Astrophysical Fractals

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Hot-Topic Presentation of Astrophysical Dynamics

December 9th 2010

Cosmological Principle

Basic Assumptions:

- homogeneity
- isotropy

What Is A Fractal?

- Shape or distribution, where each part of it is a version of the whole, reduced in scale.

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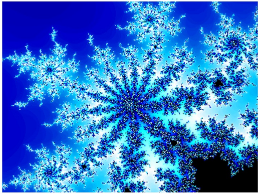
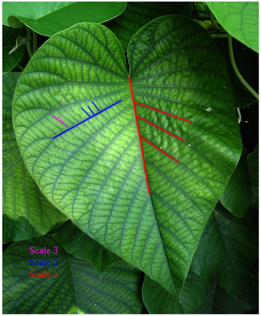




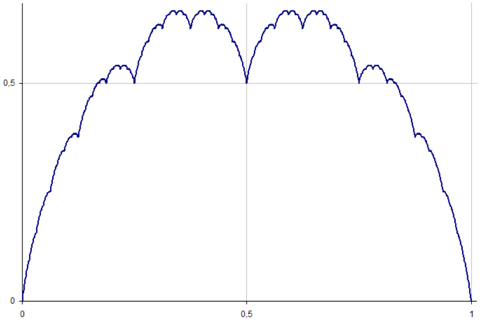
Fractal Dimension

- statistical property
- real number $\mathbb{R}$, not just natural number $\mathbb{N}$
- topological dimension $\leq$ D $\leq$ embedded space

Fractal Dimension

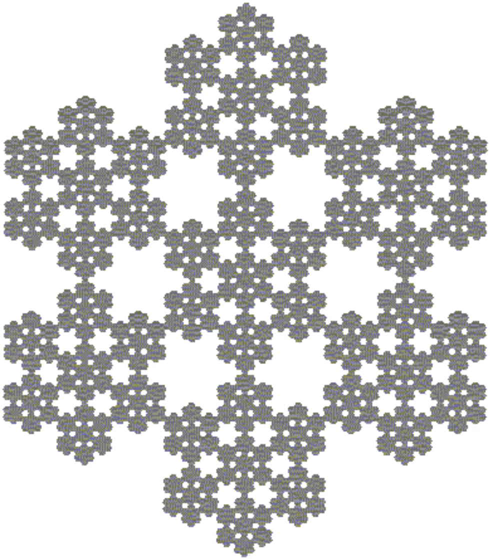
Takagi curve

$\mathrm{D} = 1$



Fractal Dimension

Hexaflake
D = 1.77

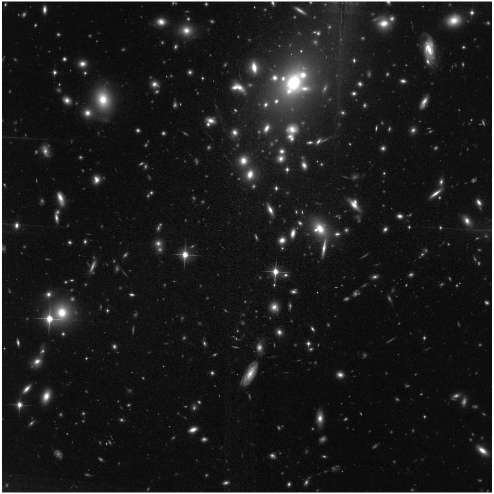


Fractal Dimension

Distribution of galaxy clusters

D $\sim$ 2

(meassured by SDSS)



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- $D = 3$ if the distribution is homogenous
- D is used to characterize the degree of clustering

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- number of galaxies within a distance R:

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- correlation function: $\xi(\vec{r}) = \frac{<n(\vec{r}_i)\ n(\vec{r}_i + \vec{r})>}{<n>^2} - 1$

  $\xi(\vec{r}) = 0$ in case of homogeneity

  $\xi(r) \propto r^{-\gamma}$ with $D = 3 - \gamma$

Astrophysical Fractals

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- $D = 1.6$ - 2.0 in both cases

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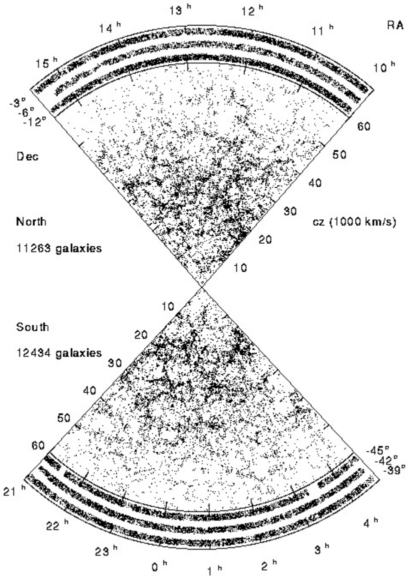

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- distinction between luminous objects and total matter

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- ISM
  - Observations of molecular clouds reveal hierarchical structures.
- Galaxies
  - not homogeneously distributed
  - hierarchical structure: groups, clusters & superclusters
  - Galaxies & clusters show scaling property $\xi(r) \propto r^{-\gamma}$

Origin of Astrophysical Fractals

- Formation based on **turbulence** and/or **self-gravity**?

Turbulence

- Reynolds number: $R_e = \frac{\sigma\, l}{\nu} \overset{ISM}{\approx} 10^9$

  ($R_e \gg 10^3$: *turbulent*)

  σ : velocity dispersion
  l : typical dimension for cloud cloud collision
  ν : viscosity (negligible)

- ISM is governed by strong fluctuations in density and velocity

- Characteristic for media with 2nd order phase transition.

Turbulence

- Simulations require enormous computational requirements.
- only 3 levels for pressure-less turbulent flows without self-gravity
- inducing heating and cooling (time scales $<$ dynamical)
  - polytropic eq. of state: $P \propto \rho^{\gamma}, \gamma$=0-2 ($\gamma = 1:$ isothermal)
  - logatropic eq. of state: $P \propto log(\rho)$
- This does not represent dynamical processes in the ISM

Gravitational Instability

- Simulations of a gravitational unstable media shows filaments and clusters over long time scales.
- Non-linear gravitational instability leads to long range fractal order (scaling laws).
- Cosmic expansion supports galactic fractals
- Galaxy differential rotation supports ISM fractals
- Scaling laws persist as long as the larger scale is driving the gravitational instability.

Results

	Sample	R (h^{-1} Mpc)	D_2
Guzzo et al.	Perseus-Pisces	1.0–3.5	1.2
		3.5–20	2.2
Martinez & Coles	QDOT	1.0–10	2.25
		10–30	2.77
Lemson & Sanders	CfA	1.0–30	2.0
Martínez et al.	Stromlo-APM	30–60	2.7–2.9
Scaramella et al.	ESP	300–400	2.93
X-Ray Background		~ 500	$3 - D_2 = 10^{-4}$
			with $\sigma_8 = 2$, $\Gamma = 0.5$
COBE 4 year normalisation		~ 1000	$3 - D_2 = 2 \times 10^{-5}$
			with $\sigma_8 = 1.4$, $\Gamma = 0.5$

Conclusions

- distribution of ISM and galaxies is self-similar
- galaxy fractals are still growing to larger scales
- num. simulations are confined by practical constraints
- quasi-station fractals due to self-gravity and energy injection
- turbulence is more likely to be a consequence
- smooth transition to homogeneity