# Review questions II

1. What is the condition for propagation of electromagnetic waves in conductors. Give estimate for the plasma frequency for typical metals.

2. What is the skin effect? Derive formula for the skin depth in low- and high-frequency limits.

3. What is the plasma oscillation?

4. Explain a dielectric function approach to the problem of screening.

5. Discuss static screening of electron-electron Coulomb interaction. Derive formula for the electron dielectric function in Thomas-Fermi approximation. What is the form of the screened Coulomb potential? What is the screening radius?

6. Calculate spatial distribution of the screened Coulomb potential $\varphi(\mathbf{k}) = 4\pi e^2/(k^2+k_0^2)$.

7. Discuss dynamics of bare ions. Derive formula for the dielectric function of bare ions and discuss the plasma oscillations. What is the ion plasma frequency? Compare it with the electron plasma frequency.

8. Discuss dynamics of ions and electrons within the Jellium model. What is the dispersion law of the collective mode? What is this mode, why is it different from the plasma oscillation? Derive formula for the sound velocity.

9. Discuss total dielectric function of the electrons and ions. Focus on the sign of the screened Coulomb potential. explain the mechanism of electron-electron attraction.

10. Consider electron in 1D potential $U(x) = 2U[1 - cos(2\pi x/a)]$ and calculate the dispersion relation E(k) for first energy band using tight-binding approximation. Assume that the potential U is large and use parabolic form $U(x) = m(wx)^2/2$ in the vicinity of the potential minimum. Consider the lowest level of the oscillator that broadens into a band).

11. Discuss the lattice dynamics. What is the harmonic approximation and when is it valid? What is meant by the adiabatic approximation?

12. Explain the concepts of normal modes and dispersion curves. What are acoustic and optical branches? Explain their origin and their characteristic dispersion curves.

13. What is the dynamical matrix?

14. How many phonon states are in the 1BZ? How is the number of phonon branches related to dimensionality and the basis of a crystal?

15. Derive the normal modes and the dispersion curve for a mono-atomic 1D Bravais lattice with nearest neighbour interactions in the harmonic approximation. Same for the lattice with basis.

16. Explain concept of phonons and how they are related to normal modes. Derive distribution function for phonons.

17. Derive Drude-Petit formula for the specific heat of a lattice. When and why does this formula disagree with the experiment?

18. Derive formula for the energy of the phonon gas. Use this formula for derivation of the phonon specific heat at low temperature.

19. What is the Debye approximation and in what limit does it become exact? Define Debye temperature, wave vector and frequency.

20. Derive formula for the phonon density of states. How is this formula approximated in Debye's theory?

21. Derive temperature dependence for a total number of phonons (use Debye approximation).

22. Discuss the Einstein model for the specific heat. In which temperature range it is useful?

23. Consider 1D Bravais lattice with basis with nearest neighbour interaction; the lattice constant is $a$ and the interaction constants are $K$ and $G$. Show that in the limit of equal interaction constants $K = G$ the dispersion relation for the lattice vibrations reduces to that for a monoatomic chain with lattice constant $a/2$. (Hint: you have to check the behavior of the amplitude ratio of the acoustic and optical branches.)

24. Consider the phonon specific heat at low temperature. Show that
(i) the low-frequency density of normal modes in $d$-dimensional crystal varies as $\omega^{d-1}$,
(ii) the specific heat of $d$-dimensional crystal vanishes as $T^d$ at low temperature,
(iii) if the phonon dispersion equation changes to $\omega \sim k^\nu$, the specific heat vanishes as $T^{d/\nu}$.

25. Discuss the role of phonons in the heat transport. What is the thermal conductivity of perfect crystal? Why?

26. Write down Boltzmann kinetic equation for phonons in a relaxation time approximation. Caclulate thermal conductivity. Discuss temperature dependence of thermal conductivity.

27. Discuss mechanisms of thermal resistance.

28. Discuss phonon-phonon scattering. What is the physical mechanism behind it? What is the Umklapp process?

29. How do lattice vibrations affect the electrons? Derive correction to the Hamiltonian of the Bloch electron due to lattice vibrations.

30. Discuss a matrix element of electron-phonon interaction, focus on the conservation of crystal momentum and dependence on number of phonons. What is the Umklapp process?

31. Is the electron-phonon relaxation time temperature dependent? Present arguments and estimate temperature dependence of the electron-phonon relaxation time at high temperature, at low temperature. What is the criterion for distinguishing these regions?

32. Why does resistance of metals depend on temperature? Which are the dominating scattering mechanisms in real crystals at low and high temperatures, respectively?

33. What is Fermi liquid? Explain the concept of quasiparticle, what is the role of the Fermi statistics? Estimate a quasiparticle lifetime near the Fermi surface.

34. Discuss Bose condensation, give examples.

35. Calculate critical temperature of the Bose condensation. Find number of particles in the Bose condensate as the function of temperature.

36. Explain mechanism of the Cooper instability. What is the relation between binding of electrons by weak attraction and the Fermi statistics? Derive formula for the binding energy of the Cooper pair.

37. Calculate average radius $\rho$ of a Cooper pair. Answer: $\rho = const(\hbar v_F/|E|)$. Hint: calculate average square radius,

$$\rho^2 = \frac{<\psi|\rho^2|\psi>}{<\psi|\psi>},$$

using wave function $\psi(\mathbf{p}) = const/(2E_{\mathbf{p}} + |E|)$; use momentum representation, $\rho \to i\hbar(\partial/\partial\mathbf{p})$.)

38. How to prove in practice that resistance of a superconductor is zero?

39. What is a critical temperature of a superconductor? Critical magnetic field? Why superconductivity is destroyed by magnetic field?

40. What is the Meissner effect? Derive the London equation for the supercurrent. Derive equation for the penetration depth of the magnetic filed.

41. Derive quantization of the magnetic flux. What is the magnitude of the flux quantum?

42. Explain physical meaning of the module and the phase of the superconducting order parameter. What is a current state of a superconductor?

43. What is the origin of magnetic moment in an atom? Explain the concept of the $g$-factor and the Bohr magneton. List the three Hund's rules.

44. What is the main mechanism for magnetic interaction in a solid? Explain the concept of exchange interaction. What is meant by direct, super-, indirect, and itinerant exchange?

45. What is the Heisenberg model for magnetic interaction?

46. What are the characteristics of ferromagnetic, antiferromagnetic, and ferrimagnetic ordering?

47. Explain the concept of spin-waves. What is meant by magnons? Derive magnon spectrum. What is the low-temperature dependence of the ferromagnetic magnetization?

48. Calculate the carrier density in intrinsic semiconductors. Find position of the chemical potential.

49. Discuss the effect of impurities on the carrier concentration. What is the difference between donors and acceptors? What is the difference between n-type and p-type semiconductors? How does the position of the chemical potential change in the presence of impurities?