HARMONIC OSCILLATOR

CONT'D

HARMONIC OSCILLATOR :

THE $|n\rangle$ BASIS IS SUPERIOR TO THE $|x\rangle$ BASIS !

EXAMPLE

WHAT IS $\langle 3|X^3|2\rangle$?

$|x\rangle$ BASIS

$$\mathbb{1} = \int |x\rangle\langle x|\, dx$$

$$\langle 3|X^3|2\rangle = \int \langle 3|X^3|x\rangle\langle x|2\rangle\, dx$$

$$= \left(\frac{m\omega}{\pi\hbar}\right)^{1/2}\left(\frac{1}{2^3 3!}\,\frac{1}{2!\,2^2}\right)^{1/2}\int_{-\infty}^{\infty}\Big\{\exp\left(-\frac{m\omega x^2}{2\hbar}\right)$$

$$\times H_3\left[\left(\frac{m\omega}{\hbar}\right)^{1/2}x\right]x^3\exp\left(-\frac{m\omega x^2}{2\hbar}\right)H_2\left[\left(\frac{m\omega}{\hbar}\right)^{1/2}x\right]\Big\}dx$$

$|n\rangle$ BASIS

$$\langle 3|X^3|2\rangle = \left(\frac{\hbar}{2m\omega}\right)^{3/2}\langle 3|(a+a^\dagger)^3|2\rangle$$

$$= \left(\frac{\hbar}{2m\omega}\right)^{3/2}\langle 3|(a^3 + a^2a^\dagger + aa^\dagger a + aa^\dagger a^\dagger$$

$$+ a^\dagger aa + a^\dagger aa^\dagger + a^\dagger a^\dagger a + a^\dagger a^\dagger a^\dagger)|2\rangle$$

only non-zero contributions from $a^\dagger a^\dagger a$, $aa^\dagger a^\dagger$, $a^\dagger aa^\dagger$

$$= \left(\frac{\hbar}{2m\omega}\right)^{3/2}\left[2\sqrt{3} + 4\sqrt{3} + 3\sqrt{3}\right]$$

A more general version of **Postulate II** :

The independent variables x and p of classical mechanics become Hermitian operators X and P defined by the canonical commutator $[X, P] = i\hbar$. Dependent variables $\omega(x,p)$ are given by operators $\Omega = \omega(x \to X, p \to P)$.

This is a basis independent, general version of postulate II (still adapted to a single particle in 1D).

PATH INTEGRALS

Lagrangian formalism		Hamiltonian formalism
The state of a system with n degrees of freedom is described by n coordinates $(q_1, \ldots, q_n)$ and n velocities $(\dot{q}_1, \ldots, \dot{q}_n)$, or in a more compact notation by $(q, \dot{q})$.	(1)	The state of a system with n degrees of freedom is described by n coordinates and n momenta $(q_1, \ldots, q_n; p_1, \ldots, p_n)$ or, more succinctly, by (q, p).
The state of the system may be represented by a point moving with a definite velocity in an *n-dimensional configuration space*.	(2)	The state of the system may be represented by a point in a *2n-dimensional phase space*, with coordinates $(q_1, \ldots, q_n; p_1, \ldots, p_n)$.
The n coordinates evolve according to n second-order equations.	(3)	The $2n$ coordinates and momenta obey $2n$ first-order equations.
For a given $\mathscr{L}$, several trajectories may pass through a given point in configuration space depending on $\dot{q}$.	(4)	For a given $\mathscr{H}$ only one trajectory passes through a given point in phase space.

In 1933 Dirac published a paper in *Physikalische Zeitschrift der Sowjetunion* on "The Lagrangian in Quantum Mechanics." He begins by saying:

"Quantum mechanics was built up on a foundation of analogy with the Hamiltonian theory of classical mechanics. This is because the classical notion of canonical coordinates and momenta was found to be one with a very simple quantum analogue....

Now there is an alternative formulation for classical dynamics, provided by the Lagrangian. This requires one to work in terms of coordinates and velocities instead of coordinates and momenta. The two formulations are, of course, closely related, but there are reasons for believing that the Lagrangian one is the more fundamental.

In the first place the Lagrangian method allows one to collect together all the equations of motion and express them as the stationary property of a certain action function. (This action function is just the time integral of the Lagrangian.) There is no corresponding action principle in terms of the coordinates and momenta of the Hamiltonian theory. [This is not true, but it doesn't matter.] Secondly the Lagrangian method can easily be expressed relativistically, on account of the action function being a relativistic invariant; while the Hamiltonian method is essentially nonrelativistic in form, since it marks out a particular time variable....

For these reasons it would seem desirable to take up the question of what corresponds in the quantum theory to the Lagrangian method of the classical theory."

IN 1933 PAUL DIRAC WROTE A PAPER ...

... EXPANDING ON IDEAS HE HAD ALREADY DISCUSSED IN HIS BOOK ON QUANTUM MECHANICS

THE BOOK ON Q.M.!

The Principles of Quantum Mechanics

FOURTH EDITION

P. A. M. DIRAC

THIS GOT FEYNMAN STARTED AS HE READ DIRAC'S BOOK (AS A STUDENT AT PRINCETON IN THE 1940s)

Path integral formulation of q. m.

Dirac 1928, **Feynman 1949**

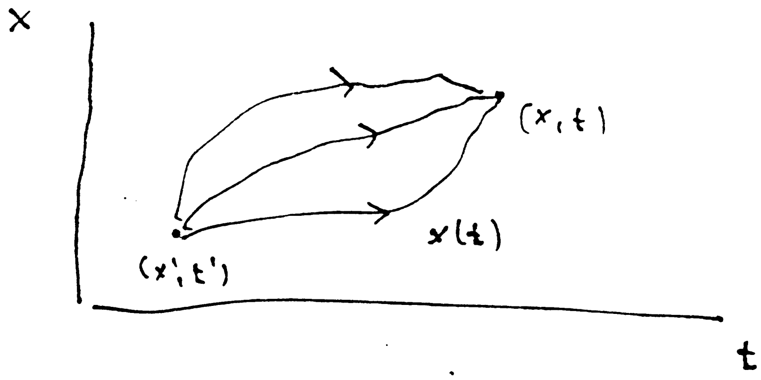
Central object in quantum mechanics = the propagator $U(t)$

$$U(t)\,|\psi(0)\rangle = |\psi(t)\rangle$$

Feynman:

For a single particle in 1D, to find $U(x,t;x',t') = \langle x|U(t-t')|x'\rangle$ do the following:

(i) Draw all paths in the x-t plane connecting (x',t') and (x,t)



(ii) Find the action $S[x(t)]$ for each path $x(t)$

Then,

(iii) $U(x,t;x',t') = \underset{\substack{\uparrow \\ \text{normalization factor}}}{A} \underbrace{\sum}_{\text{ALL PATHS}} e^{iS[x(t)]/\hbar}$

DIRAC FEYNMAN

... SOME YEARS LATER