PULSARS

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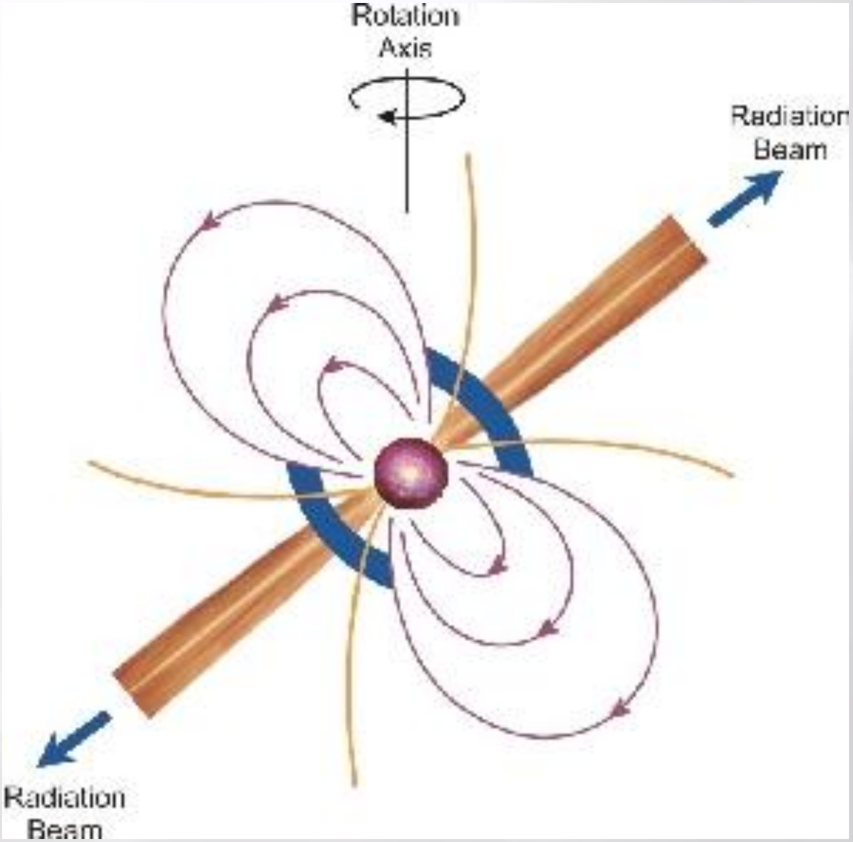
Astrophysical Dynamics

PRESENTATION

The lighthouse effect



Number in our galaxy : around 1200

Radiation powered by rotation

Small periods of rotation

High magnetic field

Small radius

High mass density

A magnetosphere around it, but wait…

Mass : $1.4\ M_{\odot}\ to\ 2.1 M_{\odot}$

Increase of the mass density during the collapse by a factor of: 10^{15}
(new density comparable to the atomic nucleus density)

Contraction of the radius by a factor of: $\boxed{10^5\ \ (R_{pulsar} \approx 10km)}$

Conservation of the angular momentum: $M\Omega_{in}R_{in}^2 = M\Omega_{final}R_{final}^2$

Estimated angular velocity: $\boxed{\Omega_{final} \approx \frac{2\pi}{10^{-4}} s^{-1}}$

Measured period: $\boxed{P_{pulsar} \approx 10^{-3} \sim 10^{0}\ s}$

Alfvén's theorem of flux freezing : 'The magnetic flux linked with a surface that moves with the fluid is constant'.

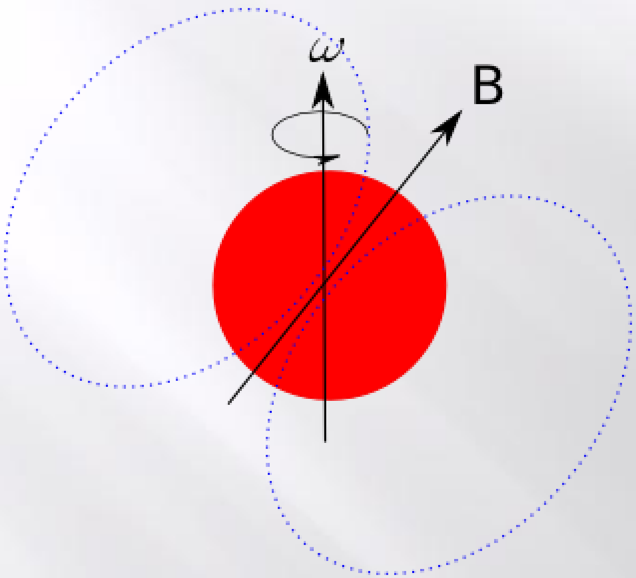
The proof is based on the induction equation : $\frac{\partial \boldsymbol{B}}{\partial t} = \nabla \times (\boldsymbol{v} \times \boldsymbol{B}) + \eta \nabla^2 \boldsymbol{B}$
and the approximation negligible resistivity.

Then, $\phi \; \alpha \; BR^2 = constant$

$$B \approx 10^8 T$$

ROTATING MAGNETIC DIPOLE

A FIRST MODEL



Assumptions:

Rotating magnetic dipole moment emitting magnetic dipole radiation

The radiation is powered by the kinetic energy of the pulsars

The region surrounding the pulsar is a vaccum and the emission is free

We ignore the relativistic effects near the neutron star

We do not bother solving the electromagnetic fields in the whole space

$$\frac{dE}{dt} = I\Omega\frac{d\Omega}{dt} = -\frac{2}{c^3}|\ddot{\boldsymbol{m}}|^2$$

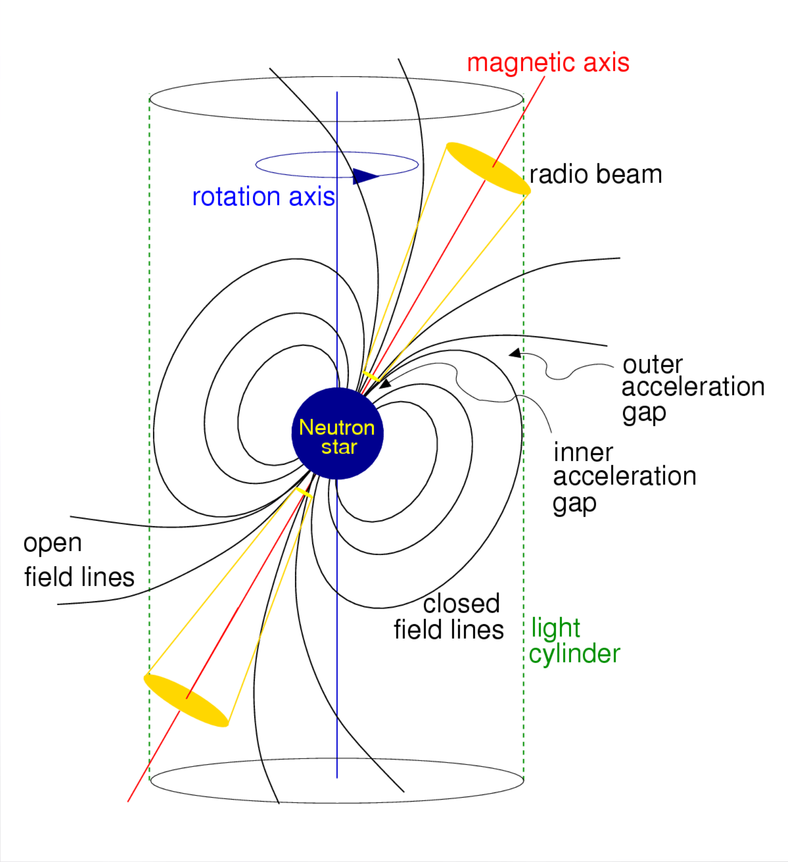
$$I\Omega\frac{d\Omega}{dt} = -\frac{B^2R^6\Omega^4\sin^2\alpha}{6c^3}$$

$$\boxed{\Omega = \Omega_0\left(1+\frac{t}{\tau}\right)^{-\frac{1}{2}}} \quad with\ \tau = \frac{3c^3I}{B^2R^6\sin^2\alpha\,\Omega_0^2}:$$

$$\tau \sim 10^6 yr$$

THE ALIGNED ROTATOR

A MORE ACCURATE MODEL



Assumptions

Neutron star: conducting rotating sphere.

Uniform internal magnetic-field:

$$\boldsymbol{B} = B_0 \boldsymbol{e_z}$$

The axis of rotation and the magnetic axis are aligned

The particles move to compensate for the magnetic force:

$$\boldsymbol{E} = -\frac{\boldsymbol{v} \times \boldsymbol{B}}{c}$$

Calculus of the electric potential

Inside the neutron star

$$\boldsymbol{E}_{in} = -\frac{\Omega B_0 r sin\theta}{c}(sin\theta\,\boldsymbol{e}_r + cos\theta\,\boldsymbol{e}_\theta) = -\nabla\phi_{in}$$

$$\phi_{in} = \frac{\Omega B_0}{2c} r^2 \sin^2\theta + \ constant$$

Outside the neutron star

$$\Delta\phi_{out} = 0$$

$$\phi_{out} = -\frac{\Omega B_0 R^5}{6cr^3}(3\cos^2\theta - 1) \quad +constant$$

$$\boldsymbol{E}_{out} = -\frac{\Omega B_0 R^5}{3cr^3}(3\cos^2\theta - 1)\boldsymbol{e}_r - \frac{\Omega B_0 R^5}{cr^4} sin\theta cos\theta\,\boldsymbol{e}_\theta$$

$$\boldsymbol{B}_{out} = \frac{B_0 R^3}{2r^3}(2\cos\theta\,\boldsymbol{e}_r + sin\theta\,\boldsymbol{e}_\theta)$$

Magnetic field lines : $r = Ksin^2\theta$

Deduction of the densities and electric currents

$$\rho = \frac{\nabla \boldsymbol{E}}{4\pi} \quad and \quad \boldsymbol{j} = \frac{c}{4\pi} \nabla \times \boldsymbol{B}$$

$$\rho_{in} = \frac{\Omega B_0}{2\pi c} \quad and \quad \boldsymbol{j}_{in} = 0$$

$$\rho_{out} = 0 \quad and \quad \boldsymbol{j}_{out} = 0$$

Apparition of surfacic charges

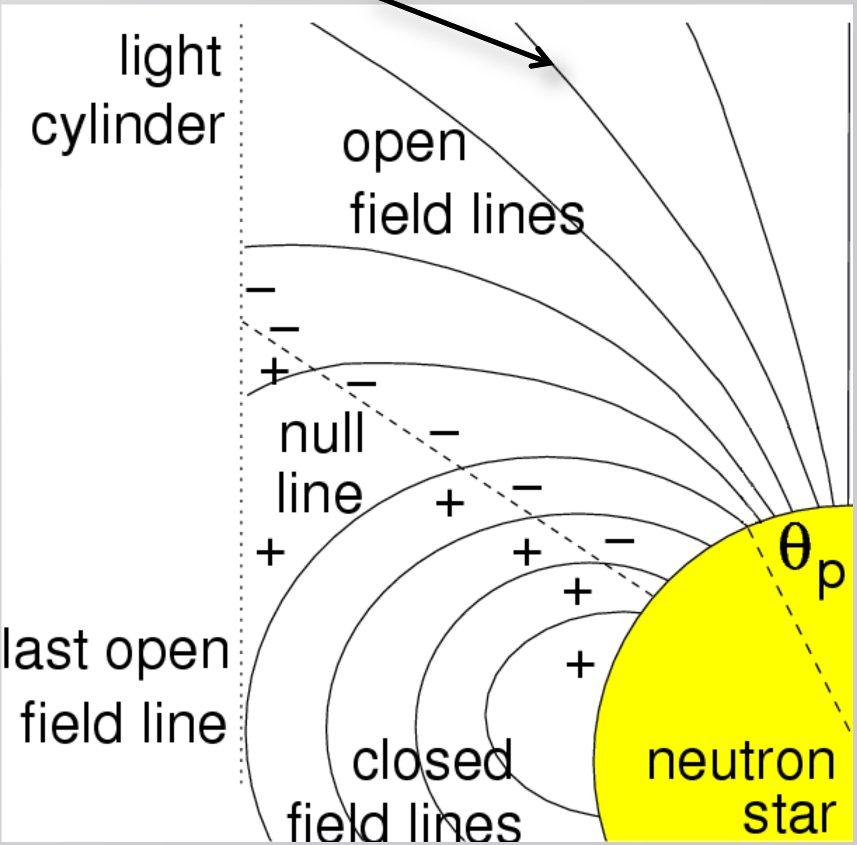
$$\rho_{sur} = \frac{\Omega B_0 R}{24\pi}(9 - 15\cos^2\theta) \;\; and \quad \boldsymbol{j}_{sur} = \frac{cB_0}{8\pi} sin\theta \, \boldsymbol{e_\theta}$$

Pulling out particules?

$$\phi_{sur} \approx \frac{\Omega B_0 R^2}{2c} \approx 10^{16} V \quad and \quad E \approx \frac{\Omega R}{c} B_0 \approx 10^8 esu$$

...depending on the solid state of the crust!
But let us assume it happens.

$$r = K sin^2 \theta$$



Field capable to give energy to the particules

$$\boldsymbol{E}_{//} = \frac{\boldsymbol{E}.\boldsymbol{B}}{|\boldsymbol{B}|} = -\frac{\Omega R}{c} B_0 \cos^3 \theta$$

Why do the particules follow the field lines??

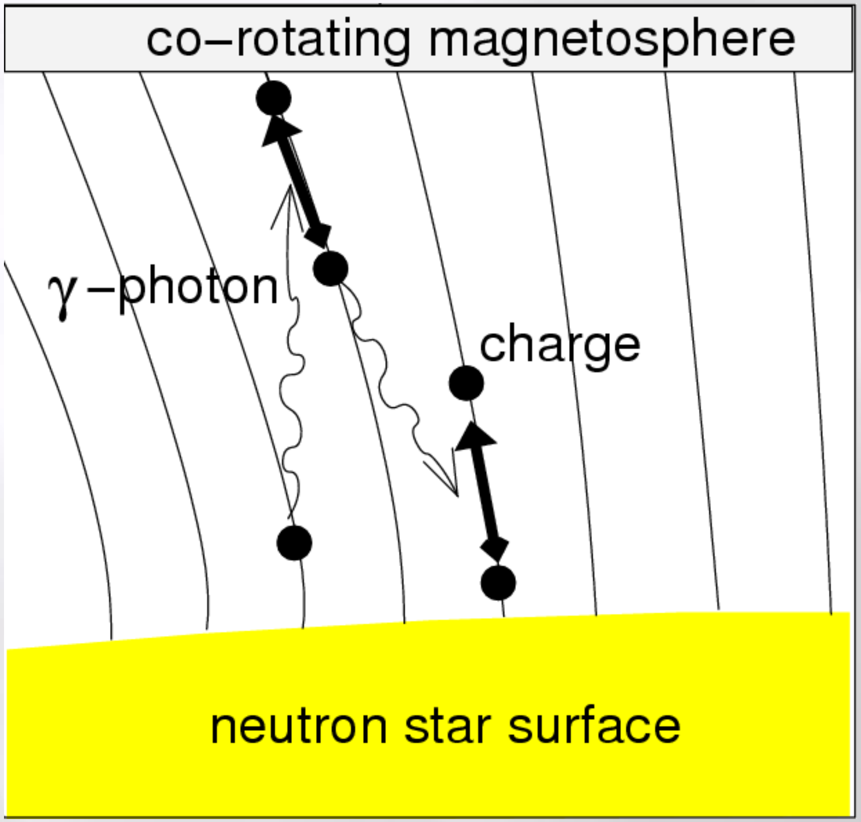
Light cylinder: $R_L \approx \dfrac{c}{\Omega}$

So, emission through the polar cap of radius:

$$R_P \approx R sin\theta_P \approx R \left(\frac{R}{R_L}\right)^{\frac{1}{2}} \approx 10^4 cm$$

What happens to the field lines near the light cylinder? Derive the derivative of angular velocity!

PAIR PRODUCTION IN A PULSAR ATMOSPHERE



Accelerated charged particules

Highly energetic photons in a high electric field

Cascade…

What only electrons and not protons?

Near the surface, acceleration of electrons to high energies

$$\gamma m c^2 = e\Delta V \approx e\frac{B\Omega^2 R^3}{2c^2} \quad \rightarrow \quad \gamma \approx 10^7$$

Radiation following a synchrotron spectrum

Condition of pair-production by a photon: $\gamma^3 \frac{\hbar c}{R} \approx \hbar\omega > 2m_e c^2$

which gives: $\frac{B}{10^8 T}\left(\frac{P}{1s}\right)^{-2} \geq 0.025$

Another condition to be respected for the cascade: $l < R$

which gives: $\frac{B}{10^9 T}\left(\frac{P}{1s}\right)^{-\frac{3}{2}} > 31$

CONCLUSION

Glitches

Binary pulsars