Theoretical Physics
Chalmers & Göteborg University

Written Exam in Quantum Mechanics, FKA081

January 15, 2001, 8.45 - 13.45

Examiner: Henrik Johannesson, phone: 3185.
Allowed references: J. Sakurai, "Modern Quantum Mechanics", mathematical tables, one sheet of hand-written formulas

Please put your name on *each solution sheet*, and don't forget to also put your e-mail address on the cover sheet.

Structure your solutions carefully. State precisely which assumptions, theoretical results, approximations, etc. you use. The logic of your arguments must be transparent, and you should strive for optimal readability! All problems are equally weighted (10p/problem.)

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1. Time evolution and expectation values

Let $|\,n\rangle$ (with $n = 0, 1, 2, \ldots$) be the eigenstates to the one-dimensional harmonic oscillator Hamiltonian $(a^\dagger a + 1/2)\hbar\omega$, where $a^\dagger$ and a are the creation- and destruction operators with commutator $[\,a, a^\dagger\,] = 1$. A collection of independent harmonic oscillators are described by a mixed ensemble where at time $t = 0$ 1/3 of the oscillators are in the state $|\,\psi_1(0)\rangle = (\,|\,0\rangle + |\,1\rangle)/\sqrt{2}$, while 2/3 are in the state $|\,\psi_2(0)\rangle = (\,|\,0\rangle + |\,2\rangle)/\sqrt{2}$.

(a) How does the ensemble evolve with time?

(b) Calculate the expectation value for a measurement of the momentum of an oscillator at time t.

2. EPR and Bell's inequalities

(a) Summarize (roughly!) the argument that lead Einstein, Podolsky and Rosen (EPR) to conclude that quantum mechanics is an "incomplete" theory.

(b) The considerations of EPR were concerned with two observers who make spin measurements along the same axis.[1] John Bell found his famous inequalities by asking what happens if the observers measure the spin of the particles along different axes. What is the relevance of Bell's work? In particular, how does it address the issue raised by EPR?

(c) A so called "EPR state" is an example of a quantum entangled state. Give a precise definition of what is meant by *quantum entanglement* and discuss some of its consequences and potentials for future technologies.

[1] Strictly speaking, this is true only in the simplified version of the original EPR argument, presented some years later by David Bohm.

3. Stern-Gerlach

Consider a polarized beam P of atoms with spin quantum number 1/2 and zero orbital angular momentum, all with angular momentum $+\hbar/2$ along the $x-$axis. U is a beam of similar but unpolarized atoms.

(a) What is the spin state of the atoms in beam P in terms of the eigenstates of S_z?

(b) If the two beams, P and U, are passed separately through a Stern-Gerlach apparatus with its magnetic field along the z-axis, is there any difference between the emerging beams in the two cases? What if the magnetic field is along the x-axis?

(c) The beam U can be represented as an incoherent mixture of the eigenstates $|\uparrow\rangle$ and $|\downarrow\rangle$ of S_z. The spin states of the atoms in U may thus be written as $|\psi\rangle \sim a_1|\uparrow\rangle + a_2|\downarrow\rangle$, where a_1 and a_2 are complex numbers of modulus unity with random relative phases, i.e. $< a_1^* a_2 > = < a_2^* a_1 > = 0$, where the brackets indicate the average over all values of the relative phases. By expanding $|\psi\rangle$ in terms of the eigenstates of S_x, show that this state has the required physical properties, consistent with your answer in (b).

4. Perturbation theory

A particle of mass m and charge e oscillates in a one-dimensional harmonic potential with angular frequency ω. Show, *using perturbation theory*, that the effect of an applied electric field E is to lower all the energy levels by $e^2E^2/2m\omega^2$. Compare this with the classical result. Any surprise? Discuss!

5. Identical particles

Two non-interacting particles, with the same mass m are confined to a one-dimensional "box" with impenetrable walls, inside which they can move freely. The box has length a.

(a) What are the values of the four lowest energies of the system?

(b) What are the degeneracies of these energies if the two particles are

(i) identical, with spin 1/2,
(ii) distinguishable, but both have spin 1/2,
(iii) identical, with spin 1.