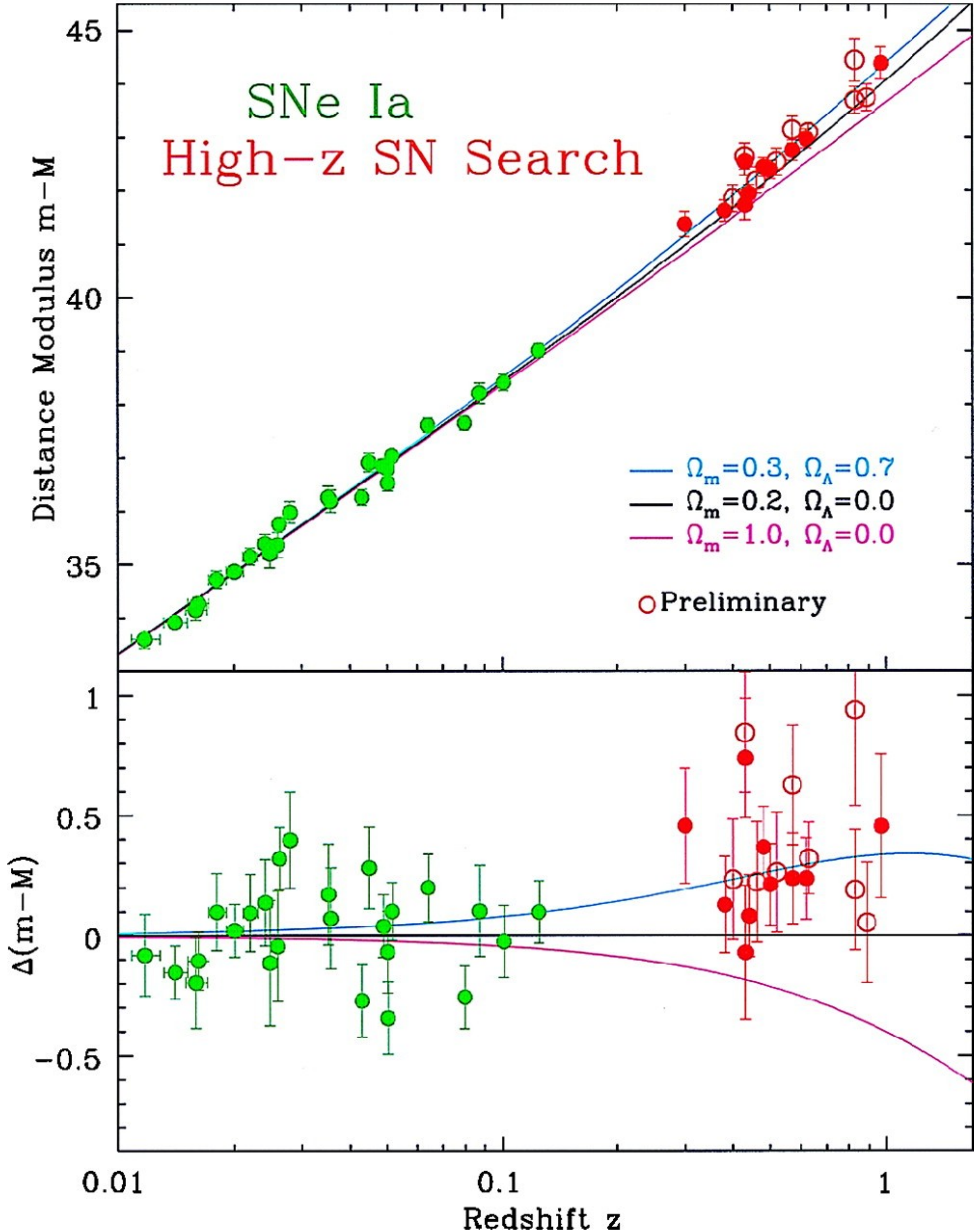
And furthermore the density parameter which relates the measured density to the critical one.

$$\Omega = \frac{\rho}{\rho_c}$$

From measurements of the cosmic microwave background radiation we have that k = 0, or at least very, very close to 0. But estimates of the total matter of the galaxy yields only $\Omega_M = 0.3$ (of which almost all is dark matter). Enter dark energy!

Before going deeper into this we take a look at another sign of the dark energy. The Hubble law has been known since 1929 and is an empirical formula relating the speed with which galaxies seem to move away to their distance.

$$v = H_0 d$$



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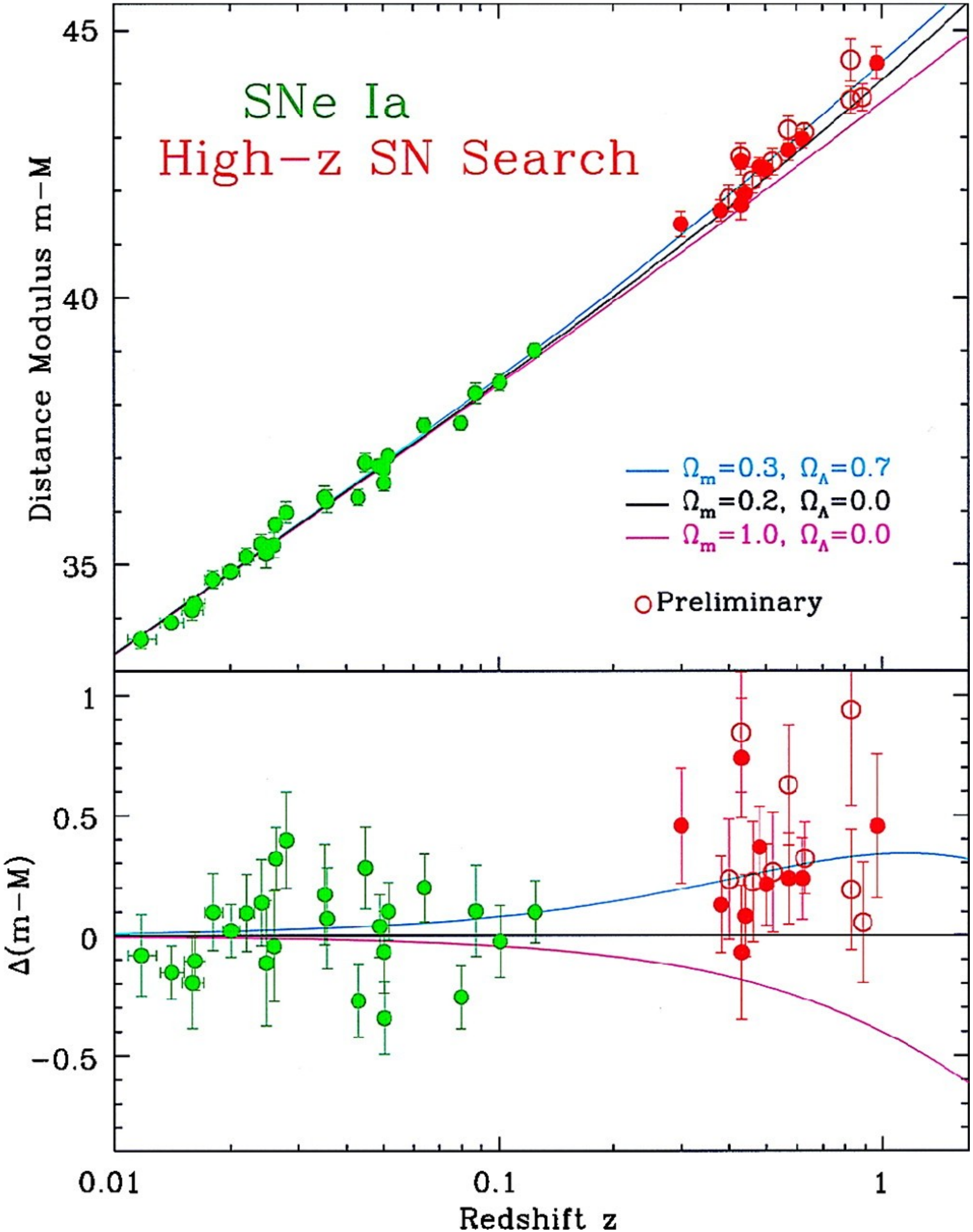
When measuring you use the redshift instead. For a plane universe the luminosity depends on the redshift linearly

$$d = \frac{z}{H_0}$$

In 1998 two independent teams discovered that the linearity of the law seems not to hold...



Now we try to explain this fact. Include a cosmological constant to make the EE take the form

$$R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}R+\Lambda g_{\mu\nu}=8\pi G T_{\mu\nu}$$

But this is the same thing as working with a modified energy momentum tensor on the form

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If we find a Λ so that $\Omega_\Lambda = 0.7$ and we're done. This corresponds to a vacuum energy density of

$$\rho_\Lambda = (10^{-3} eV)^4 = 10^{-8} erg/cm^3$$

The 'not-so-nice' problems

In GR Λ is a free parameter but if interpreting it as an vacuum energy density then we can estimate it from QM (or rather QFT).

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A more intuitive way to get the same result is to note that the unit of Λ is m^{-2}. The natural thing to do would be to relate this scale length to some other fundamental length, e.g. the planck length $\Lambda \sim L_p^{-2}$. In terms of the density instead

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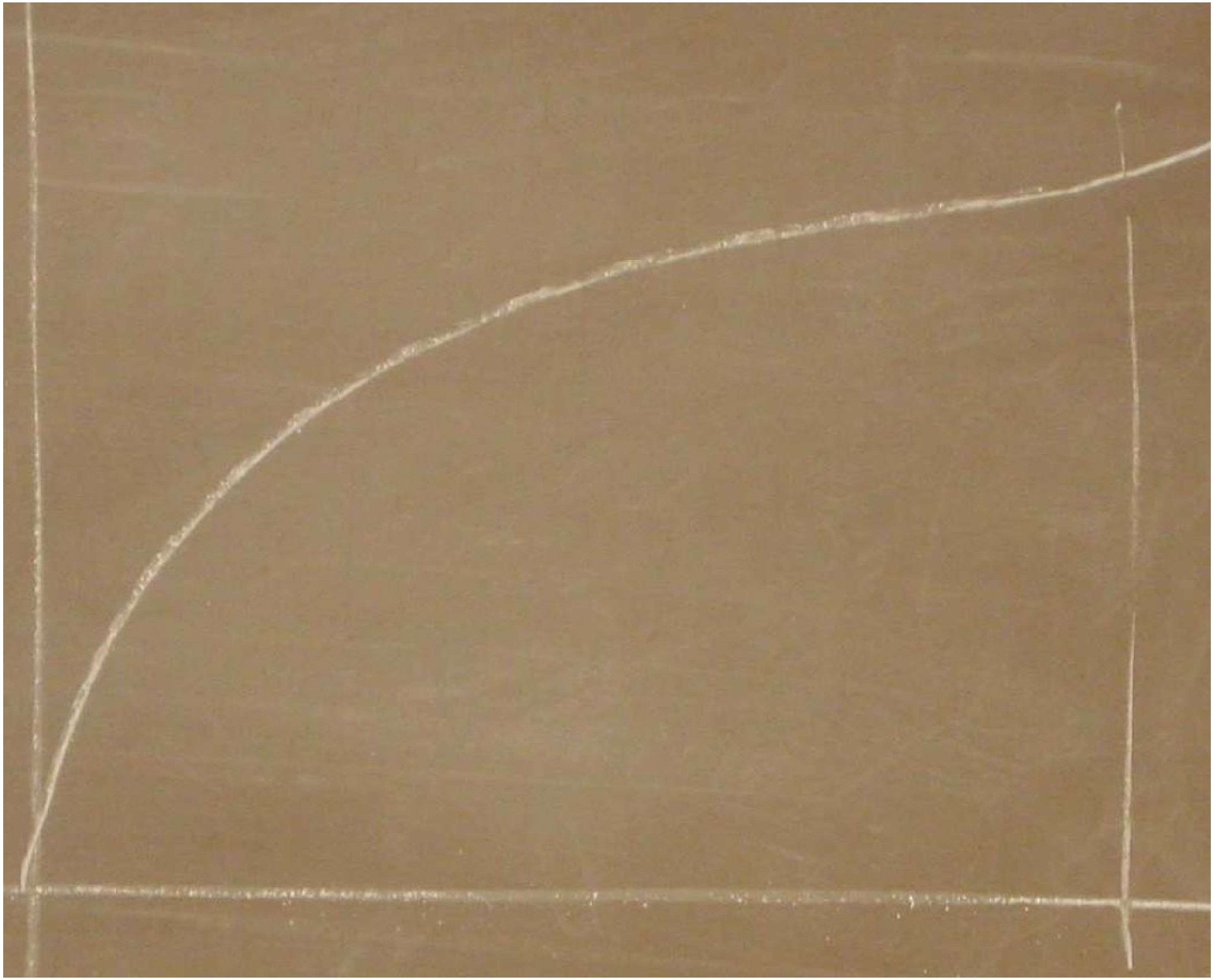
'The worst theoretical prediction in the history of physics.'

The coincidence problem. Why do we live in a 'special' time?



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$$\rho = \frac{1}{2}\dot{Q}^2 + V \qquad\qquad p = L$$

And the eqm

$$\ddot{Q} + 3H\dot{Q} + \frac{\partial V}{\partial Q} = 0$$

The interesting parameter here is the potential V(Q). Letting it be big at small t insures that the solutions to the FRLW-metric is exponential and inflation is possible. Then it must fall of to let expansion continue at a slower pace. There are still problems with huge differences in 'observed' and guessed values though.