

## Program midterm 2004

1

Equilibrium ensembles. N-particle distribution function, reduced distribution functions. Statistical independence. Self-averaging. Liouville equation; and coarse graining. Microcanonical ensemble. Gibbs entropy; maximum entropy principle. Canonical distribution. Partition function and free energy. Ideal gas; Maxwell and Boltzmann distributions.

2

Exploration of vicinity of equilibrium. Local equilibrium; non-equilibrium entropy. Equilibrium fluctuations; Gaussian distribution. Correlation functions for several Gaussian variables. Non-equilibrium thermodynamics, thermodynamic flows and forces, kinetic coefficients, Onsager relations.

3

Stochastic variables, stochastic processes. Joint probabilities, conditional probabilities, moments, cumulants, characteristic function. Stationary process. Markov process. Smolukhovsky equation. Random walk. Binomial distribution. First and second moments.

4

Continuous random walk: Poissonian distribution. Shot noise. Central limit theorem Gaussian distribution. Random walk model for diffusion. Continuous Markov process: master equation. Application of master equation. Binary chemical reaction – exponential relaxation. Birth-death problem: master equation, stationary solution for constant rates, relaxation to stationary state.

5

Boltzmann equation for ideal gas with impurities. Derivation of collision term from master equation. Microscopic scattering properties: detailed balance and conservation laws. Validity: scattering time /length  $\ll$  evolution (relaxation) time /length. Derivation of convection term (Liouville equation). Stationary state (isotropic distribution). Relaxation process (= isotropization), relaxation time approximation. H-theorem. Weak non-equilibrium, linear response, transport coefficients.

6

Boltzmann equation for ideal gas with particle-particle scattering.

Collision term (master equation for two-particle distribution)  
Detailed balance and conservation laws.  
Statistical independence: from two- to one-particle distribution (coarse graining).  
H-theorem.  
Stationary state – Maxwell distribution.  
Relaxation, relaxation time approximation.  
Validity: scattering time  $\ll$  relaxation time.  
Local equilibrium; relaxation of small gradients: new stage of relaxation.  
Macroscopic conservation laws; hydrodynamic equations; formulas for flows.

7

Relaxation of small gradients in ideal gas with impurities: diffusion.  
Derivation of diffusion equation from Boltzmann equation.  
Binary (AB) ideal gas of light/heavy particles.  
Relaxation of B particles, hierarchy of relaxation rates: AA-rate  $\gg$  AB-rate  $\gg$  BB-rate:  
Brownian motion.  
Derivation of Fokker-Planck equation: elimination of A-distribution, approximation of small-angle scattering.

8

Langevin equation. White noise.  
Calculation of correlation functions and mean square deviations.  
Application to electrical circuits. Nyquist theory for current fluctuations  
Spectral analysis of noise, spectral function.  
Response function, fluctuation-dissipation theorem. Nyquist formula for current noise.  
Measuring spectral function, Wiener-Khinchine theorem.

9

Methods of non-equilibrium statistical physics: relation between stochastic equations, thermodynamics and kinetics.  
Derivation of Fokker-Planck equation from Langevin equation.  
Derivation of Langevin equation from microscopic particle – oscillator-bath model

## **Recitations and Home Problems**

1

Derivation of Liouville equation.  
Calculation of correlation functions for thermodynamic variables.

2

Rubber band problem (1D walk).

3

Transport coefficients and linear response.  
Current flow and calculation of ac conductivity.

Energy flow and derivation of thermal conductivity.  
Calculation of magnetoresistance and Hall coefficient

4

Tourist-mosquitoes problem.  
Generalized Langevin equation.  
Fluctuation of a rotor (Fokker-Planck equation).

5

Fluctuation of harmonic oscillator without damping (Langevin equation).  
Calculation of spectral function.