Quantum Mechanics

First lecture

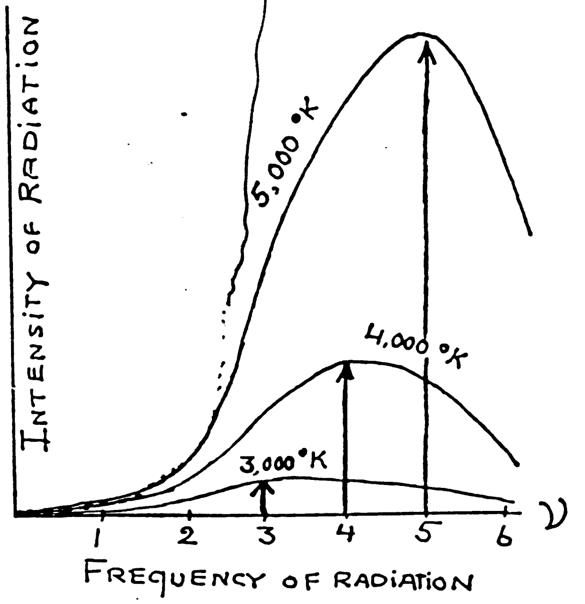
Sept. 2 2003

QUANTUM MECHANICS

How it all started...

UV catastrophe

Max Planck
German Physical Society, Dec 14, 1900



Planck's hypothesis

"Any physical entity whose 'coordinate' is a sinusoidal function of time (i.e. undergoes simple harmonic oscillations) and which is capable of emitting and absorbing electromagnetic radiation can possess only total energies ε_n which satisfy the relation

$$\varepsilon_n = nh\nu \qquad n = 0, 1, 2, \ldots$$

where ν is the frequency of the oscillations, and h is a universal constant..."

⇓

agreement with experiments

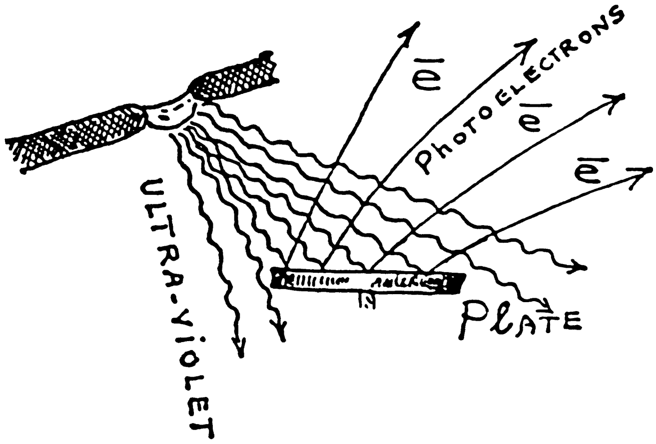
*"My hypothesis was an act of desperation...
I knew that the problem of (of the equilibrium of matter and radiation) is of fundamental significance for physics; I knew the formula that reproduces the energy distribution in the normal spectrum; a theoretical interpretation had to be found at any cost, no matter how high."*

Max Planck

Next step...

Albert Einstein, 1905 (Annus Mirabilis)

Lenard's experiment (1902)



K_{max} independent of the intensity of the UV light

Photo electrons ejected only for $\nu > \nu_0$

Einstein's hypothesis

"...electromagnetic radiation propagates in small bundles (photons) with energy $\varepsilon = h\nu$, where ν is the frequency. A photon is completely absorbed by an electron in the metal plate..."

$\Downarrow$

explains the experimental results

"There is hardly one among the great problems - in which modern physics is so rich - to which Einstein has not made an important contribution. That he may have sometimes missed the target in his speculations, as for example in his hypothesis of 'photons', cannot really be held too much against him, for it is not possible to introduce fundamentally new ideas, even in the exact sciences, without occasionally taking a risk."

statement of the Prussian Academy of Sciences, 1907

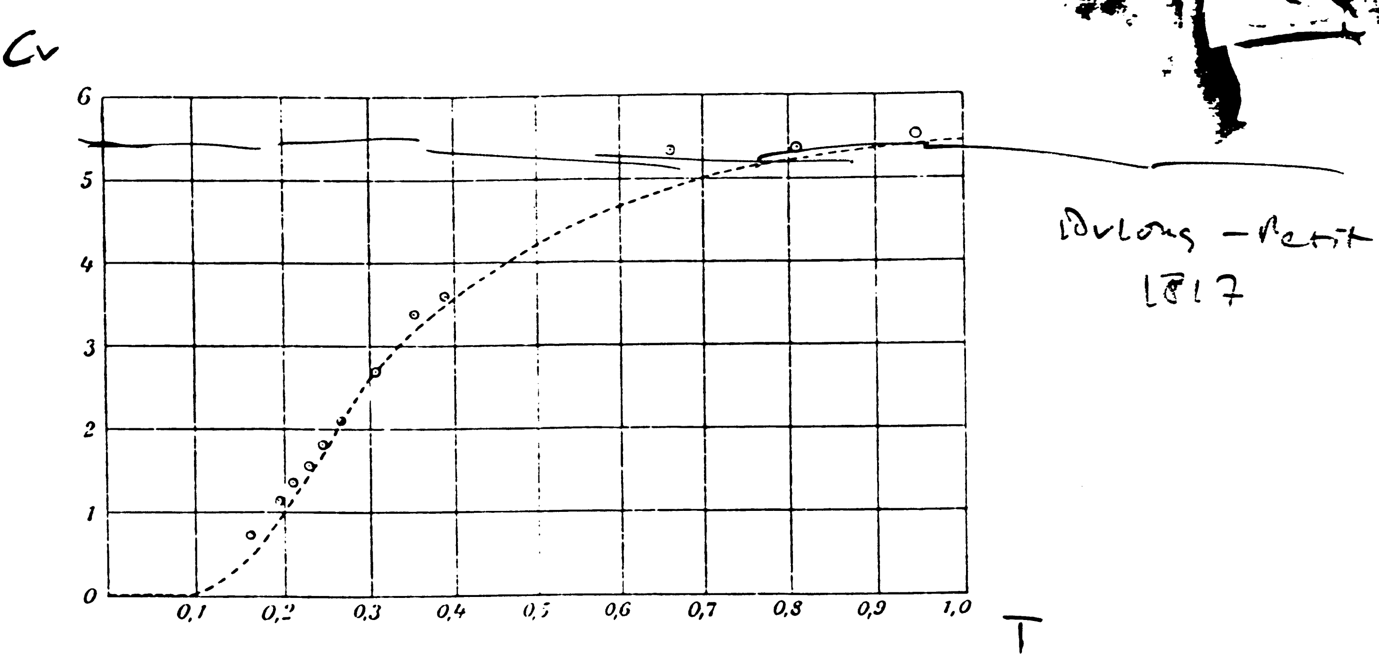
"50 years of hard thinking... and I am still not any closer to the question 'What really is a photon?' "

Einstein 1951

A breakthrough...

Einstein 1907

Quantum theory of the solid state...



Einstein's idea

" ***...Any harmonic oscillations, like atomic vibrations in a solid , are quantized with energies*** $\varepsilon_n = nh\nu$, ***n = 0, 1, 2, ..*** "

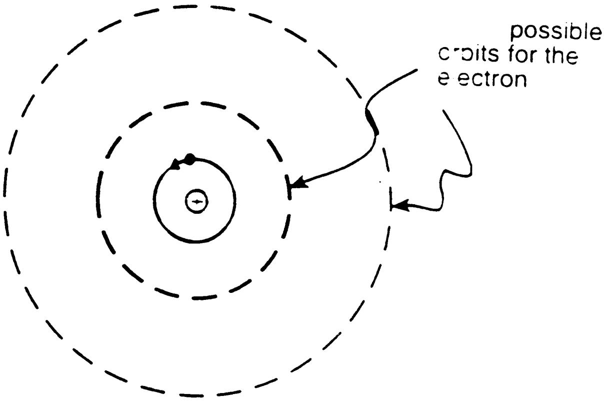
⇓

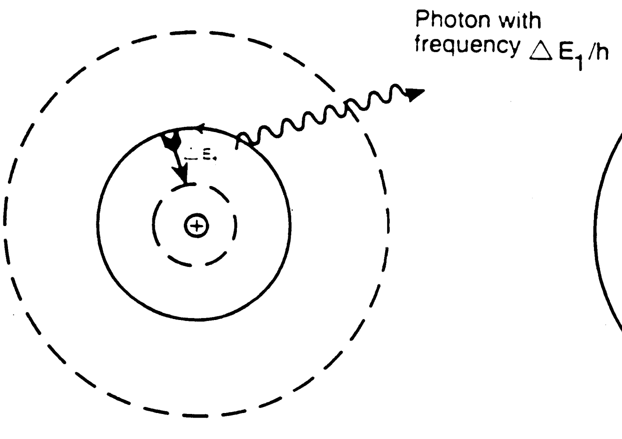
agreement with experiments

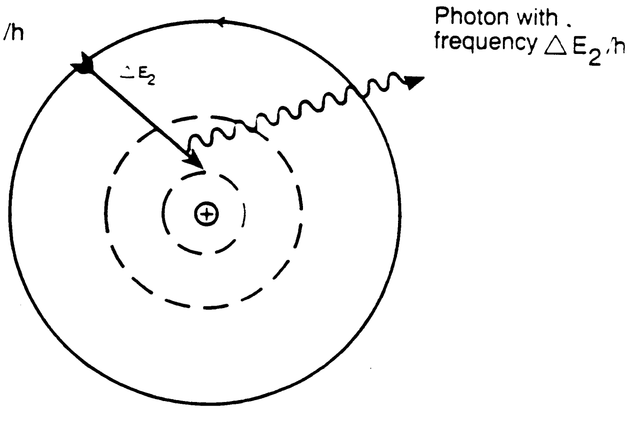
Important extension (beyond electromagnetism) -----> people started to listen!

One of them...

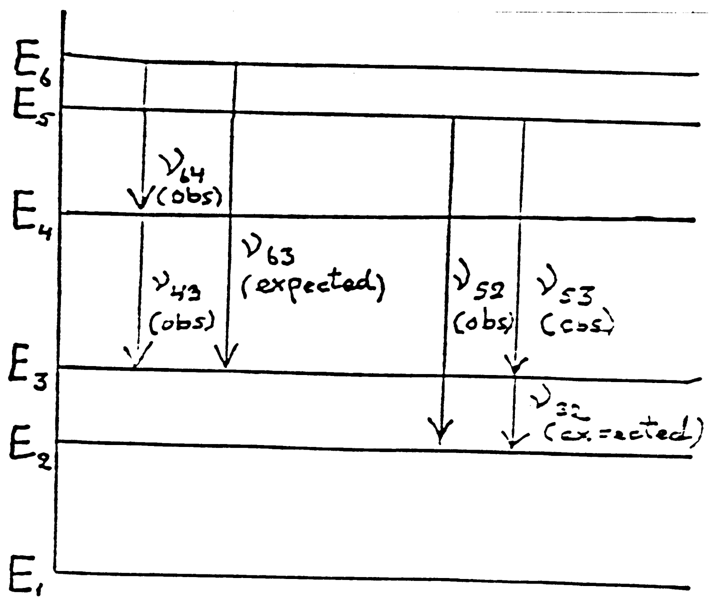
Niels Bohr, 1913







$$(1/\lambda) = R\left(1/n_1^2 - 1/n_2^2\right)$$

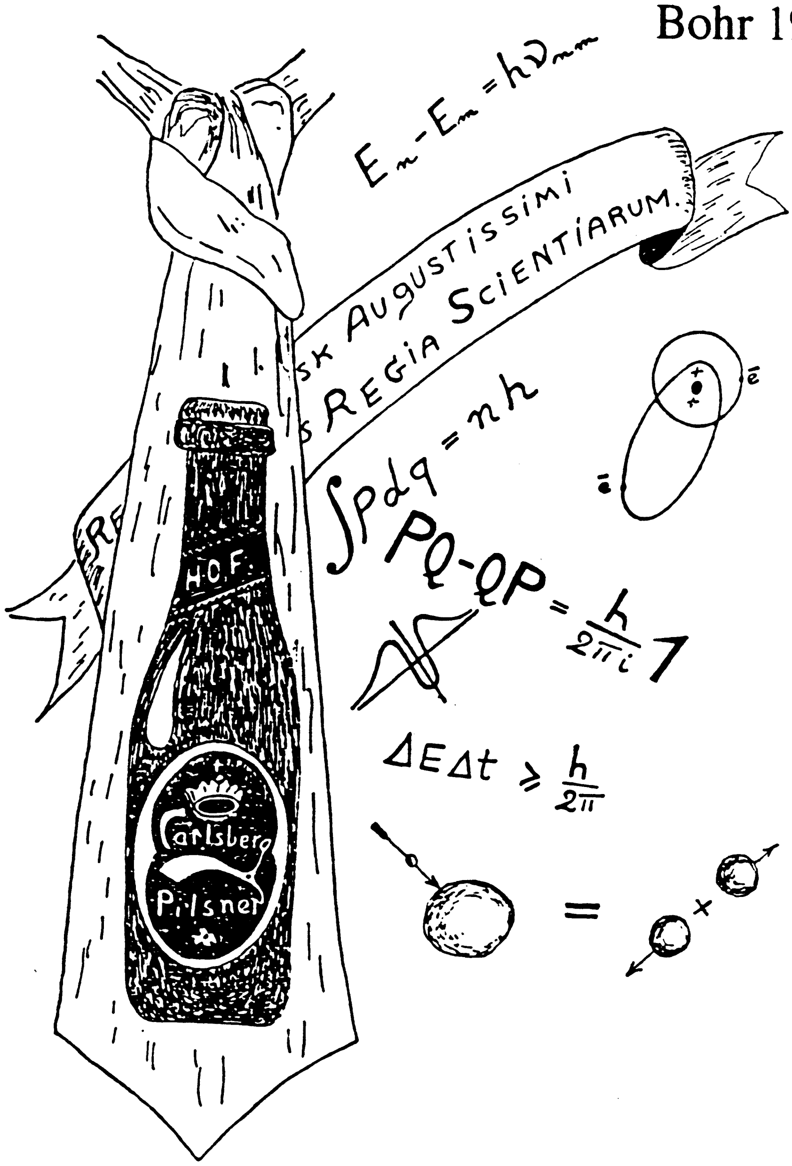


"The decade that followed Bohr's 1913 paper was rather peculiar... physics turned into art"

Isidor Rabi, 1941

"There is no great truth where the opposite is not also a great truth..."

Bohr 1923



Carlsberg Beer and its consequences.

One of the Carlsberg fellows, *Werner Heisenberg,* left for a vacation in 1925,...

On the island of Helgoland, June 6-7 1925, Heisenberg invented " **matrix mechanics**":

"It was about three o'clock at night when the final result of my calculation lay before me... At first I was deeply shaken... I was so excited I couldn't sleep. So I left the house... and awaited the sunrise on top of a rock."

"The present paper seeks to establish a basis for theoretical quantum mechanics founded exclusively upon relationships between quantities which are in principle observable."

From the abstract of Heisenberg paper in *Zeitschrift für Physik*, 1926

About the same time *Erwin Schrödinger* formulated an alternative formalism, "**wave mechanics**", which was soon shown to be equivalent to Heisenberg's theory.

With the contributions of *Louis de Broglie, Max Born, Wolfgang Pauli, Paul Dirac* and a few others

quantum mechanics

was born. ~ 1928

TOP *W. Heisenberg playing it cool. (Photographer unknown)*

The most succesful scientific theory ever!

Ex. The g factor of the electron

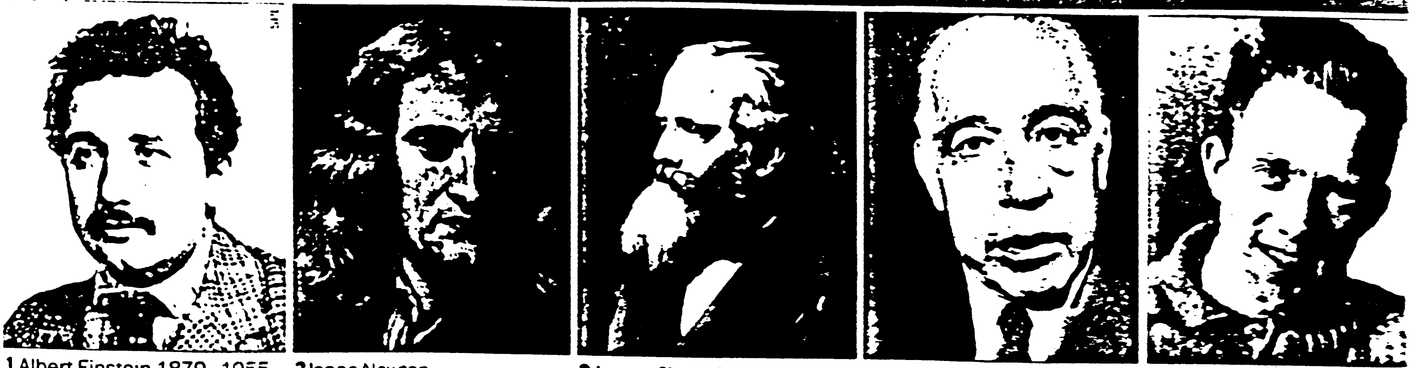
Experimental:	2.0023193048(8)
Calculated by QED:	2.0023193048(4)

Quantum mechanics has spread into biology, micro technology, cosmology, drug design, materials science....

The consequences are felt everywhere, from solid-state electronics to fiber optics, from basic chemistry and molecular biology to the pharmaceutical industry, from the understanding of superconductivity to the behavior of black holes and the fundamental forces between elementary particles, from the design of advanced medical diagnostics and treatments, to computers and the IT "revolution", to lasers, nanoscale engineering and nuclear technology.... to future quantum technologies in computing, cryptography, and artificial matter and life....

and yet...

The top physicists of all time



1 Albert Einstein 1879–1955 German/Swiss/American 119 votes

2 Isaac Newton 1642–1727 British 96 votes

3 James Clerk Maxwell 1831–1879 British 67 votes

4 Niels Bohr 1885–1962 Danish 47 votes

5 Werner Heisenberg 1901–1976 German 30 votes

6 Galileo Galilei 1564–1642 Italian 27 votes

7 Richard Feynman 1918–1988 American 23 votes

8= Paul Dirac 1902–1984 British 22 votes

8= Erwin Schrödinger 1887–1961 Austrian 22 votes

10 Ernest Rutherford 1871–1937 New Zealander 20 votes

"The more succesful the quantum theory is, the sillier it looks."

Einstein

"Anyone who thinks that he can understand quantum mechanics without getting into a state of confusion has not understood anything of the theory."

Bohr

"No one has ever understood quantum mechanics."

Feynman

"I regret that I had anything to do with it..."

Schrödinger

"What it all means? It is simple. You take what you need. you do what you have to do, and that's it..."

Heisenberg

QUANTUM MECHANICS
HAS CHANGED THE WAY
WE LOOK AT THE WORLD

What's wrong with classical physics?

Classical physics

PARTICLES = LOCALIZED BUNDLES OF ENERGY AND MOMENTUM

$$\left(\vec{q}(0), \vec{p}(0)\right) \xrightarrow{\vec{F} = m\vec{a}} \left(\vec{q}(t), \vec{p}(t)\right)$$

WAVES = DISTURBANCES SPREAD OVER SPACE TRANSPORTING ENERGY AND MOMENTUM

$$\left(\psi(\vec{r},0), \dot{\psi}(\vec{r},0)\right) \longrightarrow \left(\psi(\vec{r},t), \dot{\psi}(\vec{r},t)\right)$$

$$\nabla^2\psi = \frac{1}{c^2}\frac{\partial^2\psi}{\partial t^2}$$

PLANE WAVES IN D = 1 : $\psi(x,t) = A \exp\left(i\,(kx - \omega t)\right)$

$$k = \frac{2\pi}{\lambda} \qquad \omega = \frac{2\pi}{T}$$

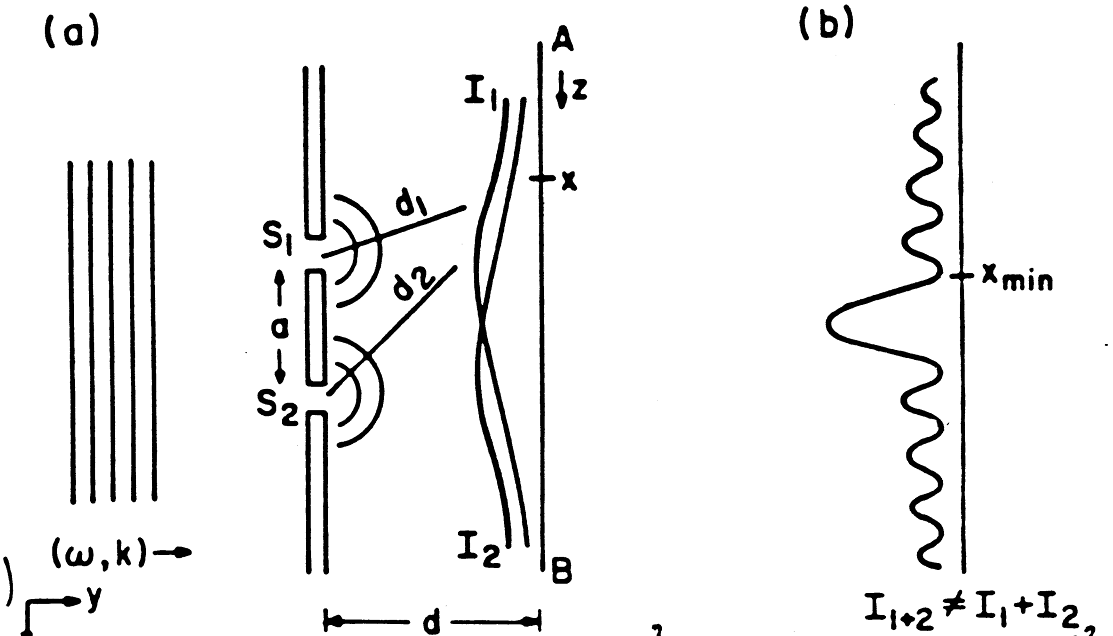
WAVES EXHIBIT INTERFERENCE ...

PARTICLES DON'T !

The double-slit experiment

("*the* problem of quantum mechanics", Feynman)

WAVES



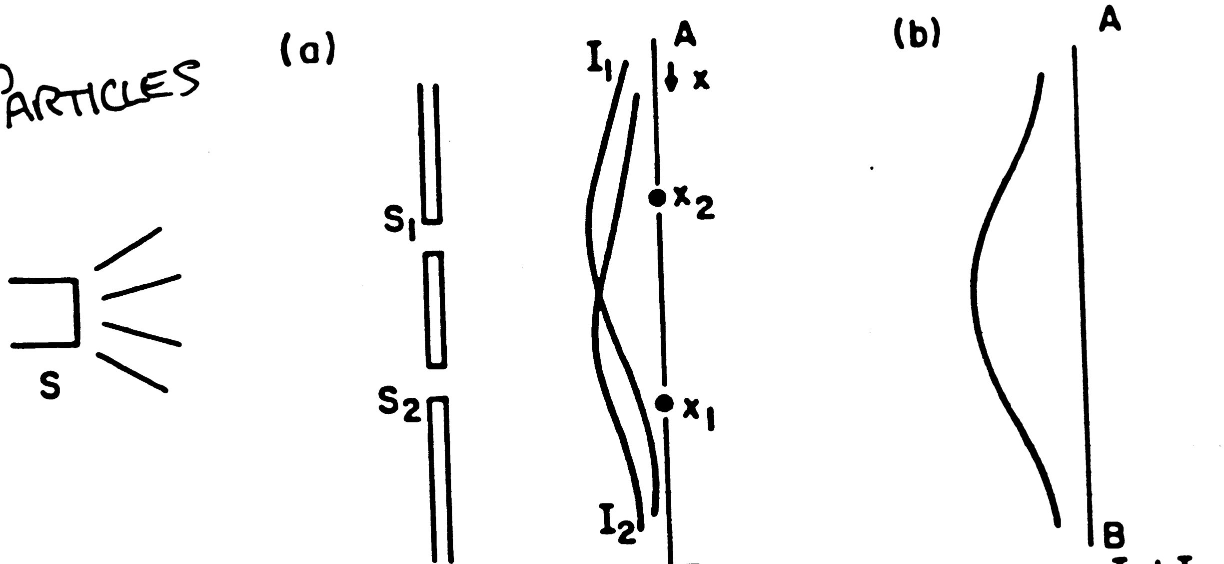
$\psi = \exp(i(ky - \omega t))$

$I_1 = |\psi_1|^2 \quad I_2 = |\psi_2|^2$

$I_{1+2} \neq I_1 + I_2$

$I_{1+2} = |\psi_1 + \psi_2|^2$

PARTICLES



$I_{1+2} = I_1 + I_2$

CLASSICAL PHYSICS

↓

DO THE EXPERIMENT WITH LIGHT = ELECTROMAGNETIC WA[VES]

POLARIZATION ALONG $\hat{z}$ → SCALAR WAVE FUNCTION OK

MATHEMATICAL FOUNDATION:

The theory of linear vector spaces

Def A linear vector space is a collection of objects $\mathbb{V}^n$

"kets" — $|1\rangle, |2\rangle, \ldots |V\rangle, |W\rangle, \ldots$ called ***vectors,***

for which there exists a rule for forming a vector sum, $|V\rangle + |W\rangle$, and for multiplication with scalars a, b, c, ..., denoted $a|V\rangle$

$= |aV\rangle$

satisfying

- closure $|v\rangle + |w\rangle \in \mathbb{V}^n$
- scalar multiplication is distributive in vectors and scalars $a(|v\rangle + |w\rangle)$ $(a+b)|v\rangle = a|v\rangle + b|v\rangle$
- scalar multiplication is associative $a(b|v\rangle) = ab|v\rangle$
- addition is commutative and associative $|v\rangle + (|w\rangle + |z\rangle) = (|v\rangle + |w\rangle) + |z\rangle$
- there exists a null vector $|0\rangle$, $|v\rangle + |0\rangle = |v\rangle$
- every vector has an inverse under addition $|v\rangle + |v\rangle^{-1} = |0\rangle$

Do what comes naturally!

$\mathbb{V}^n(R)$ REAL

Def. The numbers a, b, c,... are called the *field* over which the vector space is defined.

$\mathbb{V}^n(C)$ COMPLEX

Def. Linear independence of $|1\rangle, |2\rangle, \ldots |n\rangle$ if

$$\sum_{i=1}^{n} a_i |i\rangle = |0\rangle \Rightarrow \forall a_i = 0$$

Def. Dimension n = maximum number of linearly independent vectors

Examples: $V^3(R)$ --> arrows in 3D real space

$V^4(C)$ --> all 2x2 matrices with complex elements

- What about all real functions defined on $0 \leq x \leq L$?

Vectors of the form (a,b,1), with a,b complex numbers?

$|V\rangle$ DENOTES A GENERIC VECTOR.

<u>Theorem</u> ANY VECTOR $|V\rangle$ IN AN n-DIMENSIONAL SPACE CAN BE WRITTEN AS A LINEAR COMBINATION OF n LINEARLY INDEPENDENT VECTORS $|1\rangle, \ldots |n\rangle$

<u>Definition</u> Basis = set of n linearly independent vectors in an n-dimensional space

<u>Definition</u> $\sum_{i=1}^{n} v_i |i\rangle = |V\rangle$

v_i ↑ COMPONENTS (BASIS DEPENDENT)

<u>Theorem</u> The expansion $|V\rangle = \sum_{i=1}^{n} v_i |i\rangle$ is unique ($|i\rangle$ ↙ BASIS VECTORS)

Proof: $|V\rangle = \sum_i v_i |i\rangle = \sum_i v_i' |i\rangle \Rightarrow \sum_i (v_i - v_i')|i\rangle = |0\rangle$

$\Rightarrow v_i = v_i'$ SINCE $\{|i\rangle\}$ A BASIS

To add two vectors, add their components!

To multiply a vector by a scalar, multiply all its components by the scalar!

"bra" "ket" $|v\rangle$

<u>Definition</u> An *inner product* $\langle v|w\rangle$ between two vectors $|v\rangle$ and $|w\rangle$ is a number (generally complex) satisfying

- skew-symmetry $\langle v|w\rangle = \langle w|v\rangle^*$
- positive semidefiniteness $\langle v|v\rangle \geq 0$
- linearity in the ket $\langle v|aw+bz\rangle = \langle v|aw\rangle + \langle v|bz\rangle = a\langle v|w\rangle + b\langle v|z\rangle$

⇓

- antilinearity in the first factor

<u>Definition</u> $|v\rangle, |w\rangle$ *orthogonal* vectors if $\langle v|w\rangle = 0$

<u>Definition</u> $\sqrt{\langle v|v\rangle} = |v|$ = *norm* of the vector

<u>Definition</u> Pairwise orthogonal basis vectors of unit norm = *orthonormal (ON)* basis

→ INNER PRODUCT IN TERMS OF COMPONENTS

<u>Theorem</u> Any linearly independent basis can be transformed to an ON basis *(Gram-Schmidt)*